ON THE THIRD SMARANDACHE CONJECTURE ABOUT PRIMES

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Abstract. In this paper we basically verify the third Smarandache conjecture on prime.

Key words. Smarandache third conjecture, prime, gap.

For any positive integer \( n \), let \( P(n) \) be the \( n \)-th prime. Let \( k \) be a positive integer with \( k > 1 \), and let

(1) \[ c(n,k) = (P(n+1))^{1/k} - (P(n))^{1/k} \]

Smarandache [3] has been conjectured that

(2) \[ c(n,k) < \frac{2}{k} \]

In [2], Russo verified this conjecture for \( P(n) < 2^{25} \) and \( 2 \leq k \leq 10 \). In this paper we prove a general result as follows.

Theorem. If \( k > 2 \) and \( n > C \), where \( C \) is an effectively computable absolute constant, then the inequality (2) holds.

Proof. Since \( k > 2 \), we get from (1) that

(3) \[ C(n,k) = \frac{P(n+1) - P(n)}{(P(n+1))^{(k-1)/k} + (P(n+1))^{(k-2)/k}(P(n))^{1/k} + \ldots + (P(n))^{1/k}} \]

By the result of [1], we have

(4) \[ P(n+1) - P(n) < C(a)(P(n))^{1/2} + a \]

for any positive number \( a \), where \( C(a) \) is an effectively
computable constant depending on \( a \). Put \( a = 1/20 \). Since \( k \geq 3 \) and \( (k-1)/k \geq 2/3 \), we see from (3) and (4) that

\[
C(n,k) < \frac{2 C(1/20)}{k \left( 2(P(n))^{1/15} \right)}.
\]

Since \( C(1/20) \) is an effectively computable absolute constant, if \( n > C \), then \( 2(P(n))^{1/15} > C(1/20) \). Thus, by (5), the inequality (2) holds. The theorem is proved.

References


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