

ON SMARANDACHE ALGEBRAIC STRUCTURES. II:THE SMARANDACHE SEMIGROUP

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Abstract . In this paper we prove that $A(a,n)$ is a Smarandache semigroup.

Key words . Smarandache algorithm , Smarandache semigroup .

Let G be a semigroup . If G contains a proper subset which is a group under the same operation , then G is called a Smarandache semigroup (see [2]) . For example , $G=\{18,18^2,18^3,18^4,18^5\} \pmod{60}$ is a commutative multiplicative semigroup . Since the subset $\{18^2,18^3,18^4,18^5\} \pmod{60}$ is a group , G is a Smarandache semigroup .

Let a,n be integers such that $a \neq 0$ and $n > 1$. Further , let $A(a,n)$ be defined as in [1] . In this paper we prove the following result .

Theorem . $A(a,n)$ is a Smarandache semigroup .

Proof . Under the definitions and notations of [1] , let $A'(a,n)=\{a^e, a^{e+1}, \dots, a^{e+f-1}\} \pmod{n}$. Then $A'(n,a)$ is a proper subset of $A(a,n)$.

If $e=0$, then $a^e = 1 \in A'(a,n)$. Clear , 1 is the unit of $A'(a,n)$. Moreover , for any $a^i \in A(a,n)$ with $i > 0$, a^i is the inverse element of a^i in $A'(a,n)$.

If $e > 0$, then $a^f \in A'(a,n)$. Since $a^f \equiv 1 \pmod{n}$, a^f is the unit of $A'(a,n)$ and a^{ft-i} is the inverse element of a^i in $A'(a,n)$, where t is the integer satisfying $e < ft-i \leq e+f-1$. Thus , under the Smarandache algorithm , $A'(a,n)$ is a group . It implies that $A(a,n)$ is a Smarandache

semigroup. The theorem is proved.

References

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