ON SMARANDACHE CONCATENATED SEQUENCES I: PRIME POWER SEQUENCES

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Abstract. Let \( A = \{p^n\}_{n=1}^\infty \), where \( p \) is a prime. Let \( C(A) = \{c_n\} \) denote the Smarandache concatenated sequence of \( A \). In this paper we prove that if \( n > 1 \) and \( p \neq 2 \) or 5, then \( c_n \) does not belong to \( A \).

Let \( A = \{a_n\}_{n=1}^\infty \) be an infinite increasing sequence of positive integers. For any positive integer \( n \), let \( a_n \) be the decimal integer such that

\[
1 \leq a_n \leq 9 \quad (1)
\]

Then sequence \( C(A) = \{c_n\}_{n=1}^\infty \) is called the Smarandache concatenated sequence of \( A \). In [1], Marimutha posed a general question as follows:

Question. How many terms of \( C(A) \) belong to \( A \)?

In this serial paper, we shall consider some interesting cases for the above question. In this part we prove the following result.

Theorem. Let \( A = \{p^n\}_{n=1}^\infty \), where \( p \) is a prime. If \( n > 1 \) and \( p \neq 2 \) or 5, then \( c_n \) does not belong to \( A \).

Proof. For any positive integer \( n \), let \( d(a) \) denote the figure number of \( a \) in the decimal system.

If \( A = \{p^n\}_{n=1}^\infty \), then from (1) we get

\[
2)c_n = p^n + p^{n-1} \times 10^{d(p^n)} + \ldots + p^1 \times 10^{d(p^2)} + p \times 10^{d(p^1)} + \ldots + p^{d(p)}
\]

Further, if \( c_n \) belongs to \( A \), then we have

\[
3)c_n = p^m,
\]

where \( m \) is a positive integer with \( m \geq n \). It implies that

\[
4)p^2 \mid c_n,
\]

if \( n > 1 \). However, if \( p \neq 2 \) or 5, then \( p \neq 10^k \) for any positive
integer $k$. Therefore, by (2), we get

\[(5) \quad p^2 \mid c_n,\]

which contradicts (4). Thus, $c_n$ does not belong to $A$ in this case. The theorem is proved.

Reference