ABSTRACT: In [1] we define SMARANDACHE FACTOR PARTITION FUNCTION (SFP), as follows:

Let \( \alpha_1, \alpha_2, \alpha_3, \ldots \alpha_r \) be a set of \( r \) natural numbers and \( p_1, p_2, p_3, \ldots p_r \) be arbitrarily chosen distinct primes then \( F(\alpha_1, \alpha_2, \alpha_3, \ldots \alpha_r) \) called the Smarandache Factor Partition of \( (\alpha_1, \alpha_2, \alpha_3, \ldots \alpha_r) \) is defined as the number of ways in which the number

\[
N = \frac{\alpha_1 \alpha_2 \alpha_3 \ldots \alpha_r}{p_1 p_2 p_3 \ldots p_r}
\]

could be expressed as the product of its' divisors. For simplicity, we denote \( F(\alpha_1, \alpha_2, \alpha_3, \ldots \alpha_r) = F'(N) \), where

\[
N = \frac{\alpha_1 \alpha_2 \alpha_3 \ldots \alpha_r \alpha_n}{p_1 p_2 p_3 \ldots p_r \ldots p_n}
\]

and \( p_r \) is the \( r \)th prime. \( p_1 = 2, p_2 = 3 \), etc.

In this note another result pertaining to SFPs has been derived.

DISCUSSION:

Let

\[
N = \frac{\alpha_1 \alpha_2 \alpha_3 \ldots \alpha_r}{p_1 p_2 p_3 \ldots p_r}
\]

(1) \( L(N) = \) length of that factor partition of \( N \) which contains the maximum number of terms. In this case we have
\[ L(N) = \sum_{i=1}^{r} \alpha_i \]

(2) \[ A_{L(N)} = \text{A set of } L(N) \text{ distinct primes.} \]

(3) \[ B(N) = \{ p: p \mid N \text{, } p \text{ is a prime.} \} \]

\[ B(N) = \{ p_1, p_2, \ldots, p_r \} \]

(4) \[ \Psi[N, A_{L(N)}] = \{ x \mid d(x) = N \text{ and } B(x) \subseteq A_{L(N)} \} \], where \( d(x) \) is the number of divisors of \( x \).

To derive an expression for the order of the set \( \Psi[N, A_{L(N)}] \) defined above.

There are \( F'(N) \) factor partitions of \( N \). Let \( F_1 \) be one of them.

\[ F_1 \rightarrow N = s_1 \times s_2 \times s_3 \times \ldots \times s_t. \]

if

\[ \theta = \begin{array}{ccccccc}
9 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 \\
p_1 & p_2 & p_3 & \ldots & p_t & p_{t+1} & p_{t+2} & \ldots & p_{L(N)}
\end{array} \]

where \( p_t \in A_{L(N)} \), then \( \theta \in \Psi[N, A_{L(N)}] \) for

\[ d(\theta) = s_1 \times s_2 \times s_3 \times \ldots \times s_t \times 1 \times 1 \times 1 \times \ldots = N \]

Thus each factor partition of \( N \) generates a few elements of \( \Psi \).

Let \( E(F_1) \) denote the number of elements generated by \( F_1 \).

\[ F_1 \rightarrow N = s_1 \times s_2 \times s_3 \times \ldots \times s_t. \]

multiplying the right member with unity as many times as required to make

the number of terms in the product equal to \( L(N) \).

\[ N = \prod_{k=1}^{L(N)} s_k \]

281
where \( s_{t+1} = s_{t+2} = s_{t+3} = \ldots = s_{L(N)} = 1 \)

Let \( x_1 \)'s are equal

\( x_2 \)'s are equal

\( \ldots \)

\( x_m \)'s are equal

such that \( x_1 + x_2 + x_3 + \ldots + x_m = L(N) \). Where any \( x_i \) can be unity also.

Then we get

\[
E(F_1) = \frac{\{L(N)\}!}{\{(x_1)!(x_2)!(x_3)! \ldots (x_m)!\}}
\]

summing over all the factor partitions we get

\[
O(\Psi[ N, A_{L(N)}]) = \sum_{k=1}^{F'(N)} E(F_k) \quad \text{(7.1)}
\]

Example:

\( N = 12 = 2^2 \cdot 3, L(N) = 3, F'(N) = 4 \)

Let \( A_{L(N)} = \{2, 3, 5\} \)

\( F_1 \rightarrow \) \( N = 12 = 12 \times 1 \times 1, \quad x_1 = 2, \quad x_2 = 1 \)

\( E(F_1) = \frac{3!}{\{(2!)(1!)\}} = 3 \)

\( F_2 \rightarrow \) \( N = 12 = 6 \times 2 \times 1, \quad x_1 = 1, \quad x_2 = 1, \quad x_3 = 1 \)

\( E(F_2) = \frac{3!}{\{(1!)(1!)(1!)\}} = 6 \)

\( F_3 \rightarrow \) \( N = 12 = 4 \times 3 \times 1, \quad x_1 = 1, \quad x_2 = 1, \quad x_3 = 1 \)

\( E(F_3) = \frac{3!}{\{(1!)(1!)(1!)\}} = 6 \)

\( F_4 \rightarrow \) \( N = 12 = 3 \times 2 \times 2, \quad x_1 = 1, \quad x_2 = 2 \)

\( E(F_4) = \frac{3!}{\{(2!)(1!)\}} = 3 \)
\[ O(\Psi[ N, A_{L(N)}]) = \sum_{k=1}^{F'(N)} E(F_k) = 3 + 6 + 6 + 3 = 18 \]

To verify we have
\[ \Psi[ N, A_{L(N)}] = \{ 2^{11}, 3^{11}, 5^{11}, 2^5 \times 3, 2^5 \times 3, 3^5 \times 2, 3^5 \times 5, 5^5 \times 2, \\
5^5 \times 3, 2^3 \times 3^2, 2^3 \times 5^2, 3^3 \times 2^2, 3^3 \times 5^2, 5^3 \times 2^2, 5^3 \times 3^2, 2^2 \times 3 \times 5, \\
3^2 \times 2 \times 5, 5^2 \times 2 \times 3 \} \]

REFERENCES:


[2] "The Florentine Smarandache " Special Collection, Archives of American Mathematics, Centre for American History, University of Texas at Austin, USA.