NOTES ON PRIMES SMARANDACHE PROGRESSIONS

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Abstract. In this note we discuss the primes in Smarandache progressions.

For any positive integer n, let \( p_n \) denote the \( n^{th} \) prime.

For the fixed coprime positive integers \( a, b \), let \( P(a,b) = \{ a p_n + b \}_{n=1}^{\infty} \).

Then \( P(a,b) \) is called a Smarandache progression.

In [1, Problem 17], Smarandache posed the following questions:

Questions. How many primes belong to \( P(a,b) \)?

It would be seen that the answers of Smarandache's question is different from pairs \( (a,b) \). We now give some observable examples as follows:

Example 1. If \( a, b \) are odd integers, then \( a p_n + b \) is an even integer for \( n > 1 \). It implies that \( P(a,b) \) contains at most one prime. In particular, \( P(1,1) \) contains only the prime 3.

Example 2. Under the assumption of twin prime conjecture that there exist infinitely many primes \( p \) such that \( p + 2 \) is also a prime, then the progression \( P(1,2) \) contains infinitely
many primes.

Example 3. Under the assumption of Germain prime conjecture that there exist infinitely many primes \( p \) such that \( 2p+1 \) is also a prime, then the progression \( P(2,1) \) contains infinitely many primes.

Reference