OPEN PROBLEMS AND CONJECTURES ON THE
FACTOR /RECIPROCAL PARTITION THEORY:

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(1.1) To derive a formula for SFPs of given length $m$ of $p^aq^a$ for any value of $a$.

(1.2) To derive a formula for SFPs of

$$N = p_1^{2} p_2^{2} p_3^{2} \ldots p_r^{2}$$

(1.3) To derive a formula for SFPs of given length $m$ of

$$N = p_1^{a} p_2^{a} p_3^{a} \ldots p_r^{a}$$

(1.4) To derive a reduction formula for $p^aq^a$ as a linear combination of $p^{a-r}q^{a-r}$ for $r = 0$ to $a-1$.

Similar reduction formulae for (1.2) and (1.3) also.

(1.5) In general, in how many ways a number can be expressed as the product of its divisors?

(1.6) Every positive integer can be expressed as the sum of the reciprocal of a finite number of distinct natural numbers. (in infinitely many ways.)

Let us define a function $R_m(n)$ as the minimum number of natural numbers required for such an expression.

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(1.7). Every natural number can be expressed as the sum of the reciprocals of a set of natural numbers which are in Arithmetic Progression.

(1.8). Let 
\[ \sum \frac{1}{r} \leq n \leq \sum \frac{1}{(r+1)} \]
where \( \sum 1/r \) stands for the sum of the reciprocals of first \( r \) natural numbers and let \( S_1 = \sum 1/r \)
let \( S_2 = S_1 + 1/(r+k_1) \) such that \( S_2 + 1/(r+k_1+1) > n \geq S_2 \)
let \( S_3 = S_2 + 1/(r+k_2) \) such that \( S_3 + 1/(r+k_2+1) > n \geq S_3 \)
and so on, then there exists a finite \( m \) such that 
\[ S_{m+1} + 1/(r+k_m) = n \]
Remarks: The veracity of conjecture (1.6) is deducible from conjecture (1.8).

(1.9). (a) There are infinitely many disjoint sets of natural numbers sum of whose reciprocals is unity.
(b) Among the sets mentioned in (a), there are sets which can be organised in an order such that the largest element of any set is smaller than the smallest element of the next set.

DEFINITION: We can define Smarandache Factor Partition Sequence as follows: \( T_n = \text{factor partition of } n = F'(n) \)
\( T_1 = 1 \), \( T_2 = 3 \), \( T_{12} = 4 \) etc.

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SFPS is given by
\[ 1,1,1,2,1,2,1,3,2,2,1,4,1,2,2,5, 1,4,1,4,2,2,1,7,2, \ldots, \]

**DEFINITION:** Let \( S \) be the smallest number such that \( F'(S) = n \).

We define \( S \) a **Vedam Number** and the sequence formed by Vedam numbers as the **Smarandache Vedam Sequence**.

Smarandache Vedam Sequence is given as follows:
\[ T_n = F'(S) \]
\[ 1, 4, 8, 12, 16, \ldots, 24, \ldots \]

**Note:** There exist no number whose factor partition is equal to 6, hence a question mark at the sixth slot. We define such numbers as **Dull numbers**. The readers can explore the distribution (frequency) and other properties of dull numbers.

**DEFINITION:** A number \( n \) is said to be a **Balu number** if it satisfies the relation \( d(n) = F'(n) = r \), and is the smallest such number.

1, 16, 36 are all Balu numbers.

\[ d(1) = F'(1) = 1 \quad d(16) = F'(16) = 5, \quad d(36) = F'(36) = 9. \]

Each Balu number \( \geq 16 \) generates a **Balu Class** \( C_B(n) \) of numbers having the same canonical form satisfying the equation
\[ d(m) = F'(m). \text{e.g.} \quad C_B(16) = \{ x \mid x = p^4, \text{p is a prime.} \} = \{ 16, 81, 256, \ldots \}. \]

Similarly \( C_B(36) = \{ x \mid x = p^2q^2, \text{p and q are primes.} \} \)
Conjecture

(1.10): There are only finite number of Balu Classes.

In case Conjecture (1.10) is true, to find out the largest Balu number.

REFERENCES


[8] "The Florentine Smarandache " Special Collection, Archives of American Mathematics, Centre for American History, University of Texax at Austin, USA.