Open Questions For The Smarandache Function

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Let \( S(n) \) be the Smarandache function. I propose the following open questions:

1) Solve the following equation in integers:
\[
1/ S^2(a) = 1/ S^2(b) + 1/ S^2(c).
\]

2) Solve the following equation in integers:
\[
S^2(\phi(a)) = S^2(\phi(b)) + S^2(\phi(c)).
\]

3) Solve the following equation in integers:
\[
S(d(n) + \sigma(n)) = d(S(n)) + \sigma(S(n)).
\]

4) Solve the following equation in integers:
\[
S(a*d(n) + b*\sigma(n) + c*\phi(n) + d*\psi(n)) = a*d(S(n)) + b*\sigma(S(n)) + c*\phi(S(n)) + d*\psi(S(n)).
\]

5) Solve the following equation in integers:
\[
S(\sum_{k=1}^{n} n^k) = \prod_{k=1}^{n} S(k)^{\phi(n)}
\]

6) Solve the following equation in integers:
\[
\pm S(1) \pm S(2) \pm \ldots \pm S(n) = \phi((n(n+1))/2).
\]

7) Solve the following equation in integers:
\[
S(\pm 1^2 \pm 2^2 \pm \ldots \pm n^2) = \pm S^2(1) \pm S^2(2) \pm \ldots \pm S^2(n).
\]

8) Solve the following equation in integers:
9) Solve the following equation in integers:
\[ S(\pm d(1) \pm d(2) \pm \ldots \pm d(n)) = S(d(n(n+1))/2). \]

10) Solve the following equation in integers:
\[ S(\pm \sigma(1) \pm \sigma(2) \pm \ldots \pm \sigma(n)) = S(\sigma(n(n+1))/2). \]

11) Solve the following equation in integers:
\[
\frac{1}{S(1)} + \frac{1}{S(2)} + \ldots + \frac{1}{S(n)} = \frac{n}{S((n(n+1))/2)}.
\]

12) Solve the following equation in integers:
\[ S(1*2) + S(2*3) + \ldots + S(n(n+1)) = S((n(n+1)(n+2))/3). \]

13. Let \( \alpha_k(n) \) be the first \( k \) digits of \( n \) and \( \beta_p(n) \) the last \( p \) digits of \( n \). Determine all integer 5-tuples \( (n,m,r,k,p) \) for which:
\[ S^2(\alpha_k(n)) = S^2(\alpha_k(m)) + S^2(\alpha_k(r)) \]
and
\[ S^2(\beta_p(n)) = S^2(\beta_p(m)) + S^2(\beta_p(r)). \]

14) Determine all integer 5-tuples \( (n,m,r,k,p) \) for which:
\[ \alpha_k^2(S(n)) = \alpha_k^2(S(m)) + \alpha_k^2(S(r)) \]
and
\[ \beta_k^2(S(n)) = \beta_k^2(S(m)) + \beta_k^2(S(k)). \]

15) Determine all integer pairs \( (n,k) \) for which:
\[ \alpha_{k+2}^2(S(n)) = \alpha_{k+1}^2(S(n)) + \alpha_k^2(S(n)). \]
16) Determine all integer pairs \((n, p)\) for which:
\[
\beta_{p+1}^2(S(n)) = \beta_{p+1}^2(S(n)) + \beta_p^2(S(n)).
\]

17) Find all integer pairs \((n, k)\) such that
\[
S(\alpha_k(n)) + S(\alpha_k(n+2)) = 2 \cdot S(\alpha_k(n+1))
\]

18) Find all integer pairs \((n, p)\) such that
\[
S(\beta_p(n)) + S(\beta_p(n+2)) = 2 \cdot S(\beta_p(n+1))
\]

19) Let \(p_n\) be the \(n\)-th prime number. Determine all integer triples \((n, k, p)\) for which
\[
S(\alpha_k(p_n)) + S(\beta_p(p_n)) = 2 \cdot S(\alpha_{(k+p)/2}(p_n))
\]
and
\[
S(\alpha_k(p_n)) + S(\beta_p(p_n)) = 2 \cdot S(\beta_{(k+p)/2}(p_n)).
\]

20) Find all integer pairs \((a, b)\) such that
\[
\frac{a \cdot S(b) + b \cdot S(a)}{a + b} = S\left(\frac{a^2 + b^2}{a + b}\right)
\]

21) Solve the following in integers:
\[
\pm S(\sigma(1)) \pm S(\sigma(2)) \pm \ldots \pm S(\sigma(n)) = \pm \sigma(\pm S(1) \pm S(2) \pm \ldots \pm S(n)).
\]

22) Solve the following in integers:
\[
\alpha_k(n) = S(n) \quad \text{and} \quad \beta_p(n) = S(n).
\]

23) Solve the following in integers:
\[
\alpha_k(n!) = S(m) \quad \text{and} \quad \beta_p(n!) = S(m).
\]