ON THE SMARANDACHE FUNCTION AND THE FIXED-POINT THEORY OF NUMBERS

by

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This brief note points out several basic connections between the Smarandache function, fixed-point theory [1] and prime-number theory. First recall that fixed-point theory in function spaces provides elegant, if not short, proofs of the existence of solutions to many kinds of differential equations, integral equations, optimization problems and game-theoretic problems. Further, fixed-point theory in the ring of rational integers and fixed-lattice-point theory provide many results on the existence of solutions in diophantine theory. Here are four fundamental examples of fixed-point theory in number theory. (1) There is the well-known basic result that for p>4, p is prime iff S(p) = p. (2) Recall that the present author defined [2] the number-theoretic function Ψ(n) as the product of the primes alone in the mosaic of n, where the mosaic of n is obtained from n by recursively applying the unique factorization theorem/fundamental theorem of arithmetic to itself. Now the asymptotic density of fixed points of Ψ(n) is 7/π², just as the asymptotic density of square-free numbers is 6/π². Indeed, (3) the theory of perfect numbers is also connected to fixed-point theory, since if one puts f(n) = δ(n) - n, where δ(n) is the sum of the divisors on n, then n is perfect iff f(n) = n. Finally, (4) the present author defined [2] the number-theoretic function Ψ*(n) as the sum of the primes alone in the mosaic of n. Here we have a striking similarity to the Smarandache function itself (see example (1) above), since Ψ*(n) = n if n = 4 or n = p for some prime p; i.e., if p > 4, n is prime iff Ψ*(n) = n. Thus, the distribution function for the fixed points of S(n) or of Ψ*(n) is essentially the distribution function for the primes, Π(n).

Problems

(1) Put S'(n) = S(S(n)) and define S''(n) recursively, where S(n) is the Smarandache function. (Note: This approach aligns Smarandache function theory more closely with recursive function theory/computer theory.) For each n, determine the least m for which S''(n) is prime.
(2) Prove that S(n) = S(n+3) for only finitely many n.
(3) Prove that S(n) = S(n+2) for only finitely many n.
(4) Prove that S(n) = S(n+1) for no n.

References


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