In this paper we investigate some properties of Smarandache sequences of the 2nd kind and demonstrate that these numbers are near prime numbers. In particular, we establish that prime numbers and Smarandache numbers of the 2nd kind (a) may be computed from the similar analytical expressions, (b) may be used for constructing Magic squares 3x3 or Magic squares 9x9, consisted of 9 Magic squares 3x3.

Key words: prime numbers, Smarandache numbers of the 2nd kind, density of numerical sequences, Magic squares 3x3 and 9x9.

1 Introduction

We remind [2, 3], that in the general case Magic squares represent by themselves numerical or analytical square tables, whose elements satisfy a set of definite basic and additional relations. The basic relations therewith assign some constant property for the elements located in the rows, columns and two main diagonals of a square table, and additional relations, assign additional characteristics for some other sets of its elements.

Let it be required to construct Magic squares \( n \times n \) in size from a given set of numbers. Judging by the mentioned general definition of Magic squares, there is no difficulty in understanding that the foregoing problem consists of the four interrelated problems

1. Elaborate the practical methods for generating the given set of numbers;
2. Look for a concrete family of \( n^2 \) elements, which would satisfy both the basic and all the additional characteristics of the Magic squares;
3. Determine how many Magic squares can be constructed from the chosen family of \( n^2 \) elements;
4. Elaborate the practical methods for constructing these Magic squares.

For instance, as we demonstrated in [5],

a) every \((n+1)\)-th term \( a_{n+1} \) of Smarandache sequences of 1st kind may be formed by subjoining several natural numbers to previous terms \( a_n \) and also may be computed from the analytical expression
\[ a_{\text{S}}(n) = \sigma(a_n 10^{\omega(n)} + \xi(\psi(n))), \]  

where \( \psi(n) \), \( \psi(a_n) \) and \( \xi(\psi(n)) \) are some functions; \( \sigma \) is an operator. In other words, for generating Smarandache sequences of 1st kind, the set of analytical formulae may be used (see the problem 1);

\begin{center}
\begin{tabular}{|c|c|c|}
\hline
22232425262728 & 15161718192021 & 2021222324252627 \\
17181920212223 & 19202122232425 & 21222324252627 \\
18192021222324 & 23242526272829 & 16171819202122 \\
\hline
\end{tabular}
\end{center}

(1)

\begin{center}
\begin{tabular}{|c|c|c|}
\hline
171819191817 & 101112121110 & 151617171615 \\
121314141312 & 141516161514 & 161718181716 \\
131415151413 & 181920201918 & 111213131211 \\
\hline
\end{tabular}
\end{center}

(2)

\begin{center}
\begin{tabular}{|c|c|c|}
\hline
17181920191817 & 10111213121110 & 15161718171615 \\
12131415141312 & 14151617161514 & 16171819181716 \\
13141516151413 & 181920201918 & 11121314131211 \\
\hline
\end{tabular}
\end{center}

(3)

Figure 1. Magic squares 3x3 from k-truncated Smarandache numbers of 1st kind.

b) it is impossible to construct Magic squares 3x3 from Smarandache numbers of 1st kind without previous truncating these numbers. Consequently, if the given set of numbers consists only of Smarandache numbers of 1st kind, then one releases from care on solving problems, mentioned above in items 2 - 4;

c) there is a set of analytical formulae available for constructing Magic squares 3x3 in size from k-truncated Smarandache numbers of 1st kind (examples of Magic squares 3x3, obtained by these formulae, are shown in figure 1). In this case the foregoing set of analytical formulae is also the desired practical method for constructing Magic squares 3x3 from k-truncated Smarandache numbers of 1st kind (see the problem 4).

The main goal of this paper is to investigate some properties of Smarandache sequences of the 2nd kind [6, 9] and to demonstrate that these numbers are near prime numbers. In particular, we establish in the paper, that prime numbers and Smarandache numbers of the 2nd kind

127
a) may be computed from the similar analytical expressions (see Section 2 and 3);
b) may be used for constructing Magic squares $3 \times 3$ or Magic squares $9 \times 9$, consisted of 9 Magic squares $3 \times 3$ (see Section 5 and 6).

2 Prime Numbers

We remind that in number theory [2, 10, 11] any positive integer (any natural number), simultaneously dividing positive integers $a$, $b$, ..., $m$, is called their common divisor. The largest of common divisors is called greatest common divisor and denoted by the symbol $\text{GCD}(a, b, ..., m)$. The existence of GCD appears from the finiteness of the number of common divisors. The numbers $a$ and $b$ for which $\text{GCD}(a, b) = 1$ are called relatively prime numbers. The analytical formula available for counting the value of $\text{GCD}(a, b)$ has form [6]

$$\text{GCD}(a, b) = b(1 - \text{sign}(r)) + k \text{sign}(r), \quad r = a - b[a/b],$$

$$k = \text{MAX}_{i=2}^{[\sqrt{a}]} \{i(1 - d)\}, \quad d = \text{sign}\{a - \lceil a/i \rceil\} + \text{sign}\{b - \lceil b/i \rceil\},$$

where the function $\text{MAX}(a_1, a_2, ..., a_i)$ gives the greatest from numbers $a_1, a_2, ..., a_i$; $\text{sign}(x) = 1$ if $x > 0$ and $\text{sign}(0) = 0$.

It is easy to prove, that any natural number larger than a unit, has no less than two divisors: the unit and itself. Any natural number $p > 1$, having exactly two divisors, is called prime. If the number of divisors is more than 2, then the number is called composite (for example, the number 11, having divisors 1 and 11, is the prime number, whereas the number 10, having the divisors 1, 2, 5 and 10, is the composite number). In this paper we shall consider the number 1 as the least prime number. The analytical formula, generating $n$-th prime number $p_n$ has form [6]

$$p_n = \sum_{m=0}^{[\sqrt{n}]} \text{sg}(n - 1 - \sum_{i=2}^{m} \chi_i), \quad \chi_i = \prod_{p=2}^{[\sqrt{i}]} \{\text{sg}(i/p)\},$$

where $p_2 = 2, p_3 = 3, p_4 = 5, ...; \text{sg}(x) = 1$ if $x > 0$ and $\text{sg}(x) = 0$ if $x \leq 0$.

It is proved in the number theory [2, 10, 11], that any natural number larger than a unit can be represented as a product of prime numbers and this representation is unique (we assume that products, differing only by the order of cofactors, are identical). For solving the problem on decomposing the natural number $a$ in simple cofactors, it is necessary to know all the prime numbers $p_n < \sqrt{a}$.

Let $m = \lfloor \sqrt{a} \rfloor$, where the notation $[b]$ means integer part from $b$. Then, for finding all the prime number $p_n$ one may use the following procedure (Eratosthenes sieve) [2, 10, 11]:

1. Write out all the successive numbers from 2 to $m$ and put $p = 2;$
2. In the series of the numbers 2, 3, 4, ..., m, cross out all the numbers having the form $p + kp$, where $k = 1, 2, \ldots$;

3. If, in the series of the numbers 2, 3, 4, ..., m, all the numbers larger than $p$ have been crossed out, then pass to step 4. If there still remain the numbers larger than $p$, which have not been crossed out, then the first of these ones we denote by $p_1$. If $p_1^2 \geq m$, then pass to step 4. Otherwise, put $p = p_1$ and pass to step 2;

4. The end of the procedure: primes are all the numbers of the series 1, 2, 3, 4, ..., m, which have not been deleted.

If an arithmetical progression from $n$ prime numbers is found then it should be known that \[2, 101\]

*The difference of any arithmetical progression, containing $n$ prime numbers larger than $n$, is divisible by all the prime numbers $\leq n$ (Cantor theorem).*

From the series of the consecutive prime numbers one may reveal subsequences of numbers, possessed the different interesting properties. For instance

a) two prime numbers are called *reversed*, if each is obtained from other by reversing of its digits. If $p < 1000$ then such numbers are

\[1, 2, 3, 5, 7, 11, 13, 17, 31, 37, 71, 73, 79, 97, 101, 107, 113, 131, 149, \ldots (4)\]

\[151, 157, 167, 179, 181, 191, 199, 311, 313, 337, 347, 353, 359, 373,\]

\[383, 389, 701, 709, 727, 733, 739, 743, 751, 757, 761, 769, 787, 797,\]

\[907, 919, 929, 937, 941, 953, 967, 971, 983, 991.\]

b) among the numbers of (4) one may reveal the *symmetric* prime numbers:

\[1, 2, 3, 5, 7, 11, 131, 151, 181, 191, 313, 337, 373, 383, 727, 757, \ldots (5)\]

\[787, 797, 919, 929;\]

c) two prime numbers are called *mirror-reversed*, if each is obtained from other by reflecting in the mirror, located above the number. If $p < 3000$ then such numbers are:

\[1, 2, 3, 5, 11, 13, 23, 31, 53, 83, 101, 131, 181, 227, 251, 311, 313, \ldots (6)\]

\[331, 383, 521, 557, 811, 823, 853, 881, 883, 1013, 1021, 1031, 1033,\]

\[1051, 1103, 1123, 1153, 1181, 1223, 1231, 1283, 1301, 1303, 1381,\]

\[1331, 1553, 1583, 1811, 1831, 2003, 2011, 2053, 2081, 2113, 2203,\]

\[2251, 2281, 2333, 2381, 2531, 2851.\]
3 Smarandache Numbers of the 2nd Kind

In this section we consider 4 different Smarandache sequences of the 2nd kind \([6, 9]\) and demonstrate that the value of \(n\)-th numbers \(a_n\) in these sequences may be computed by the universal analytical formula (compare with formula (3))

\[
a_n = \sum_{m=0}^{U_n} \sum_{i=1}^{X_i} (n + 2 - b - \sum_{j=1}^{i} X_j),
\]

where \(X_i\) are the characteristic numbers for the described below Smarandache sequences of the 2nd type and \(U_n = 10 + (n+1)^2\).

3.1 Pseudo-Prime Numbers

a) Smarandache \(P_1\)-series

1, 2, 3, 5, 7, 11, 13, 14, 16, 17, 19, 20, 23, 29, 30, 31, 32, 34, ... (8)

contains the only such natural numbers, which are or prime numbers itself or prime numbers can be obtained from \(P_1\)-series numbers by a permutation of digits (for instance, the number 115 is the pseudo-prime of \(P_1\)-series because the number 151 is the prime).

It is clear from the description of \(P_1\)-series numbers that they may be generated by the following algorithm

1. Write out all the successive prime numbers from 1 to 13: 1, 2, 3, 5, 7, 11, 13 and put \(n=8\); \(a_n = 13\);
2. Assume \(p = a_n + 1\).
3. Examine the number \(p\). If \(p\) is a prime or a prime number can be obtained from \(a_n\) by a permutation of digits, then increase \(n\) by 1, put \(a_n = p\) and go to step 2. Else increase \(p\) by 1 and go to the beginning of this step.

To convert the foregoing algorithm into a computer-oriented method (see problem 1 in Section 1), we evidently to translate this description into one of special computer-oriented languages. There is a set of methods to realise such translation [6]. The most simplest among ones is to write program code directly from the verbal description of the algorithm without any preliminary construction. For instance, Pascal program identical with the verbal description of the algorithm under consideration are shown in Table 1. In this program the procedure \(Pd\), the functions \(PrimeList\) and \(PseudoPrime\) are used for generating respectively permutations, primes numbers and pseudo-prime numbers; the meaning of the logical function \(BelongToPrimes\) is clear from its name.

In the case, when verbal descriptions are complex, babelized or incomplete, the translation of these descriptions into computer languages may be performed sometimes in two stages [7]: firstly, verbal descriptions of computational algorithms are translated into analytical ones and then analytical descriptions are translated into computer languages. To demonstrate how this scheme is realised in practice, let us apply it to the algorithm, generating \(P_1\)-series numbers.
Table 1. Pascal program 1 for generating Smarandache \( P \)-series

```pascal
Type Ten = Array[1..10] of Integer;

Procedure Pd(Var m4, n1, n: Integer; Var nb3, nb4, nb5: Ten);
Label A28, A29, A30; Var nt, k, m: Integer;
Begin
  If m4 = 1 Then
  Begin
    m4 := 0; n := n1;
    For k := 2 to n do
      Begin
        Nb4[k] := 0;
        Nb5[k] := 1;
      End;
    Exit;
  End;
  k := 0; n := n1;
A28: m := Nb4[n] + Nb5[n]; Nb4[n] := m;
  If m = n Then
  Begin
    Nb5[n] := -1;
    Goto A29;
  End;
  If Abs(m) > 0 Then Goto A30;
  Nb5[n] := 1;
  Inc(k);
A29: If n > 2 Then
  Begin
    Dec(n);
    Goto A28;
  End;
  Inc(m);
  m4 := 1;
A30: m := m + k;
  nt := nb3[m];
  nb3[m] := nb3[m + 1];
  nb3[m + 1] := nt
End;

Const Mn = 10000;
MaxN: Integer = Mn;
Type int = Array[1..Mn] of Integer; pint = Array[1..Mn] of Integer;
Var pI: pint;

Function Primelist(Var MaxN: Integer): pint;
Var i, j, k: Integer; p: pint; Ok: Boolean;
Begin
  GetMem(p, MaxN);
  pA[1] := 2;
  i := 3;
  k := 1;
  Repeat
    j := (i + r) shr 1;
    If num < Primelist[j] Then
      Begin
        BelongToPrimes := False;
      End;
    Exit;
  Until r = 0;
  BelongToPrimes := False;
End;

Function BelongToPrimes(num: Integer): Boolean;
Var I, r, j: Integer;
Begin
  BelongToPrimes := True;
  i := 1;
  r := MaxN;
  Repeat
    j := (i + r) shr 1;
    If num < Primelist[j] Then r := j;
    Exit;
  Until r = 0;
  BelongToPrimes := False;
End;
```

Else if num < Primelist[j] Then
  l := j + 1
  Else Exit;
  Until r = BelongToPrimes = False;
End;

Function
PseudoPrime(Num: Integer): Boolean;
Var g, nb3, nb4, nb5: Ten;
nd, m, r, mn, m4, n1, mm, i, j, d, k, n: Integer;
Begin
  PseudoPrime := True;
  [Decomposition number num on digits]
  d := Num; c := 0;
  Repeat
    d := d mod 10;
    r := d;
    d := d div 10;
    Until r = 0;
    [Examination whether numbers, composed from digits are prime]
    m4 := 1;
    m := 1;
    For i = 1 to n do
      Nb3[i] := g[i];
    Repeat
      Pd(m4, n1, n, nb3, nb4, nb5);
      Inc(m);
    Until False;
    PseudoPrime := False;
End;

Var Ind, Num, i: Integer; Listpint;
Begin
  pi := Primelist(MaxN);
  [Generating list of primes up to MaxN]
  Ind := 0;
  For Num := 10 to MN do
    [If number is pseudoprime then add it to list]
    Begin
      Inc(Ind);
      Listpint[Ind] := Num;
    End;
  End;
  Assign(Output, 'Sp1'); Rewrite(Output);
  For i := 1 to Ind do
    Write(Listpint[i] - 1,
         'Sp1' file, 10 values per row)
  Output Close(output);
End;
```

131
Table 2. Pascal program 2 for generating Smarandache $P_1$-series

```
Const MaxG=5;
Var c,d,r:Array[1..MaxG]Of Integer;
g:Integer;
Function Sg(x:Integer):Integer;
Begin (function returns unit if argument is
greater than zero)
  If x>0 Then Sg:=1 Else Sg:=0;
End;
Function Fact(x:Integer):LongInt;
Var i:Integer; f:LongInt;
Begin (function calculates factorial of
given number)
  f:=1; For i:=1 to x do f:=f*i; Fact:=f;
End;
Function Lg(x:Extended):Extended;
Begin (function returns decimal logarithm of
given number)
  Lg:=Ln(x)/Ln(10);
End;
Function Power(x: Extended; Deg: Integer): Extended;
Var p:Extended; i:Integer;
Begin (function returns argument in 'deg'
power)
  p:=1; For i:=1 to Deg do p:=p*x;
  Power:=p;
End;
Function Mu(p,g:Integer):Integer;
Var m,q:Integer;
{this is an auxiliary function}
Begin m:=1;
  For q:=1 to p do m:=m*(g-q+1); Mu:=m;
End;
Function GetPos(k,p:Integer):Integer;
Var i:Integer;
Begin (function returns location of element 'p'
in 'k'th permutation of 'g' objects)
  c[p]=(k div Mu(p,g)) mod 2;
f:=(k div Mu(p-1,g)) mod (g-p+1);
d[p]:=p-1+(1-c[p])+(g-p-f);
r[p]:=d[p];
For i:=p downto 1 do
  r[p]:=r[p]-Byte(d[i]>=rfp)); GetPos:=rfp;
End;
Function MXi(i:Integer):Integer;
Var k,q,p,s,Pro:Integer;
Sum,c:Extended;
Begin (function returns unit if examined value 'i'
belongs to set of Smarandache numbers)
  S:=0; g:=Trunc(Lg(i))+1;
  For k:=0 to Fact(g)-1 do
    Begin (Construction number 'c' from permutated
digits of number 'i')
      sum:=0; For p:=1 to g do
        sum:=sum+(Int(i*Power(10,g-p))-
      10*Int((Power(10,g-p+1))/
        Power(10,GetPos(k,p)));
      c:=Power(10,g-1)*sum;
      Pro:=1; {If 'c' is prime number}
      For q:=2 to Trunc(sqrt(c)) do
        Pro:=Pro*Sg(Round(c) mod q);
      Sum:=Sum+Pro;
    End;
  MXi:=Sg(Sum);
End;
Var xi,n,M:Integer;
Function BuildAn(n:Integer):Integer;
Var i,dx,a:Integer;
Un,SumXi:LongInt;
Begin (function returns 'n'th element of
Smarandache sequence)
  a:=0; Un:=Sqr(LongInt(n));
  For m:=0 to Un do
    Begin ("SumXi" is quantity of Smarandache numbers
which are less than number 'm')
      SumXi:=0; For i:=1 to m do
        SumXi:=SumXi+MXi(i);
      a:=a*Sg(n-SumXi);
    End;
End;
Begin (Output of the first 'M' Smarandache
numbers)
  M:=30;
  For n:=1 to M do Write(BuildAn(n):5);
WriteLn;
End.
```

132
The analytical formula available for determining $n$-th number in the $P_1$-series is obtained from (7) when [6]
\[ b = 2, \quad \chi_i = \text{sg} \left\{ \sum_{k=0}^{r-1} \prod_{q=2}^{\sigma_1} (c - q / c) \right\}, \tag{9} \]
and $g$, $c$ and $r_p$ are calculated by the formulae
\[ g = \lfloor \log d \rfloor + 1, \quad c = 10 + \sum_{j=1}^{\sigma_1} \left\{ \lfloor (j / 10^\varphi_p) - 10 \rfloor / 10^\varphi_p \right\}, \tag{10} \]
\[ r_p = z_1, \quad d_p = p - 1 + f(1 - c_p) + c_p (g - p - 1), c_p = \lfloor (-1)^{p-1} - 1 / 2, \]
\[ f = t_{p+1} - (g - p + 1) \lfloor t_{p+1} / (g - p + 1), \quad t_p = \lfloor k / 10^\varphi_p (g - q + 1) \rfloor, \]
\[ z_1 = z_2 = \text{sg}(1 + d_1 - z_2), \quad z_2 = z_3 = \text{sg}(1 + d_2 - z_3), \ldots, \]
\[ z_{p-1} = z_p - \text{sg}(1 + d_{p-1} - z_p), \quad z_p = d_p - \text{sg}(1 + d_{p-1} - d_p). \]

Pascal program identical with the analytical description (9) – (10) of the algorithm, generating $P_1$-series numbers, takes the form, shown in Table 2.

It should be noted that most part of Pascal text of program 2 consists of formulae (9) – (10). In other words, translating analytical descriptions of computative algorithms into computer languages requires noticeably less efforts than the translation of verbal descriptions. Therefore, our conclusion is that

\textit{if it is possible, one should provide the verbal descriptions of computational algorithms with the analytical ones, constructed, for instance, by using logical functions [5 - 7].}

b) Smarandache $P_2$-series
\[ 14, 16, 20, 30, 32, 34, 35, 38, 50, 70, 74, 76, 91, 92, 95, 98, \ldots \tag{11} \]
contains the only such natural numbers, which are the composite numbers itself, but the prime numbers can be obtained from $P_2$-series numbers by a permutation of digits. The analytical formula available for determining $n$-th number in the $P_2$-series has the same form as for $P_1$-series numbers, but in this case the value of $\chi_i$ from (9) is computed by the formula
\[ \chi_i = (1 - w_0) \text{sg} \left( \sum_{k=1}^{\sigma_1} w_k \right), \quad w_k = \prod_{q=2}^{\sigma_1} (c - q / c). \tag{12} \]

3.2 Some Modifications of Eratosthenes Sieve

a) Smarandache $T_1$-series
\[ 7, 13, 19, 23, 25, 31, 33, 37, 43, 47, 49, 53, 55, 61, 63, \ldots \tag{13} \]
is obtained from the series of natural numbers by deleting all even numbers and all such odd numbers $t_i$ that the numbers $t_i+2$ are primes. The analytical formula for the determination of $n$-th number in the $T_1$-series has the form (7) with

133
\[ b = 2, \quad \chi_i = (i - 2[i/2]) \{1 - \prod_{k=3}^{x^2/2} \sgn(i + 2 - k((i + 2)/k))\}, \quad (14) \]

b) Smarandache \( T_2 \)-series

1, 3, 5, 9, 11, 13, 17, 21, 25, 27, 29, 33, 35, 37, 43, 49, ...

(15)

This series may be obtained from the series of natural numbers by the following step-procedure:

On \( k \)-th step each \( 2^k \)-th number is deleted from the series of numbers constructed on \((k-1)\)-th step.

The analytical formula for the determination of \( n \)-th number in the \( T_2 \)-series has the form (7) with

\[ \chi_i = \sgn \left( \prod_{k=1}^{\log_2 n} \{x_k - 2^k \lfloor x_k/2^k \rfloor\} \right), \quad x_1 = i, \quad x_{k+1} = x_k - \lfloor x_k/2^k \rfloor, \quad (16) \]

where \( \log a \) is the logarithm of the number \( a \) to the base 2.

4 Algorithms for Solving Problems on Constructing Magic Squares 3x3 from Given Class of Numbers

**Proposition 1.** A set of nine numbers is available for constructing Magic squares 3x3 only in the case if one succeeds to represent these nine numbers in the form of such three arithmetic progressions from 3 numbers whose differences are identical and the first terms of all three progressions are also forming an arithmetic progression.

**Proof.** The general algebraic formula of Magic squares 3x3 is shown in figure 1(3) [2, 4]. The table 1(4) is obtained from table 1(3) by arranging its symbols. It is noteworthy that arithmetic progressions with the difference \( b \) are placed in the rows of the table 1(4), whereas ones, having the difference \( c \), are located in its columns. Thus, the proof of Proposition 1 follows directly from the construction of tables 1(3) and/or 1(4).

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<td></td>
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<td></td>
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<td></td>
<td></td>
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</tr>
<tr>
<td>a+c</td>
<td>a+b+c</td>
<td>a+2b+c</td>
<td></td>
<td></td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>(2)</td>
<td>(4)</td>
<td>a+2c</td>
<td>a+b+2c</td>
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</tbody>
</table>

Figure 1. To proofs of correctness of Proposition 1 and Algorithm 1:

(3) — the general algebraic formula of Magic squares 3x3; (4) — additional table of Magic squares 3x3; (1) \((c > 2b)\) and (2) \((b < c < 2b)\) — two possible arrangements of the nine increasing numbers in cells of the additional table (4).
By Proposition 1 and two possible arrangements of the nine increasing numbers in cells of the additional table 1(4), which are shown in figures 1(1) and 1(2), we may elaborate algorithm 1 available for constructing Magic squares 3x3 from an arbitrarily given set of nine increasing numbers [2]:

1. Take two square tables 3x3 and arrange 9 testing numbers in them so as it is shown in figures 1(1) and 1(2).
2. Check whether three arithmetic progressions of Proposition 1 are in one of these square tables 3x3.

It should be noted, if the problem on constructing the Magic square 3x3 from the given set of nine increasing numbers has the solution, then this solution is always unique with regard for rotations and mappings.

For finding all Magic squares 3x3 from a given class of numbers with the number \( f \) in its central cell, one may use the following algorithm 2 [2, 4]

a) write out the possible decompositions of the number 2\( f \) in the two summands of the following form:

\[
2f = x_1(j) + x_2(j),
\]

where \( j \) is the number of a decomposition and \( x_1(j), x_2(j) \) are the two numbers such that \( x_1(j) < x_2(j) \) and both these numbers belong to the given class of numbers;

b) in the complete set of various decompositions (17), fix one, having, for instance, the number \( k \) and, for this decomposition, determine the number \( d(k) = f - x_1(j) \);

c) find all possible arithmetic progressions from 3 numbers with differences equal \( d(k) \) among a set of numbers \( \{x_i(j)\} \) without \( x_1(k) \). If there are \( m \) such arithmetic progressions then there are \( m \) Magic squares 3x3 with the numbers \( x_1(k) \) and \( x_2(k) \) in its cells;

d) repeat items (b) and (c) for other values of \( k \).

5 Magic Squares 3x3 and 9x9 from Prime Numbers

**Proposition 2.** A Magic square 3x3 can be constructed from prime numbers only in the case if the parameters \( b \) and \( c \) of the general algebraic formula 1(3) and/or additional table 1(4) are the numbers multiple of 6.

**Proof.** The truth of Proposition 2 follows from Proposition 1 and Cantor theorem of Section 2.

**Corollaries** from Proposition 2 [2]:

1. By using prime numbers one cannot construct a Magic square 3x3 with one of the cells containing numbers 2 or 3.
2. All nine prime numbers of a Magic square $3 \times 3$ are either numbers of the form $6k - 1$ or have the form $6k + 1$.

**Proposition 3** [2]. *With regard for rotations and mappings, the last digits of the prime numbers may be arranged in the cells of the additional table of a Magic square $3 \times 3$ only in such variants, which are shown in figure 2.*

**Proof.** To prove the truth of Proposition 3, we need the two more easily verified properties of the additional table 1(4).

1. In this table the sums of the symbols of the central row, central column and both diagonals are identical and coincide with the Magic constant of the general algebraic formula 1(3).

2. An arithmetic progression, consisting of three numbers, occurs not only in the rows and columns but also in each diagonal of the additional table.

Now let us place a prime number, for instance, ending by 1, into the central cell of the additional table 1(4). It is clear, that in this case the last digits of all other prime numbers of the additional table of a Magic square $3 \times 3$ must be such that their sums in the central column, central row and both diagonals would terminate by 3. Thus, only certain arrangements of the last digits of prime numbers are possible in the remaining cells of the additional table and all such variants are shown in figure 2.

![Figure 2. All possible arrangements of the last digits of the prime numbers in cells of the additional table 1(4).](image)

**Corollaries** from Proposition 3 [2]:

1. Since 5 is a prime number having the form $6k - 1$, only the prime numbers of the form $6k - 1$ can be placed in cells of the additional table 1(4) with arrangements 2(3), 2(6), 2(9) and 2(12).
2. The arithmetic progression from three prime numbers $a_k - 30m, a_k, a_k + 30m$ may be found among nine prime numbers of any Magic square $3 \times 3$, where the number $a_k$ is located in the central cell of the Magic square and $m$ is some integer number. Hence it appears that

no Magic square $3 \times 3$ may be constructed from prime numbers if $a_k < 30$.

Let us consider some results of [2], obtained for prime numbers by computer.

1. Magic squares $3 \times 3$, shown in figure 3, are the least ones, constructed only from prime numbers.

2. Let it be required to construct a Magic square $3 \times 3$ only from prime numbers with the number $a_k$ in its central cell. This problem cannot be solved only for the following prime numbers $a_k > 30$:
   a) having the form $6k - 1$: 41, 101; 53, 83, 113, 233; 47, 107, 197, 317; 569;
   b) having the form $6k + 1$: 31, 61, 181, 331; 43, 163, 223, 313, 433; 67, 97, 277, 457; 79, 199, 229, 439, 859.

3. The results of the item 2 make it possible to assume that, for any $a_k$ larger than some prime number $P_{\text{max}}$, one can always construct a Magic square $3 \times 3$ with Magic sum $S = 3a_k$ and the prime numbers, ending by the same digit as the number $a_k$, $P_{\text{max}}$ equals the following prime numbers:
   a) having the form $6k - 1$: 5081 (281); 3323 (683); 6257 (557); 3779 (359);
   b) having the form $6k + 1$: 3931 (601); 3253 (523); 4297 (307); 7489 (769),
where in brackets we indicate the least prime numbers $a_k$, for which one can construct a Magic square $3 \times 3$ with $S = 3a_k$ and the prime numbers, ending by the same digit as $a_k$.

4. Let it be required from prime numbers to construct a Magic square $9 \times 9$, which contains the number $a_k$ in its central cell and consists of 9 Magic squares $3 \times 3$.

The example of the least Magic square $9 \times 9$, constructed only from prime numbers and consisted of 9 Magic squares $3 \times 3$, is shown in figure 4.

If $a_k > 1019$, then the problem on constructing Magic squares $9 \times 9$, discussed in this item, cannot be solved only for following prime numbers $a_k$:

\begin{align*}
1021, 1031, 1033, 1039, 1049, 1051, 1061, 1069, 1087, 1091, 1093,
\end{align*}
1097, 1109, 1117, 1123, 1129, 1153, 1171, 1181, 1193, 1201, 1213, 1217, 1229, 1231, 1237, 1249, 1259, 1279, 1283, 1303, 1307, 1321, 1327, 1439, 1453, 1481, 1483, 1489, 1511, 1531, 1543, 1567, 1783.

Figure 4. The example of the least Magic square 9x9, constructed only from prime numbers and consisted of 9 Magic squares 3x3.

<table>
<thead>
<tr>
<th>2531</th>
<th>17</th>
<th>1409</th>
<th>1097</th>
<th>71</th>
<th>863</th>
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<td>107</td>
<td>491</td>
<td>823</td>
<td>257</td>
<td>1031</td>
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<td>53</td>
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<tr>
<td>1433</td>
<td>29</td>
<td>821</td>
<td>1811</td>
<td>137</td>
<td>1109</td>
<td>2153</td>
<td>311</td>
<td>1367</td>
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<tr>
<td>149</td>
<td>761</td>
<td>1373</td>
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<td>1019</td>
<td>1721</td>
<td>491</td>
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<td>89</td>
<td>929</td>
<td>1901</td>
<td>227</td>
<td>1187</td>
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<td>401</td>
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<tr>
<td>1487</td>
<td>431</td>
<td>1013</td>
<td>2339</td>
<td>173</td>
<td>1571</td>
<td>1307</td>
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<td>839</td>
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<td>2549</td>
<td>383</td>
<td>599</td>
<td>1427</td>
<td>131</td>
</tr>
</tbody>
</table>

6 Magic Squares 3x3 and 9x9 from Smarandache Numbers of the 2nd Kind

6.1 Magic Squares 3x3 and 9x9 from P₁-Series Numbers

Let the notation \( A_{C_j}(N) \) means the quantity of all \( C_j \)-series numbers, whose values are less than \( N \), and the notation \( P_0 \)-series means the prime numbers series.

**Proposition 4.** For any natural number \( N \) the following inequality

\[
A_{P_1}(N) \geq A_{P_0}(N)
\]

is fulfilled

*Proof.* The truth of Proposition 4 follows from the description of \( P_1 \)-series numbers (see Section 3.1). Namely, \( P_0 \)-series numbers is subset of \( P_1 \)-series numbers at any \( N \) and agree with a set of \( P_1 \)-series numbers only if \( N \leq 13 \).

**Proposition 5.** \( P_1 \)-series numbers are available for constructing Magic squares 3x3.

*Proof.* The truth of Proposition 5 follows from Proposition 4 and that the prime numbers are available for constructing Magic squares 3x3 (see Section 5).

Solving the problems on constructing Magic squares 3x3 from \( P_1 \)-series numbers by computer, we find that
1. Magic squares $3 \times 3$, shown in figure 5, are the least ones, constructed from $P_1$-series numbers.

\[
\begin{array}{ccc}
47 & 5 & 35 \\
17 & 29 & 41 \\
23 & 53 & 11
\end{array}
\]  
\[
\begin{array}{ccc}
50 & 11 & 35 \\
17 & 32 & 47 \\
29 & 53 & 14
\end{array}
\]  
\[
\begin{array}{ccc}
53 & 11 & 41 \\
23 & 35 & 47 \\
29 & 59 & 17
\end{array}
\]  
\[
\begin{array}{ccc}
50 & 17 & 38 \\
23 & 53 & 11 \\
32 & 53 & 20
\end{array}
\]

(1) (2) (3) (4)

Figure 5. The least Magic squares $3 \times 3$, constructed from $P_1$-series numbers.

2. Let it be required from $P_1$-series numbers to construct a Magic square $3 \times 3$ with the number $a_k$ in its central cell.

If $a_k > 35$, then this problem cannot be solved only for the following $P_1$-series numbers: 38, 43, 47, 50 and 61.

3. Let it be required from $P_1$-series numbers to construct a Magic square $9 \times 9$, which contains the number $a_k$ in its central cell and consists of 9 Magic squares $3 \times 3$.

Magic square $9 \times 9$, shown in figure 6, is the least such one, constructed from $P_1$-series numbers.

We note, that

a) in the Magic square $9 \times 9$, shown in figure 6, the numbers 215, 35, 143, 59, 203, 119, 227 and 47 may be replaced respectively by 203, 47, 143, 71, 191, 119, 215 and 59;

\[
\begin{array}{cccccccc}
413 & 101 & 329 & 137 & 20 & 92 & 383 & 2 & 269 \\
197 & 281 & 365 & 38 & 83 & 128 & 104 & 218 & 332 \\
233 & 461 & 149 & 74 & 146 & 29 & 167 & 434 & 53
\end{array}
\]

\[
\begin{array}{cccccccc}
215 & 35 & 143 & 293 & 11 & 278 & 380 & 17 & 374 \\
59 & 131 & 203 & 179 & 194 & 209 & 251 & 257 & 263 \\
119 & 227 & 47 & 110 & 377 & 95 & 140 & 497 & 134
\end{array}
\]

\[
\begin{array}{cccccccc}
317 & 5 & 188 & 323 & 272 & 320 & 182 & 14 & 125 \\
41 & 170 & 299 & 302 & 305 & 308 & 50 & 107 & 164 \\
152 & 335 & 23 & 290 & 338 & 287 & 89 & 200 & 32
\end{array}
\]

Figure 6. The least Magic square $9 \times 9$, constructed from $P_1$-series numbers and consisted of 9 Magic squares $3 \times 3$.

b) if $a_k > 194$, then the problem on constructing Magic squares $9 \times 9$, discussed in this point, cannot be solved only for following 10 $P_1$-series numbers $a_k$: 196, 197, 199, 211, 214, 217, 223, 229, 232 and 300.
6.2 Magic Squares 3x3 and 9x9 from $P_2$-Series Numbers

**Proposition 6.** For any natural number $N$ the following inequality

$$A_{p_o}(N) < A_{p_2}(N)$$

(20)

is fulfilled

**Proof.** The truth of Proposition 6 follows from the description of $P_2$-series numbers (see Section 3.2). Namely, $P_2$-series numbers may be obtained by deleting all prime numbers from $P_1$-series numbers.

It follows from Proposition 6 that, although we know about the availability of $P_0$- and $P_2$-series numbers for constructing Magic squares 3x3, we cannot state that $P_2$-series numbers are also available for constructing Magic squares 3x3. To clear up this situation, let us consider our results, obtained for $P_2$-series numbers by computer.

1. Magic squares 3x3, shown in figure 7, are the least ones, constructed from $P_2$-series numbers.

2. Let it be required from $P_2$-series numbers to construct a Magic square 3x3 with the number $a_k$ in its central cell.

- If $a_k = 92, 125, 441, 448, 652, 766$ or $928$, then this problem has a single solution.

- If $a_k > 125$, then this problem cannot be solved only for the following $P_2$-series numbers:

  $$130, 142, 143, 145, 152, 160, 166, 169, 172, 175, 176, 190, 196, 232,$$
  $$238, 289, 292, 298, 300, 301, 304, 319, 325, 382, 385, 391, 478, 517.$$ (21)

3. Let it be required from $P_2$-series numbers to construct a Magic square 9x9, which contains the number $a_k$ in its central cell and consists of 9 Magic squares 3x3.

- If $a_k = 473$, then there are 609 the least Magic squares 9x9 with mentioned properties (the example of such Magic square is shown in figure 8).

- If $a_k > 473$, then the problem on constructing Magic squares 9x9, discussed in this item, cannot be solved only for two $P_2$-series numbers $a_k$: 478 and 517.
6.3 Magic Squares 3x3 and 9x9 from T₁-Series Numbers

**Proposition 7.** There exists such natural number $N₀$ that for any natural $N > N₀$ the following inequality

$$A₁(N) > A_{₉₋₁}(N)$$

is fulfilled.

**Proof.** As it follows from the description of $T₁$-series numbers (see Section 3.2), this series numbers may be obtained from series odd natural numbers by deleting all such odd numbers, which are prime numbers decreased by 2. Thus, we have the following relation

$$A₁(N) = \frac{(N-1)}{2} - A_{₉₋₁}(N) \quad \text{or} \quad A₁(N)/A_{₉₋₁}(N) = \frac{(N-1)}{2}/A_{₉₋₁}(N)$$

where the term $(N-1)/2$ is the quantity of all odd natural numbers, whose values are less than $N$. Since [8]

$$A_{₉₋₁}(N) = \frac{N}{\ln(N)+1} \pm \frac{N}{\ln²(N)},$$

we obtain from (23) and (24) that

$$A₁(N)/A_{₉₋₁}(N) = \ln(N)/2 - 1 > 2 \quad \text{for any} \quad N > 500.$$ 

Thus, Proposition 7 is true, if $N₀$, for instance, equals 500.

**Proposition 8.** $T₁$-series numbers are available for constructing Magic squares 3x3.

**Proof.** The truth of Proposition 8 follows from Proposition 7 and that the prime numbers are available for constructing Magic squares 3x3.

Let us consider our results, obtained for $T₁$-series numbers by computer.
1. Magic square $3\times3$, shown in figure 9(1), is the least one, constructed from $T_1$-series numbers.

<table>
<thead>
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<td>163</td>
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<td>97</td>
<td>207</td>
<td>53</td>
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</table>

Figure 9. Examples of Magic squares $3\times3$, constructed from $T_1$-series numbers.

2. Let it be required from $T_1$-series numbers to construct a Magic square $3\times3$ with the number $a_k$ in its central cell.

If $a_k = 53, 75$ or $119$, then this problem has a single solution (see figure 9(2 - 4)).

If $a_k > 31$, then this problem cannot be solved only for two $T_1$-series numbers: 33 and 47.

3. Let it be required from $T_1$-series numbers to construct a Magic square $9\times9$, which contains the number $a_k$ in its central cell and consists of 9 Magic squares $3\times3$.

If $a_k = 181$, then there are 118 the least Magic squares $9\times9$ with mentioned properties (the example of such Magic square is shown in figure 10).

If $a_k > 181$, then the problem on constructing Magic squares $9\times9$, discussed in this item, can be solved for all $T_1$-series numbers $a_k$.

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<td>201</td>
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<tr>
<td>131</td>
<td>229</td>
<td>75</td>
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</tbody>
</table>

Figure 10. The example of the least Magic square $9\times9$, constructed from $T_1$-series numbers and consisted of 9 Magic squares $3\times3$.

If $a_k = 181$, then there are 118 the least Magic squares $9\times9$ with mentioned properties (the example of such Magic square is shown in figure 10).

If $a_k > 181$, then the problem on constructing Magic squares $9\times9$, discussed in this item, can be solved for all $T_1$-series numbers $a_k$. 
6.4 Magic Squares 3x3 and 9x9 from $T_2$-Series Numbers

**Proposition 9.** There exists such natural number $N_0$ that for any natural $N > N_0$ the following inequality

\[ A_{T2}(N) > A_{Po}(N) \]  

is fulfilled

**Proof.** As it follows from the description of $T_2$-series numbers (see Section 3.2), this series numbers may be obtained from series natural numbers by deleting all $2^k$-th numbers on each $k$-th step of step-procedure (sieve). Thus, we have the following relation

\[ A_{T2}(N) = N - \prod_{k=4}^{[\log_2 N]} 1/2^k = N(1-2/\{\log(N) \log(N)+1)) = N(1-2.9/\{\ln(N) (1.44 \ln(N) + 1)). \]

We obtain from (24) and (27) that

\[ A_{T2}(N)/A_{Po}(N) = \ln(N) > 2 \text{ for any } N > 20. \]  

Thus, Proposition 9 is true, if $N_0$, for instance, equals 20.

**Proposition 10.** $T_2$-series numbers are available for constructing Magic squares 3x3.

**Proof.** The truth of Proposition 10 follows from Proposition 9 and that the prime numbers are available for constructing Magic squares 3x3.

Our computations give the following results:

1. Magic squares 3x3, shown in figure 11, are the least ones, constructed from $T_2$-series numbers.

2. Let it be required from $T_2$-series numbers to construct a Magic square 3x3 with the number $a_k$ in its central cell.

   If $a_k > 27$, then this problem cannot be solved only for two $T_2$-series numbers: 37 and 49.

3. Let it be required from $T_1$-series numbers to construct a Magic square 9x9, which contains the number $a_k$ in its central cell and consists of 9 Magic squares 3x3.

\[
\begin{array}{c|c|c}
29 & 1 & 21 \\
9 & 17 & 25 \\
13 & 33 & 5 \\
\hline
33 & 5 & 25 \\
13 & 21 & 29 \\
17 & 37 & 9 \\
\hline
51 & 1 & 29 \\
5 & 27 & 49 \\
25 & 53 & 3 \\
\hline
43 & 1 & 33 \\
17 & 27 & 37 \\
21 & 53 & 11 \\
\hline
43 & 5 & 33 \\
17 & 27 & 37 \\
21 & 49 & 11 \\
\end{array}
\]

**Figure 11.** The least Magic squares 3x3, constructed from $T_3$-series numbers.
If $a_k = 195$, then there are 6 the least Magic squares 9x9 with mentioned properties (the example of such Magic square is shown in figure 12).

If $a_k > 195$, then the problem on constructing Magic squares 9x9, discussed in this point, cannot be solved only for the following $P_2$-series numbers $a_k$:

\[197, 201, 205, 213, 213, 217, 221, 225, 229, 237, 245, 249, 257, 261, 269.\]

7 Concluding Remarks

As it is demonstrated in this paper, preliminary theoretical analysis of number-theoretic and combinatorial problems is always useful. In particular, the results of this analysis are able sometimes to provide investigators with valuable information, facilitating considerably the solution of all such of practical tasks, which are enumerated in Section 1. We hope, that the technique of theoretical analysis, elaborated in the paper, will become useful tool of investigators, occupied in the considered problems.

References

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