A Note On Jump Symmetric $n$-Sigraph

H. A. Malathi and H. C. Savithri

Department of Computer Science & Engineering of Rajeev Institute of Technology, Industrial Area, B-M Bypass Road, Hassan 573 201, India
Email: malathisharmas@gmail.com, savithriathreyas@gmail.com

Abstract: A Smarandachely $k$-signed graph (Smarandachely $k$-marked graph) is an ordered pair $S = (G, \sigma)$ ($S = (G, \mu)$) where $G = (V, E)$ is a graph called underlying graph of $S$ and $\sigma : E \to (\tau_1, \tau_2, \ldots, \tau_k)$ ($\mu : V \to (\tau_1, \tau_2, \ldots, \tau_k)$) is a function, where each $\tau_i \in \{+, -\}$. Particularly, a Smarandachely 2-signed graph or Smarandachely 2-marked graph is called abbreviated a signed graph or a marked graph. In this note, we obtain a structural characterization of jump symmetric $n$-sigraphs. The notion of jump symmetric $n$-sigraphs was introduced by E. Sampathkumar, P. Siva Kota Reddy and M. S. Subramanya [Proceedings of the Jangjeon Math. Soc., 11(1) (2008), 89-95].

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§1. Introduction

For standard terminology and notion in graph theory we refer the reader to West [6]; the non-standard will be given in this paper as and when required. We treat only finite simple graphs without self loops and isolates.

Let $n \geq 1$ be an integer. An $n$-tuple $(a_1, a_2, \ldots, a_n)$ is symmetric, if $a_k = a_{n-k+1}$, $1 \leq k \leq n$. Let $H_n = \{(a_1, a_2, \ldots, a_n) : a_k \in \{+, -\}, a_k = a_{n-k+1}, 1 \leq k \leq n\}$ be the set of all symmetric $n$-tuples. Note that $H_n$ is a group under coordinate wise multiplication, and the order of $H_n$ is $2^m$, where $m = \left\lceil \frac{n}{2} \right\rceil$.

A Smarandachely symmetric $n$-sigraph (Smarandachely symmetric $n$-marked graph) is an ordered pair $S_n = (G, \sigma)$ ($S_n = (G, \mu)$), where $G = (V, E)$ is a graph called the underlying graph of $S_n$ and $\sigma : E \to H_n$ ($\mu : V \to H_n$) is a function.

A sigraph (marked graph) is an ordered pair $S = (G, \sigma)$ ($S = (G, \mu)$), where $G = (V, E)$ is a graph called the underlying graph of $S$ and $\sigma : E \to \{+, -\}$ ($\mu : V \to \{+, -\}$) is a function. Thus a Smarandachely symmetric 1-sigraph (Smarandachely symmetric 1-marked graph) is a sigraph (marked graph).

The line graph $L(G)$ of graph $G$ has the edges of $G$ as the vertices and two vertices of $L(G)$

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are adjacent if the corresponding edges of $G$ are adjacent.

The jump graph $J(G)$ of a graph $G = (V, E)$ is $\overline{L(G)}$, the complement of the line graph $L(G)$ of $G$ (See [1] and [2]).

In this paper by an $n$-tuple/n-sigraph/n-marked graph we always mean a symmetric $n$-tuple/Smarandachely symmetric $n$-sigraph/Smarandachely symmetric $n$-marked graph.

An $n$-tuple $(a_1, a_2, \ldots, a_n)$ is the identity $n$-tuple, if $a_k = +$, for $1 \leq k \leq n$, otherwise it is a non-identity $n$-tuple. In an $n$-sigraph $S_n = (G, \sigma)$ an edge labelled with the identity $n$-tuple is called an identity edge, otherwise it is a non-identity edge.

Further, in an $n$-sigraph $S_n = (G, \sigma)$, for any $A \subseteq E(G)$ the $n$-tuple $\sigma(A)$ is the product of the $n$-tuples on the edges of $A$.

In [4], the authors defined two notions of balance in $n$-sigraph $S_n = (G, \sigma)$ as follows (See also R. Rangarajan and P. Siva Kota Reddy [3]):

**Definition 1.1** Let $S_n = (G, \sigma)$ be an $n$-sigraph. Then,

(i) $S_n$ is identity balanced (or $i$-balanced), if product of $n$-tuples on each cycle of $S_n$ is the identity $n$-tuple, and

(ii) $S_n$ is balanced, if every cycle in $S_n$ contains an even number of non-identity edges.

**Note** An $i$-balanced $n$-sigraph need not be balanced and conversely.

The following characterization of $i$-balanced $n$-sigraphs is obtained in [4].

**Proposition 1.1 (E. Sampathkumar et al. [4])** An $n$-sigraph $S_n = (G, \sigma)$ is $i$-balanced if, and only if, it is possible to assign $n$-tuples to its vertices such that the $n$-tuple of each edge $uv$ is equal to the product of the $n$-tuples of $u$ and $v$.

The line $n$-sigraph $L(S_n)$ of an $n$-sigraph $S_n = (G, \sigma)$ is defined as follows (See [5]):

$L(S_n) = (L(G), \sigma')$, where for any edge $ee'$ in $L(G)$, $\sigma'(ee') = \sigma(e)\sigma(e')$.

The jump $n$-sigraph of an $n$-sigraph $S_n = (G, \sigma)$ is an $n$-sigraph $J(S_n) = (J(G), \sigma')$, where for any edge $ee'$ in $J(S_n)$, $\sigma'(ee') = \sigma(e)\sigma(e')$. This concept was introduced by E. Sampathkumar et al. [4]. This notion is analogous to the line $n$-sigraph defined above. Further, an $n$-sigraph $S_n = (G, \sigma)$ is called jump $n$-sigraph, if $S_n \cong J(S'_n)$ for some signed graph $S'$. In the following section, we shall present a characterization of jump $n$-sigraphs. The following result indicates the limitations of the notion of jump $n$-sigraphs defined above, since the entire class of $i$-unbalanced $n$-sigraphs is forbidden to be jump $n$-sigraphs.

**Proposition 1.2 (E. Sampathkumar et al. [4])** For any $n$-sigraph $S_n = (G, \sigma)$, its jump $n$-sigraph $J(S_n)$ is $i$-balanced.

§2. Characterization of Jump $n$-Sigraphs

The following result characterize $n$-sigraphs which are jump $n$-sigraphs.

**Proposition 2.1** An $n$-sigraph $S_n = (G, \sigma)$ is a jump $n$-sigraph if, and only if, $S_n$ is $i$-balanced.
Proof Suppose that $S_n$ is $i$-balanced and $G$ is a jump graph. Then there exists a graph $H$ such that $J(H) \cong G$. Since $S_n$ is $i$-balanced, by Proposition 1.1, there exists a marking $\mu$ of $G$ such that each edge $uv$ in $S_n$ satisfies $\sigma(uv) = \mu(u)\mu(v)$. Now consider the $n$-sigraph $S_n' = (H, \sigma')$, where for any edge $e$ in $H$, $\sigma'(e)$ is the marking of the corresponding vertex in $G$. Then clearly, $J(S_n') \cong S_n$. Hence $S_n$ is a jump $n$-sigraph.

Conversely, suppose that $S_n = (G, \sigma)$ is a jump $n$-sigraph. Then there exists a $n$-sigraph $S_n' = (H, \sigma')$ such that $J(S_n') \cong S_n$. Hence $G$ is the jump graph of $H$ and by Proposition 1.2, $S_n$ is $i$-balanced. $\square$

References