ABSTRACT: In [1] we define SMARANDACHE FACTOR PARTITION FUNCTION (SFP), as follows:

Let \( \alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_r \) be a set of \( r \) natural numbers and \( p_1, p_2, p_3, \ldots, p_r \) be arbitrarily chosen distinct primes then \( F(\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_r) \) called the Smarandache Factor Partition of \( (\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_r) \) is defined as the number of ways in which the number

\[
N = \prod_{i=1}^{r} p_i^{\alpha_i}
\]

could be expressed as the product of its' divisors. For simplicity, we denote \( F(\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_r) = F'(N) \), where

\[
N = \prod_{i=1}^{n} p_i^{\alpha_i}
\]

and \( p_r \) is the \( r^{th} \) prime. \( p_1 = 2, p_2 = 3 \) etc.

In this note an algorithm to list out all the SFPs of a number without missing any is developed.

DISCUSSION:

DEFINITION: \( F'_x(y) \) is defined as the number of those SFPs of \( y \) which involve terms not greater than \( x \).

If \( F_1 \) be a factor partition of \( y \):

\[
F_1 \longrightarrow x_1 \times x_2 \times x_3 \times \ldots \times x_r
\]

then \( F_1 \) is included in \( F'_x(y) \) iff
clearly $F'_x(y) \leq F'(y)$, The equality holds good iff $x \geq y$. 

Example: $F'_8(24) = 5$. Out of 7 only the last 5 are included in $F'_8(24)$.

(1) 24
(2) 12 $\times$ 2
(3) 8 $\times$ 3
(4) 6 $\times$ 4
(5) 6 $\times$ 2 $\times$ 2
(6) 4 $\times$ 3 $\times$ 2
(7) 3 $\times$ 2 $\times$ 2 $\times$ 2.

ALGORITHM: Let $d_1, d_2, d_3, \ldots, d_r$ be the divisors of $N$ in descending order. For listing the factor partitions following are the steps:

(A) (1) Start with $d_1 = N$.

(2) Write all the factor partitions involving $d_2$ and so on.

(B) While listing care should be taken that the terms from left to right should be written in descending order.

** At $d_k \geq N^{1/2} \geq d_{k+1}$, and onwards, step (B) will ensure that there is no repetition.

Example: $N = 36$, Divisors are 36, 18, 12, 9, 6, 4, 3, 2, 1.

36 $\rightarrow$ 36
18 $\rightarrow$ 18 $\times$ 2
12 $\rightarrow$ 12 $\times$ 3
9 $\rightarrow$ 9 $\times$ 4
9 $\times$ 2 $\times$ 2
6 $\rightarrow$ 6 $\times$ 6
6 $\rightarrow$ 6 $\times$ 3 $\times$ 2
--------------------------------- $d_k = N^{1/2}$
4 $\rightarrow$ 4 $\times$ 3 $\times$ 3
FORMULA FOR $F'(N)$

$$F'(N) = \sum_{d_r|N} F'_{d_r}(N/d_r) \quad \text{(8.1)}$$

Example:

$N = 216 = 2^33^3$

(1) $216 \quad \Rightarrow F_{216}(1) = 1$
(2) $108 \times 2 \quad \Rightarrow F_{108}(2) = 1$
(3) $72 \times 3 \quad \Rightarrow F_{72}(3) = 1$
(4) $54 \times 4 \quad \Rightarrow F_{54}(4) = 2$
(5) $54 \times 2 \times 2 \quad \Rightarrow F_{27}(8) = 3$
(6) $36 \times 6 \quad \Rightarrow F_{36}(6) = 2$
(7) $36 \times 3 \times 2 \quad \Rightarrow F_{36}(6) = 2$
(8) $27 \times 8 \quad \Rightarrow F_{27}(8) = 3$
(9) $27 \times 4 \times 2 \quad \Rightarrow F_{27}(8) = 3$
(10) $27 \times 2 \times 2 \times 2 \quad \Rightarrow F_{27}(8) = 3$
(11) $24 \times 9 \quad \Rightarrow F_{24}(9) = 2$
(12) $24 \times 3 \times 3 \quad \Rightarrow F_{24}(9) = 2$
(13) $18 \times 12 \quad \Rightarrow F_{18}(12) = 4$
(14) $18 \times 6 \times 2 \quad \Rightarrow F_{18}(12) = 4$
(15) $18 \times 4 \times 3 \quad \Rightarrow F_{18}(12) = 4$
(16) $18 \times 3 \times 2 \times 2 \quad \Rightarrow F_{18}(12) = 4$
(17) $12 \times 9 \times 2 \quad \Rightarrow F_{12}(18) = 3$
(18) $12 \times 6 \times 3 \quad \Rightarrow F_{12}(18) = 3$
(19) $12 \times 3 \times 3 \times 2 \quad \Rightarrow F_{12}(18) = 3$
(20) $9 \times 8 \times 3 \quad \Rightarrow F_{9}(24) = 5$
(21) $9 \times 6 \times 4 \quad \Rightarrow F_{9}(24) = 5$
(22) $9 \times 6 \times 2 \times 2 \quad \Rightarrow F_{9}(24) = 5$
(23) $9 \times 4 \times 3 \times 2 \quad \Rightarrow F_{9}(24) = 5$
(24) $9 \times 3 \times 2 \times 2 \quad \Rightarrow F_{9}(24) = 5$
(25) $8 \times 3 \times 3 \times 3 \quad \Rightarrow F_{8}(27) = 1$
(26) $6 \times 6 \times 6 \quad \Rightarrow F_{6}(36) = 4$
(27) $6 \times 6 \times 3 \times 2 \quad \Rightarrow F_{6}(36) = 4$
(28) $6 \times 4 \times 3 \times 3 \quad \Rightarrow F_{6}(36) = 4$
(29) $6 \times 3 \times 3 \times 2 \times 2 \quad \Rightarrow F_{6}(36) = 4$
(30) $4 \times 3 \times 3 \times 3 \times 2 \times 2 \quad \Rightarrow F_{4}(54) = 1$
(31) $3 \times 3 \times 3 \times 2 \times 2 \times 2 \quad \Rightarrow F_{1}(216) = 0$

288
\[ F'(216) = \sum_{d_r/N} F'_d(216/d_r) = 31 \]

**Remarks:** This algorithm would be quite helpful in developing a computer program for the listing of SFPs.

**REFERENCES:**


[2] "The Florentine Smarandache " Special Collection, Archives of American Mathematics, Centre for American History, University of Texas at Austin, USA.