Abstract: In this paper, we prove that there exist infinitely many positive integers $n$ satisfying $S(Z(n)) > Z(S(n))$ or $S(Z(n)) < Z(S(n))$.

Key words: Smarandache function, Pseudo-Smarandache function, composite function, difference.

For any positive integer $n$, let $S(n)$, $Z(n)$ denote the Smarandache function and the Pseudo-Smarandache function of $n$ respectively. In this paper, we prove the following results:

**Theorem 1:** There exist infinitely many $n$ satisfying $S(Z(n)) > Z(S(n))$.

**Theorem 2:** There exist infinitely many $n$ satisfying $S(Z(n)) < Z(S(n))$.

The above mentioned results solve Problem 21 of [1].

Proof of Theorem 1.
Let $p$ be an odd prime. If $n = (1/2)p(p+1)$, then we have

(1) $S(Z(n)) = S(Z((1/2)p(p+1))) = S(p) = p$

and

(2) $Z(S(n)) = Z(S((1/2)p(p+1))) = Z(p) = p-1$. 

We see from (1) and (2) that $S(Z(n)) > Z(S(n))$ for any odd prime $p$. It is a well-known fact that there exist infinitely many odd primes $p$. Thus, the theorem is proved.

Proof of Theorem 2.
If $n = p$, where $p$ is an odd prime, then we have

(3) $S(Z(n)) = S(Z(p)) = S(p-1) < p-1$

and

(4) $Z(S(n)) = Z(S(p)) = Z(p) = p-1$.

By (3) and (4), we get $S(Z(n)) < Z(S(n))$ for any $p$. Thus, the theorem is proved.

Reference


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