ON THE DIOPHANTINE EQUATION $S(n) = n$

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Abstract. Let $S(n)$ denote the Smarandache function of $n$. In this paper we prove that $S(n) = n$ if and only if $n = 1, 4$ or $p$, where $p$ is a prime.

Let $N$ be the set of all positive integers. For any positive integer $n$, let $S(n)$ denote the Smarandache function of $n$ (see[1]). It is an obvious fact that $S(n) \leq n$. In this paper we consider the diophantine equation

(1) $S(n) = n, \ n \in N.$

We prove a general result as follows:

Theorem. The equation (1) has only the solutions $n = 1, 4$ or $p$, where $p$ is a prime.

Proof. If $n = 1, 4$ or $p$, then (1) holds. Let $n$ be an another solution of (1). Then $n$ must be a composite integer with $n > 4$. Since $n$ is a composite integer, we have $n = uv$, where $u,v$ are integers satisfying $u \geq v \geq 2$. If $u \neq v$, then we get $n | uv$. It implies that $S(n) \leq u = n / v < n$, a contradiction.
If \( u = v \), then we have \( n = u^2 \) and \( n \mid (2u)! \).

It implies that \( S(n) \leq 2u \). Since \( n > 4 \), we get \( u > 2 \) and
\( S(n) \leq 2u < u^2 = n \), a contradiction. Thus, (1) has only the
solution \( n = 1, 4 \) or \( p \). The theorem is proved.

Reference

1. F Smarandache, A function in the number theory, Smarandache
function J. 1 (1990), No.1, 3 - 17.