TWO CONJECTURES CONCERNING EXTENTS OF SMARANDACHE FACTOR PARTITIONS

Maohua Le

Abstract . In this paper we verify two conjectures concerning extents of Smarandache factor partitions .

Key words . Smarandache factor partition , sum of length .

Let $p_1, p_2, \ldots, p_n$ be distinct primes , and let $a_1, a_2, \ldots, a_n$ be positive integers . Further , let

$$t = p_1^{a_1} p_2^{a_2} \cdots p_n^{a_n} , \tag{1}$$

and let $F(a_1, a_2, \cdots, a_n)$ denote the number of ways in which t could be expressed as the product of its divisors . Furthermore ,let

$$F(1\#n) = F(\underbrace{1, 1, \cdots, 1}_{n \text{ ones}}) . \tag{2}$$

If $d_1, d_2, \ldots, d_r$ are divisors of t and

$$t = d_1 d_2 \ldots d_r , \tag{3}$$

then (3) is called a Smarandache factor partition representation with length r . Further , let Extent $(F(1\#n))$ denote the sum of lengths of all Smarandache factor partition representations of $p_1 p_2 \cdots p_n$. In [2] , Murthy proposed the following two conjectures .

Conjecture 1 .

$$\text{Extent}(F(1\#n)) = F(1\#(n+1)) - F(1\#n) . \tag{4}$$

Conjecture 2 .

$$(5) \qquad \sum_{k=0}^{n} \text{Extent}\,(F(1\#n)) = F(1\#(n+1)) .$$

In this paper we verify the mentioned conjectures as follows.

Theorem. For any positive integer n, the identities (4) and (5) are true.

Proof. Let $Y(n)$ be the n-th Bell number. By the definitions of $F(1\#n)$ and $Y(n)$ (see [1]), we have

$$(6) \qquad F(1\#n)=Y(n) .$$

Let $L(r)$ be the number of Smarandache factor partitions of $p_1p_2\cdots p_n$ with length r. Then we have

$$(7) \qquad L(r)=S(n,r),$$

where $S(n,r)$ is the Stirling number of the second kind with parameters n and r. Since

$$(8) \qquad Y(n)=\sum_{r=1}^{n} S(n,r),$$

by (6), (7) and (8), we get

$$(9) \qquad F(1\#n)=Y(n)=\sum_{r=1}^{n} S(n,r)$$

and

$$(10) \qquad \text{Extent}F(1\#n)=\sum_{r=1}^{n} r\,S(n,r).$$

It is a well known fact that

$$(11) \qquad rS(n,r)=S(n+1,r)-S(n,r-1),$$

for $n \geqslant r \geqslant 1$ (see [1]). Notice that $S(n,n)=1$. Therefore, by (9),(10) and (11), we obtain

$$\text{Extent}\,(F(1\#n)) = \sum_{r=1}^{n} (S(n+1,r) - S(n,r-1))$$

$$(12) \qquad = \sum_{r=1}^{n} S(n+1,r) - \sum_{r=1}^{n} S(n,r-1) = (Y(n+1)) - S(n+1,n+1)$$

$$-(Y(n)-S(n,n)) = Y(n+1) - Y(n) = F(1\#(n+1)) - F(1\#n).$$

It implies that (4) holds.

On the other hand, we get from (4) that

$$\sum_{k=0}^{n} \text{Extent}\,(F(1\#k)) = 1 + \sum_{r=1}^{n} \text{Extent}\,(F(1\#r))$$

$$(13) \qquad = \sum_{r=1}^{n} (F(1\#(r+1)) - F(1\#r)) = F(1\#(n+1)).$$

Thus, (5) is also true. The theorem is proved.

References

[1] C. Jordan, Calculus of Finite Differences, Chelsea, 1965.

[2] A. Murthy, Length/extent of Smarandache factor partitions, Smarandache Notions J. 11(2000), 275-279.

Department of Mathematics
Zhanjiang Normal College
Zhanjiang, Guangdong
P.R. CHINA