THE FIRST DIGIT AND THE TRAILING DIGIT OF ELEMENTS OF THE SMARANDACHE DECONSTRUCTIVE SEQUENCE

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Abstract. In this paper we completely determine the first digit and the trailing digit of every term in the Smarandache deconstructive sequence.

Key words. Smarandache deconstructive sequence, first digit, trailing digit.

The Smarandache deconstructive sequence is constructed by sequentially repeating the digits 1,2,...,9 in the following way:

(1) \[ 1,23,456,7891,..., \]

which first appeared in [1]. For any positive integer \( n \), let \( SDS(n) \) be the \( n \)-th element of the Smarandache deconstructive sequence. Further, let \( F(n) \) and \( T(n) \) denote the first digit and the trailing digit of \( SDS(n) \) respectively. In this paper we completely determine \( F(n) \) and \( T(n) \) for any positive integer \( n \). We prove the following result.

Theorem. For any \( n \), we have

(2) \[
F(n) = \begin{cases} 
1, & \text{if } n \equiv 0,1 \pmod{9}, \\
2, & \text{if } n \equiv 2,5,8 \pmod{9}, \\
4, & \text{if } n \equiv 3,7 \pmod{9}, \\
7, & \text{if } n \equiv 4,6 \pmod{9},
\end{cases}
\]

and

(3) \[
T(n) = \begin{cases} 
1, & \text{if } n \equiv 1,4,7 \pmod{9}, \\
3, & \text{if } n \equiv 2,6 \pmod{9}, \\
6, & \text{if } n \equiv 3,5 \pmod{9}, \\
9, & \text{if } n \equiv 0,8 \pmod{9}.
\end{cases}
\]
Proof. By (1), we get

$$F(n) = 1+2+\cdots+(n-1)+1 \equiv \frac{n^2-n}{2} + 1 \pmod{9}.$$  

(4)

Let $a$ be a positive integer with $1 \leq a \leq 9$. We see from (4) that $F(n) = a$ if and only if $n$ is a solution of the congruence

$$n^2 - n \equiv a-1 \pmod{9}.$$  

(5)

Notice that (5) has only solutions

$$n \equiv \begin{cases} 
0, 1 \pmod{9}, & \text{if } a=1, \\
2, 5, 8 \pmod{9}, & \text{if } a=2, \\
3, 7 \pmod{9}, & \text{if } a=4, \\
4, 6 \pmod{9}, & \text{if } a=7.
\end{cases}$$  

(6)

Therefore, we obtain (2) by (6) immediately.

On the other hand, since

$$T(n) = \begin{cases} 
F(n+1), & \text{if } F(n+1)>1, \\
9, & \text{if } F(n+1)=1,
\end{cases}$$  

(7)

we see from (2) that (3) holds. Thus, the theorem is proved.

Reference


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