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Smarandache-Zagreb Index on Three Graph Operators

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Abstract: Many researchers have studied several operators on a connected graph in which one make an attempt on subdivision of its edges. In this paper, we show how the Zagreb indices, a particular case of Smarandache-Zagreb index of a graph changes with these operators and extended these results to obtain a relation connecting the Zagreb index on operators.

Key Words: Subdivision graph, ladder graph, Smarandache-Zagreb index, Zagreb index, graph operators.

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§1. Introduction

A single number that can be used to characterize some property of the graph of a molecule is called a topological index. For quite some time interest has been rising in the field of computational chemistry in topological indices that capture the structural essence of compounds. The interest in topological indices is mainly related to their use in nonempirical quantitative structure property relationships and quantitative structure activity relationships. The most elementary constituents of a (molecular) graph are vertices, edges, vertex-degrees, walks and paths [14]. They are the basis of many graph-theoretical invariants referred to (somewhat imprecisely) as topological index, which have found considerable use in Zagreb index.

Suppose $G = (V, E)$ is a connected graph with the vertex set V and the edge set E. Given an edge $e = \{u, v\}$ of G. Now we can define the *subdivision graph* $S(G)$ [2] as the graph obtained from G by replacing each of its edge by a path of length 2, or equivalently by inserting an additional vertex into each edge of G.

In [2], Cvetkocic defined the operators $R(G)$ and $Q(G)$ are as follows:

the operator $R(G)$ is the graph obtained from G by adding a new vertex corresponding to each edge of G and by joining each new vertex to the end vertices of the edge corresponding to it. The operator $Q(G)$ is the graph obtained from G by inserting a new vertex into each edge of G and by joining edges those pairs of these new vertices which lie on the adjacent edges of G (See also [16]).

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The Wiener index W belongs among the oldest graph-based structure-descriptors topological indices [12,17]. Numerous of its chemical applications were reported in [6,11] and its mathematical properties are well known [3]. Another structure-descriptor introduced long time ago [4] is the Zagreb index M_1 or more precisely, the first Zagreb index, because there exists also a second Zagreb index M_2. The research background of the Zagreb index together with its generalization appears in chemistry or mathematical chemistry.

In this paper, we concentrate on Zagreb index [8] with a pair of topological indices denoted $M_1(G)$ and $M_2(G)$ [1,9,10,13,18]. The *first Zagreb index*

$$M_1(G) = \sum_{u \in V(G)} d^2(u),$$

and the *second Zagreb index*

$$M_2(G) = \sum_{uv \in E(G)} d(u)d(v).$$

Generally, let G be a graph and H its a subgraph. The *Smarandache-Zagreb index of* G *relative to* H is defined by

$$M^S(G) = \sum_{u \in V(H)} d^2(u) + \sum_{(u,v) \in E(G \setminus H)} d(u)d(v).$$

Particularly, if $H = G$ or $H = \emptyset$, we get the first or second Zagreb index $M_1(G)$ and $M_2(G)$, respectively.

A *Tadpole graph* [15] $T_{n,k}$ is a graph obtained by joining a cycle graph C_n to a path of length k and a *wheel graph* W_{n+1} [7] is defined as the graph $K_1 + C_n$, where K_1 is the singleton graph and C_n is the cycle graph [8]. A ladder graph $L_n = K_2 \Box P_n$, where P_n is a path graph. For all terminologies and notations not defined in here, we refer to Harary [5].

§2. A relation connecting the Zagreb indices on $S(G)$, $R(G)$ and $Q(G)$ for the Tadpole graph and Wheel graph

We derive a relation connecting the Zagreb index with the subdivision graph $S(G)$ and two graph operators $R(G)$ and $Q(G)$, where, n, k are integers≥ 1 in this section.

Theorem 2.1 *The first Zagreb index*

$$M_1(S(T_{n,k})) = M_1(T_{n,k}) + 4(n+k).$$

Proof The Tadpole graph $T_{n,k}$ contains $n+k-2$ vertices of degree 2, one vertex of degree 3 and a pendent vertex. Hence $M_1(T_{n,k}) = 4n+4k+2$. The subdivision graph $S(T_{n,k})$ contains $n+k$ additional subdivision vertices. Hence

$$M_1(S(T_{n,k})) = M_1(T_{n,k}) + 4(n+k)$$
$$M_1(S(T_{n,k})) = 8n + 8k + 2. \tag{2.1}$$

□

Theorem 2.2 $M_1(R(T_{n,k})) = M_1(S(T_{n,k})) + 6(2n + 2k + 1)$.

Proof Each vertex v of degree l in $T_{n,k}$ is of degree $2l$ in $R(T_{n,k})$ and all the subdivision vertices in $S(T_{n,k})$ is of the same degree l in $R(T_{n,k})$. So,

$$M_1(R(T_{n,k})) = 16(n-1) + 16(k-1) + 4(n+k) + 40$$

$$M_1(R(T_{n,k})) = M_1(S(T_{n,k})) + 6(2n+2k+1) \tag{2.2}$$

from equation (2.1). □

Theorem 2.3 $M_1(Q(T_{n,k})) = \begin{cases} M_1(T_{n,k}) + 2M_1(S(T_{n,k})) + 14, & \textit{if } k = 1; \\ M_1(T_{n,k}) + M_1(S(T_{n,k})) + 16, & \textit{if } k \geq 2. \end{cases}$

Proof If $k = 1$, the graph $Q(T_{n,k})$ contains the sub graph $T_{n,k}$. The $n + k - 2$ subdivision vertices of degree 2 in $S(T_{n,k})$ are of double the degree in $Q(T_{n,k})$ and only 2 vertices of degree 5. So,

$$\begin{aligned} M_1(Q(T_{n,k})) &= 16(n + k - 2) + 50 + M_1(T_{n,k}) \\ &= 2(8n + 8k + 2) + M_1(T_{n,k}) + 14. \end{aligned}$$

Hence $M_1(Q(T_{n,k})) = M_1(T_{n,k}) + 2M_1(S(T_{n,k})) + 14$ if $k = 1$.

For $k \geq 2$, the $n+k-4$ subdivision vertices of degree 2 in $S(T_{n,k})$ is of degree 4 in $Q(T_{n,k})$ and only 3 vertices of degree 5 and one vertex of degree 3. Hence

$$M_1(Q(T_{n,k})) = M_1(T_{n,k}) + 16(n + k) + 20$$

and

$$M_1(Q(T_{n,k})) = M_1(T_{n,k}) + M_1(S(T_{n,k})) + 16, \text{if} \quad k \geq 2.$$

□

Theorem 2.4 $M_2(S(T_{n,k})) = \begin{cases} 2M_2(T_{n,k}) - 2, & \textit{if } k = 1; \\ 2M_2(T_{n,k}) - 4, & \textit{if } k \geq 2 \end{cases}$

Proof Among the $n + k$ vertices in $T_{n,k}$, only one vertex of degree 1, one vertex of degree 3 and $n+k-2$ vertices of degree 2, among which the $n+k-4$ pairs of vertices of degree 2, the three pairs of vertices of degree 2 and 3 and a pair of vertices of degree 2 and 1 are adjacent with each other for $k \geq 2$. Hence, $k \geq 2$,

$$M_2(T_{n,k}) = 4n + 4k + 4. \tag{2.3}$$

For $k = 1$, the $n - 1$ vertices of degree 2, one vertex of degree 3 and a pendent vertex among which there will be $n - 2$ pairs of vertices of degree 2, two pairs of vertices of degree 2 and 3 and a pair of vertices of degree 3 and 1 are adjacent with each other. So when $k = 1$,

$$M_2(T_{n,k}) = 4n + 7. \tag{2.4}$$

The new $n+k$ vertices of degree 2 is inserted in $T_{n,k}$ to construct $S(T_{n,k})$.

$$M_2(S(T_{n,k})) = 4(2n-2) + 4(2k-2) + 20 = 8n + 8k + 4 \tag{2.5}$$

Hence $M_2(S(T_{n,k})) = 2M_2(T_{n,k}) - 4$, for $k \geq 2$, from equation (2.3).

$$M_2(S(T_{n,k})) = 2M_2(T_{n,k}) - 2,$$

for $k = 1$, from equation (2.4). □

Theorem 2.5 $M_2(R(T_{n,k})) = \begin{cases} 4M_2(S(T_{n,k})) + 4, & \textit{if } k = 1; \\ 4M_2(S(T_{n,k})) + 8, & \textit{if } k \geq 2. \end{cases}$

Proof If $k = 1$, the $n-2$ pairs of vertices of degree 4, $2n-2$ pairs of vertices of degree 2 and 4, two pairs of vertices of degree 4 and 6, four pairs of vertices of degree 2 and 6 and a pair of vertices of degree 2 are adjacent to each other. So, $M_2(R(T_{n,k})) = 16(n-2) + 8(2k-2) + 8(2n-2) + 100$. Hence

$$M_2(R(T_{n,k})) = 32n + 32k + 36 = 4M_2(S(T_{n,k})) + 4. \tag{2.6}$$

if $k = 1$, from equation (2.5).

The vertices which are of degree 1 in $T_{n,k}$ are of degree $2l$ in $R(T_{n,k})$ and all the subdivision vertices in $S(T_{n,k})$ remains unaltered in $R(T_{n,k})$. In $R(T_{n,k})$, the $n + k - 4$ pairs of vertices of degree 4, $2n - 1$ pairs of degree 4 and 2, three pairs of vertices of degree 4 and 6, three pairs of vertices of degree 2 and 6 and one pair of vertices of degree 2 are adjacent to each other in $R(T_{n,k})$ when $k \geq 2$. Hence

$$M_2(R(T_{n,k})) = 16(n-2) + 8(2n-2) + 16(k-2) + 8(2k-2) + 120,$$

$$M_2(R(T_{n,k})) = 32n + 32k + 24, \tag{2.7}$$

$$M_2(R(T_{n,k})) = 4(8n + 8k + 4) + 8 = 4M_2(S(T_{n,k})) + 8,$$

if $k \geq 2$ from equation (2.5). □

Theorem 2.6 $M_2(Q(T_{n,k})) = \begin{cases} M_2(R(T_{n,k})) + 39, & \textit{if } k = 1; \\ M_2(R(T_{n,k})) + 46, & \textit{if } k = 2; \\ M_2(R(T_{n,k})) + 47, & \textit{if } k \geq 3. \end{cases}$

Proof We divide the proof of this theorem into three cases.

Case 1: When $k = 1$, the $n - 3$ pairs of vertices of degree 4, $2n - 4$ pairs of vertices of degree 2 and 4, one pair of vertices of degree 5, two pairs of vertices of degree 2 and 5, two pairs of vertices of degree 3 and 5, a pair of vertices of degree 3 and 4, a pair of vertices of 4 and 1, and four pairs of vertices of degree 4 and 5 are adjacent to each other in $Q(T_{n,k})$. Hence

$$\begin{aligned} M_2(Q(T_{n,k})) &= 16(n-3) + 8(2n-4) + 91 = 32n + 91 \\ &= (32n + 16k + 36) + 39) = M_2(R(T_{n,k})) + 39 \end{aligned}$$

from equation (2.6).

Case 2: When $k = 2$, the $n-3$ pairs of vertices of degree 4, $2n-4$ pairs of vertices of degree 2 and 4, three pair of vertices of degree 5, three pairs of vertices of degree 2 and 5, 4 pairs of vertices of degree 3 and 5, a pair of vertices of degree 1 and 3, a pair of vertices of 2 and 3, and two pairs of vertices of degree 4 and 5 are adjacent to each other in $Q(T_{n,k})$. Hence

$$\begin{aligned} M_2(Q(T_{n,k})) &= 16(n-3) + 8(2n-4) + 214 = 32n + 134 \\ &= (32n + 32k + 24) + 46 = M_2(R(T_{n,k})) + 46 \end{aligned}$$

from equation (2.7).

Case 3: When $k \geq 3$, there are $n+k-6$ pairs of vertices of degree 4, $2n+2k-8$ pairs of vertices of degree 2 and 4, three pairs of vertices of degree 5, three pairs of vertices of degree 2 and 5, three pairs of vertices of degree 3 and 5, a pair of vertices of degree 3 and 1, a pair of vertices of degree 2 and 3, a pair of vertices of degree 4 and 3 and three pairs of vertices of degree 4 and 5 are neighbours to each other in $Q(T_{n,k})$, with which,

$$\begin{aligned} M_2(Q(T_{n,k})) &= 16(n+k-6) + 8(2n+2k-8) + 231 = 32n + 32k + 71 \\ &= (32n + 32k + 24) + 47 = M_2(R(T_{n,k})) + 47 \end{aligned}$$

from equation (2.7). □

Theorem 2.7 *For the wheel graph* W_{n+1}, $M_1(S(W_{n+1})) = M_1(W_{n+1}) + 8n$.

Proof In W_{n+1}, it has n vertices of degree 3 and one vertex, the center of wheel of degree n. So,

$$M_1(W_{n+1}) = 9n + n^2. \tag{2.8}$$

By inserting a vertex in each edge of W_{n+1}, $M_1(S(W_{n+1})) = M_1(W_{n+1}) + 8n$.

$$M_1(S(W_{n+1})) = n^2 + 17n. \tag{2.9}$$

□

Theorem 2.8 $M_1(R(W_{n+1})) = 4M_1(S((W_{n+1}))) - 24n$.

Proof The degrees of the subdivision vertices in $S(W_{n+1})$ remains unaltered in $R(W_{n+1})$ and a vertex of degree l in W_{n+1}, is of degree $2l$ in $R(W_{n+1})$.

$$\begin{aligned} M_1(R(W_{n+1})) &= 4n^2 + 44n = 4(n^2 + 17n) - 24n \\ &= 4M_1(S((W_{n+1}))) - 24n. \end{aligned} \tag{2.10}$$

□

Theorem 2.9 $M_1(Q(W_{n+1})) = M_1(R((W_{n+1}))) + M_1(W_{n+1}) + n(n+1)^2$.

Proof Clearly $Q(W_{n+1})$ contains the subgraph W_{n+1}. Every subdivision vertex on the edges of the subgraph C_n in $S(W_{n+1})$ is adjacent with the four subdivision vertices, two on the spoke and two on the edges of C_n. Each of the subdivision vertex on the edges of C_n is of degree 6. Also every subdivision vertex on a spoke is adjacent with the $n-1$ subdivision vertices on the other spokes and is adjacent with 2 subdivision vertices on the edges of C_n with which the subdivision vertex on the spoke is of degree $n+3$. Therefore,

$$\begin{aligned} M_1(Q(W_{n+1})) &= M_1(W_{n+1}) + 36n + (n+3)^2 n \\ &= M_1(W_{n+1}) + (4n^2 + 44n) + (n^3 + 2n^2 + n) \end{aligned}$$

and

$$M_1(Q(W_{n+1})) = M_1(R((W_{n+1}))) + M_1(W_{n+1}) + n(n+1)^2$$

by equation (2.10). □

Theorem 2.10 $M_2(S(W_{n+1})) = M_2(W_{n+1}) + (9n - n^2)$.

Proof A vertex of degree 3 is adjacent with two vertices of degree 3 and with the hub of the wheel so that

$$M_2(W_{n+1}) = 3n^2 + 9n \tag{2.11}$$

In $S(W_{n+1})$, the $2n$ additional subdivision vertices are inserted. A vertex of degree 3 is adjacent with three vertices of degree 2 and all the subdivision vertices on the spoke are adjacent to the hub.

$$\begin{aligned} M_2(S(W_{n+1})) &= 2n^2 + 18n = (3n^2 + 9n) + (9n - n^2) \\ &= M_2(W_{n+1}) + (9n - n^2) \end{aligned} \tag{2.12}$$

from equation (2.11). □

Theorem 2.11 $M_2(R(W_{n+1})) = 4M_2(S((W_{n+1}))) + 8n^2$.

Proof The degrees of the subdivision vertices in $S(W_{n+1})$ remains the same in $R(W_{n+1})$ and every vertex in W_{n+1} is of double the degree in $R(W_{n+1})$. Every vertex of degree 6 is adjacent with the hub, two vertices of degree 6 and three subdivision vertices. The subdivision vertices on the spoke is adjacent with the hub. Hence

$$\begin{aligned} M_2(R(W_{n+1})) &= 72n + 16n^2 = 4(2n^2 + 18n) + 8n^2 \\ &= 4M_2(S((W_{n+1}))) + 8n^2 \end{aligned} \tag{2.13}$$

from equation (2.12). □

Theorem 2.12 *For a wheel graph W_{n+1},*

$$M_2(Q(W_{n+1})) = \frac{2M_2(R((W_{n+1})) + 3M_2(S(W_{n+1})) + (n^4 + 7n^3 + n^2 + 27n)}{2}.$$

Proof Every subdivision vertex in $S(W_{n+1})$(other than the subdivision vertices on the spoke) is of degree 6 and is adjacent with the the two vertices of degree 3, two vertices of degree 6, two vertices of degree $n+3$. A vertex of degree 3 is adjacent with the subdivision vertices on the spokes of degree $n+3$, and the subdivision vertices on the spoke is adjacent with the hub of the wheel and the $n-1$ subdivision vertices on the remaining spokes.

$$\begin{aligned} M_2(Q(W_{n+1})) &= \left[36+36+12(n+3)+3(n+3)+n(n+3)+\frac{((n+3)^2(n-1)}{2}\right]\times n \\ &= \frac{2(16n^2+72n)+3(2n^2+18n)+(n^4+7n^3+n^2+27n)}{2} \\ &= \frac{2M_2(R((W_{n+1}))+3M_2(S(W_{n+1}))+(n^4+7n^3+n^2+27n)}{2} \end{aligned}$$

by applying equations (2.12) and (2.13). □

§3. A relation connecting the Zagreb indices on $S(G)$, $R(G)$ and $Q(G)$ for the Ladder graph

In this section, we assume n being an integer≥ 3. When $n=1$, L_1 is the path P_1 and When $n=2$, L_2 is the cycle C_4 for which the the relations on the Zagreb index are same as in the case of P_k and C_n respectively.

Theorem 3.1 *For the ladder graph L_n, $M_1(S(L_n))=M_1(L_n)+4(3n-2)$.*

Proof The ladder graph L_n contains $2n-4$ vertices of degree 3 and four vertices of degree 2. So

$$M_1(L_n)=18n-20 \tag{3.1}$$

Since there are $3n-2$ edges in L_n there is an increase of $3n-2$ subdivision vertices in $S(L_n)$.

$$M_1(S(L_n))=M_1(L_n)+4(3n-2)=30n-28. \tag{3.2}$$

□

Theorem 3.2 $M_1(R(L_n))=2M_1(S(L_n))+(24n-32)$.

Proof The subdivision vertices in $S(L_n)$ retains the same degree in $R(L_n)$ and a vertex of degree l in L_n is of degree $2l$ in $R(L_n)$. Hence,

$$M_1(R(L_n))=2^2(3n-2)+72(n-2)+64$$

and

$$M_1(R(L_n))=84n-88=2M_1(S(L_n))+(24n-32) \tag{3.3}$$

from equation (3.2). □

Theorem 3.3 $M_1(Q(L_n))=M_1(R(L_n))+42n-88$.

Proof The graph $Q(L_n)$ contains the subgraph L_n. The subdivision vertices on the top and the bottom of the ladder say v_1 and v_k in $Q(L_n)$ is of degree 4 corresponding to the adjacencies and the nearest subdivision vertices of v_1 and v_k are of degree 5 corresponding to the 3 adjacent subdivision vertices in $S(L_n)$. The remaining $3n-8$ subdivision vertices are of degree 6. So

$$M_1(Q(L_n)) = M_1(L_n) + 132 + 6^2(3n-8) = M_1(R(L_n)) + 42n - 88,$$

from equation (3.3). □

Theorem 3.4 $M_2(S(L_n)) = M_2(L_n) + 9n$.

Proof In L_n, two vertices of degree 2 are adjacent with a vertex of degree 3 and a vertex of degree 2. The $2n-8$ pairs of vertices of degree 3 are adjacent with the vertex of degree 2. Hence,

$$M_2(L_n) = 32 + 18(n-3) + 9(n-2) = 27n - 40. \tag{3.4}$$

In $S(L_n)$, eight pairs of vertices of degree 2, $6n-12$ pairs of vertices of degree 2 and three are adjacent to each other. So

$$M_2(S(L_n)) = 32 + 6(6n-12) = M_2(L_n) + 9n \tag{3.5}$$

from equation (3.4). □

Theorem 3.5 $M_2(R(L_n)) = 5M_2(S(L_n)) - 40$.

Proof The degrees of the subdivision vertices in $S(L_n)$ is unaffected in $R(L_n)$, and all the vertices in L_n become double the degree in $R(L_n)$. In $R(L_n)$, eight pairs of vertices of degree 4 and 2, $6n-12$ pairs of vertices of degree 2 and 6, two pairs of vertices of degree 4, $3n-8$ pairs of vertices of degree 6 , four pairs of vertices of degree 4 and six are adjacent to each other. So,

$$M_2(R(L_n)) = 180n - 240 = 5M_2(S(L_n)) - 40 \tag{3.6}$$

from equation (3.5). □

Theorem 18 $M_2(Q(L_n))= \begin{cases} 2M_2(R(L_n)) + (-36n-44), & \text{if } n=3; \\ M_2(R(L_n)) + (-72n+548), & \text{if } n=4; \\ 2M_2(R(L_n)) + (-36n-4), & \text{if } n \geq 5. \end{cases}$

Proof We divide the proof of this result into three cases following.

Case 1: If $n=3$, the $Q(L_n)$ contains the subgraph L_n. In $Q(L_n)$, there are four pairs of vertices of degree 4 and 5, four pairs of vertices of degree 4 and 2, four pairs of vertices of degree 2 and 5, four pairs of vertices of degree 5 and 6 , two pairs of vertices of degree 5, four pairs of vertices of degree 5 and 3, $6n-16$ pairs of vertices of degree 3 and 6, the $6n-18$ pairs of vertices of vertices of degree 6 are adjacent to each other. Hence,

$$M_2(Q(L_3)) = 382 + 18(6n-16) + 36(6n-18)$$

and

$$M_2(Q(L_n)) = 2M_2(R(L_n)) + (-36n - 44)$$

from equation (3.6).

Case 2: If $n = 4$, four pairs of vertices of degree 4 and 5, four pairs of vertices of degree 4 and 2, four pairs of vertices of degree 2 and 5, eight pairs of vertices of degree 5 and 6, four pairs of vertices of degree 5 and 3, four pairs of vertices of degree 3 and 6 and $6n - 20$ pairs of vertices of degree 6 are adjacent to each other in $Q(L_n)$. Hence,

$$M_2(Q(L_n)) = 596 + 36(6n - 16) = 308 + 108n$$

and

$$M_2(Q(L_n)) = M_2(R(L_n)) + (548 - 72n)$$

from equation (3.6).

Case 3: If $n \geq 5$, $Q(L_n)$ contains 4 pairs of vertices of degree 4 and 5, four pairs of vertices of degree 4 and 2, four pairs of vertices of degree 2 and 5, eight pairs of vertices of degree 5 and 6, four pairs of vertices of degree 5 and 3 , $6n - 16$ pairs of vertices of degree 3 and 6, $6n - 18$ pairs of vertices of degree 6 are adjacent to each other. Hence,

$$\begin{aligned} M_2(Q(L_n)) &= 452 + 18(6n - 16) + 36(6n - 18) \\ &= 2M_2(R(L_n)) + (-36n - 4) \end{aligned}$$

by equation (3.6). $\square$

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