TWO SMARANDACHE SERIES

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Abstract. In this paper we consider the convergence for two Smarandache series.

Key words. Smarandache reciprocal series, convergence.

Let \( A = \{a(n)\}_{n=1}^{\infty} \) and \( B = \{b(n)\}_{n=1}^{\infty} \) be two Smarandache sequences. Then the series
\[
S(A,B) = \sum_{n=1}^{\infty} \frac{a(n)}{b(n)}
\]
is called the Smarandache series of \( A \) and \( B \). Recently, Castillo [1] proposed the following two open problems.

**Problem 1.** Is the series
\[
S_1 = \frac{1}{1} + \frac{1}{12} + \frac{1}{123} + \frac{1}{1234} + \ldots
\]
convergent?

**Problem 2.** Is the series
\[
S_2 = \frac{1}{1} + \frac{1}{21} + \frac{1}{321} + \frac{1}{4321} + \ldots
\]
convergent?

In this paper we completely solve the mentioned problems as follows.

**Theorem.** The series \( S_1 \) is convergent and the series \( S_2 \) is divergent.

**Proof.** Let \( r(n) = \frac{1}{12 \cdots n} \) for any positive integer \( n \).

Since
by D'Alembert's criterion, we see from (3) that $S_1$ is convergent.

Let $s(n) = \frac{12 \cdots (n-1)n}{n(n-1) \cdots 21}$ for any positive integer $n$. If $n = 10^t + 1$, where $t$ is a positive integer, then we have

$$S(n) = \frac{12 \cdots (10 \cdots 01)}{(10 \cdots 01) \cdots 21} > 1.$$  

(4)

Therefore, by (4), we get from (2) that

$$S_2 = \sum_{n=1}^{\infty} s(n) > \sum_{t=1}^{\infty} s(10^t + 1) > \sum_{t=1}^{\infty} 1 = \infty.$$  

(5)

Thus, the series $S_2$ is divergent. The theorem is proved.

Reference

http://www.gallup.unm.edu/~smarandache/SERIES.TXT.

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