About a new Smarandache-type sequence

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In this paper we will discuss about a problem that I asked about 8 years ago, when I was interested mainly in computer science. The computers can operate with 256 characters and all of them has an ASCII code which is an integer from 0 to 255. If you press ALT key and you type a number, the character of the number will appear. But if you type a number that is greater than 255, the computer will calculate the remainder after division by 256, and the corresponding character will appear. "Can you show each character by pressing the same number key k-times?" - asked I.

It is quite simple to solve this problem, and the answer is no. Before proving this we generalize the problem to t-size ASCII code-tables, the codes are from 0 to t-1.

We shall use the following notations: $\mathbf{N}$ is the set of the positive integers, $\mathbf{N}_0=\mathbf{N}\cup\{0\}$, $\mathbf{Z}$ is the set of the integers and $\mathbf{Z}_t=\{0,1,\dots,t-1\}$.

Now let us see the generalized problem. Define $f:\mathbf{N}\to\mathbf{N}$ as

$$f(t)=|H_t|$$

where

$$H_t=\left\{x\in\mathbf{Z}_t : a\sum_{i=0}^{k}10^i\equiv x \pmod t \quad \text{for some } k\in\mathbf{N}_0 \text{ and } a\in\{0,1,\dots,9\}\right\}$$

Our first question was $f(256)$, and the generalized problem is to calculate $f(t)$ in generality.

It is clear that $f(t)=t$ if $t\le 10$, and $f(t)\ge 10$ if $t>10$. Now let us examine some special cases.

Let $t=2^r5^s$, $r,s\in\mathbf{N}_0$ but at least one of them is not zero. Denote by w the maximum of r and s. If $k\ge w$, then $t|10^k$, because $10^k=2^k5^k$. So

$$a\sum_{i=0}^{k}10^i\equiv a\sum_{i=0}^{w-1}10^i \pmod t,$$

thus

$$H_t=\left\{x\in\mathbf{Z}_t : a\sum_{i=0}^{k}10^i\equiv x \pmod t \quad k\in\mathbf{Z}_w \text{ and } a\in\{0,1,\dots,9\}\right\}$$

So $|H_t|\le 10w$, moreover $|H_t|\le 9w+1$, because if $a=0$, then the value of k is insignificant.

We got a sufficient condition for $f(t)<t$, that is $t>9w+1$. It is satisfied if $r\geq 6$ or $s\geq 2$ or $r=2,3,4,5$ and $s=1$. If $r=0,1$ and $s<2$, or $r=2,3$ and $s=0$ then $t\leq 10$ so we have only 2 cases to examine: $t=16$ and $t=32$. In the former, $f(16)=16$, because $10\equiv 666$, $12\equiv 44$, $13\equiv 77$, $14\equiv 222$, $15\equiv 111$ (mod 16), but in the latter $f(32)<32$; for example anybody can verify that $16\notin H_{32}$ (by the way $f(32)=26$). Specially we got the answer for our first question: $f(256)=f(2^8)<256$, because $256>9\cdot 8+1$. In fact $f(256)=60$. (Some of these results are computed by a Pascal program.)

In the next case let $t=10^r+1$, $r\geq 1$. Take a number $d=a(1+10+100+...+10^i)$. Now it is easy to see, that the remainder of d may be 0, a, $10a$, $10a+a$, $100a$, $100a+10a$, $100a+10a+a$, $...,10^{r-1}a+10^{r-2}a+...+a$, so the remainder is less than 10^r. Thus $10^r\notin H_t$, so we got $f(t)<t$.

Now we will show a simple algrithm to calculate $f(t)$. Fix a and let R_i be the remainder of $10^i a$ and S_i the sum of the first i elements of the sequence $\{R_n\}$ (mod t). It is obvious that both $\{R_n\}$ and $\{S_n\}$ are periodic, so let l be the end of the first period of $\{S_n\}$. ($S_l=S_{l'}$ for some $l'<l$.)

Then

$$H_t=\left\{x\in \mathbf{Z}_t : a\sum_{i=0}^{k}10^i\equiv x \pmod t \quad k\in \mathbf{Z}_l \text{ and } a\in\{0,1,\ldots,9\}\right\}$$

so it is easy to calculate $|H_t|$. The time complexity of this algorithm is at most $O(n^2)$.

Finally let us see a table of the values of the function f, computed by a computer.

t	1...10	11	12	13	14	15	16	17	18	19	20	...	256
$f(t)$	1...10	10	12	13	14	15	16	17	18	19	15	...	60

10≡666 (mod 12)	10≡88 (mod 13) 12≡77 (mod 13)	10≡66 (mod 14) 12≡222 (mod 14) 13≡55 (mod 14)	10≡55 (mod 15) 12≡222 (mod 15) 13≡88 (mod 15) 14≡44 (mod 15)
10≡666 (mod 16) 12≡44 (mod 16) 13≡77 (mod 16) 14≡222 (mod 16) 15≡111 (mod 16)	10≡44 (mod 17) 12≡777 (mod 17) 13≡999 (mod 17) 14≡99 (mod 17) 15≡66 (mod 17) 16≡33 (mod 17)	10≡22222 (mod 18) 12≡66 (mod 18) 13≡1111 (mod 18) 14≡8888 (mod 18) 15≡33 (mod 18) 16≡88 (mod 18) 17≡77777 (mod 18)	10≡333 (mod 19) 12≡88 (mod 19) 13≡222 (mod 19) 14≡33 (mod 19) 15≡8888 (mod 19) 16≡111 (mod 19) 17≡55 (mod 19) 18≡2222 (mod 19)

Now we still have the question: for which numbers $f(t)=t$? Are there finite or infinite many t with the property above? Is there a better (faster) algorithm to calculate $f(t)$? Is there an explicit formula? Can anyone answer?

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