Smarandache Directionally $n$-Signed Graphs — A Survey

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Abstract: Let $G = (V, E)$ be a graph. By directional labeling (or $d$-labeling) of an edge $x = uv$ of $G$ by an ordered $n$-tuple $(a_1, a_2, \cdots, a_n)$, we mean a labeling of the edge $x$ such that we consider the label on $uv$ as $(a_1, a_2, \cdots, a_n)$ in the direction from $u$ to $v$, and the label on $x$ as $(a_n, a_{n-1}, \cdots, a_1)$ in the direction from $v$ to $u$. In this survey, we study graphs, called $(n, d)$-sigraphs, in which every edge is $d$-labeled by an $n$-tuple $(a_1, a_2, \cdots, a_n)$, where $a_k \in \{+, -\}$, for $1 \leq k \leq n$. Several variations and characterizations of directionally $n$-signed graphs have been proposed and studied. These include the various notions of balance and others.

Key Words: Signed graphs, directional labeling, complementation, balance.

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§1. Introduction

For graph theory terminology and notation in this paper we follow the book [3]. All graphs considered here are finite and simple. There are two ways of labeling the edges of a graph by an ordered $n$-tuple $(a_1, a_2, \cdots, a_n)$ (See [10]).

1. Undirected labeling or labeling. This is a labeling of each edge $uv$ of $G$ by an ordered $n$-tuple $(a_1, a_2, \cdots, a_n)$ such that we consider the label on $uv$ as $(a_1, a_2, \cdots, a_n)$ irrespective of the direction from $u$ to $v$ or $v$ to $u$.

2. Directional labeling or $d$-labeling. This is a labeling of each edge $uv$ of $G$ by an ordered $n$-tuple $(a_1, a_2, \cdots, a_n)$ such that we consider the label on $uv$ as $(a_1, a_2, \cdots, a_n)$ in the direction from $u$ to $v$, and $(a_n, a_{n-1}, \cdots, a_1)$ in the direction from $v$ to $u$.

Note that the $d$-labeling of edges of $G$ by ordered $n$-tuples is equivalent to labeling the symmetric digraph $\overrightarrow{G} = (V, \overrightarrow{E})$, where $uv$ is a symmetric arc in $\overrightarrow{G}$ if, and only if, $uv$ is an edge in $G$, so that if $(a_1, a_2, \cdots, a_n)$ is the $d$-label on $uv$ in $G$, then the labels on the arcs $\overrightarrow{uv}$ and $\overrightarrow{vu}$ are $(a_1, a_2, \cdots, a_n)$ and $(a_n, a_{n-1}, \cdots, a_1)$ respectively.

Let $H_n$ be the $n$-fold sign group. $H_n = \{+, -\}^n = \{(a_1, a_2, \cdots, a_n) : a_1, a_2, \cdots, a_n \in \{+,-\}\}$ with coordinate-wise multiplication. Thus, writing $a = (a_1, a_2, \cdots, a_n)$ and $t = \{$

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(t_1, t_2, \cdots, t_n) \text{ then } at := (a_1t_1, a_2t_2, \cdots, a_nt_n). \text{ For any } t \in H_n, \text{ the action of } t \text{ on } H_n \text{ is } a' = at, \text{ the co-ordinate-wise product.}

Let \( n \geq 1 \) be a positive integer. An \( n \)-signed graph (\( n \)-signed digraph) is a graph \( G = (V, E) \) in which each edge (arc) is labeled by an ordered \( n \)-tuple of signs, i.e., an element of \( H_n \). A signed graph \( G = (V, E) \) is a graph in which each edge is labeled by + or −. Thus a 1-signed graph is a signed graph. Signed graphs are well studied in literature (See for example [1, 4-7, 13-21, 23, 24].

In this survey, we study graphs in which each edge is labeled by an ordered \( n \)-tuple \( a = (a_1, a_2, \cdots, a_n) \) of signs (i.e, an element of \( H_n \)) in one direction but in the other direction its label is the reverse: \( a' = (a_n, a_{n-1}, \cdots, a_1) \), called directionally labeled \( n \)-signed graphs (or \( (n,d) \)-signed graphs).

Note that an \( n \)-signed graph \( G = (V, E) \) can be considered as a symmetric digraph \( \overrightarrow{G} = (V, \overrightarrow{E}) \), where both \( \overrightarrow{uv} \) and \( \overrightarrow{vu} \) are arcs if, and only if, \( uv \) is an edge in \( G \). Further, if an edge \( uv \) in \( G \) is labeled by the \( n \)-tuple \( (a_1, a_2, \cdots, a_n) \), then in \( \overrightarrow{G} \) both the arcs \( \overrightarrow{uv} \) and \( \overrightarrow{vu} \) are labeled by the \( n \)-tuple \( (a_1, a_2, \cdots, a_n) \).

In [1], the authors study voltage graph defined as follows: A voltage graph is an ordered triple \( \overrightarrow{G} = (V, \overrightarrow{E}, M) \), where \( V \) and \( \overrightarrow{E} \) are the vertex set and arc set respectively and \( M \) is a group. Further, each arc is labeled by an element of the group \( M \) so that if an arc \( \overrightarrow{uv} \) is labeled by an element \( a \in M \), then the arc \( \overrightarrow{vu} \) is labeled by its inverse, \( a^{-1} \).

Since each \( n \)-tuple \( (a_1, a_2, \cdots, a_n) \) is its own inverse in the group \( H_n \), we can regard an \( n \)-signed graph \( G = (V, E) \) as a voltage graph \( \overrightarrow{G} = (V, \overrightarrow{E}, H_n) \) as defined above. Note that the \( d \)-labeling of edges in an \( (n,d) \)-signed graph considering the edges as symmetric directed arcs is different from the above labeling. For example, consider a \( (4,d) \)-signed graph in Figure 1. As mentioned above, this can also be represented by a symmetric 4-signed digraph. Note that this is not a voltage graph as defined in [1], since for example; the label on \( \overrightarrow{v_2v_1} \) is not the (group) inverse of the label on \( \overrightarrow{v_1v_2} \).

In [8-9], the authors initiated a study of \( (3,d) \) and \( (4,d) \)-Signed graphs. Also, discussed some applications of \( (3,d) \) and \( (4,d) \)-Signed graphs in real life situations.

In [10], the authors introduced the notion of complementation and generalize the notion of balance in signed graphs to the directionally \( n \)-signed graphs. In this context, the authors look upon two kinds of complementation: complementing some or all of the signs, and reversing the order of the signs on each edge. Also given some motivation to study \( (n,d) \)-signed graphs in connection with relations among human beings in society.

In [10], the authors defined complementation and isomorphism for \( (n,d) \)-signed graphs as
follows: For any \( t \in H_n \), the \( t \)-complement of \( a = (a_1, a_2, \cdots, a_n) \) is: \( a^t = at \). The reversal of \( a = (a_1, a_2, \cdots, a_n) \) is: \( a^r = (a_n, a_{n-1}, \cdots, a_1) \). For any \( T \subseteq H_n \), and \( t \in H_n \), the \( t \)-complement of \( T \) is \( T^t = \{a^t : a \in T \} \).

For any \( t \in H_n \), the \( t \)-complement of an \((n, d)\)-signed graph \( G = (V, E) \), written \( G^t \), is the same graph but with each edge label \( a = (a_1, a_2, \cdots, a_n) \) replaced by \( a^t \). The reversal \( G^r \) is the same graph but with each edge label \( a = (a_1, a_2, \cdots, a_n) \) replaced by \( a^r \).

Let \( G = (V, E) \) and \( G' = (V', E') \) be two \((n, d)\)-signed graphs. Then \( G \) is said to be isomorphic to \( G' \) and we write \( G \cong G' \), if there exists a bijection \( \phi : V \to V' \) such that if \( uv \) is an edge in \( G \) which is \( d \)-labeled by \( a = (a_1, a_2, \cdots, a_n) \), then \( \phi(u)\phi(v) \) is an edge in \( G' \) which is \( d \)-labeled by \( a \), and conversely.

For each \( t \in H_n \), an \((n, d)\)-signed graph \( G = (V, E) \) is \textit{t-self complementary}, if \( G \cong G^t \). Further, \( G \) is \textit{self reverse}, if \( G \cong G^r \).

**Proposition 1.1** (E. Sampathkumar et al. [10]) *For all \( t \in H_n \), an \((n, d)\)-signed graph \( G = (V, E) \) is \textit{t-self complementary} if, and only if, \( G^r \) is \textit{t-self complementary}, for any \( a \in H_n \).*

For any cycle \( C \) in \( G \), let \( \mathcal{P}(C) \) [10] denotes the product of the \( n \)-tuples on \( C \) given by:

\[
(\alpha_{11}, \alpha_{12}, \cdots, \alpha_{1n})(\alpha_{21}, \alpha_{22}, \cdots, \alpha_{2n}) \cdots (\alpha_{m1}, \alpha_{m2}, \cdots, \alpha_{mn})
\]

Similarly, for any path \( P \) in \( G \), \( \mathcal{P}(P) \) denotes the product of the \( n \)-tuples on \( P \) given by:

\[
(\alpha_{11}, \alpha_{12}, \cdots, \alpha_{1n})(\alpha_{21}, \alpha_{22}, \cdots, \alpha_{2n}) \cdots (\alpha_{m1}, \alpha_{m2}, \cdots, \alpha_{m1}, \alpha_{m1}, \cdots, \alpha_{m1})
\]

An \( n \)-tuple \( (\alpha_1, \alpha_2, \cdots, \alpha_n) \) is \textit{identity} \( n \)-tuple, if each \( \alpha_k = + \), for \( 1 \leq k \leq n \), otherwise it is a \textit{non-identity} \( n \)-tuple. Further an \( n \)-tuple \( a = (a_1, a_2, \cdots, a_n) \) is \textit{symmetric}, if \( a^r = a \), otherwise it is a \textit{non-symmetric} \( n \)-tuple. In \((n, d)\)-signed graph \( G = (V, E) \) an edge labeled with the identity \( n \)-tuple is called an \textit{identity edge}, otherwise it is a \textit{non-identity edge}.

Note that the above products \( \mathcal{P}(C) \) (\( \mathcal{P}(P) \)) as well as \( \mathcal{P}(C) \) (\( \mathcal{P}(P) \)) are \( n \)-tuples. In general, these two products need not be equal.

§2. Balance in an \((n, d)\)-Signed Graph

In [10], the authors defined two notions of balance in an \((n, d)\)-signed graph \( G = (V, E) \) as follows:

**Definition 2.1** Let \( G = (V, E) \) be an \((n, d)\)-sigraph. Then,

(i) \( G \) is \textit{identity balanced} (or \( i \)-balanced), if \( \mathcal{P}(C) \) on each cycle of \( G \) is the identity \( n \)-tuple, and

(ii) \( G \) is \textit{balanced}, if every cycle contains an even number of non-identity edges.

**Note:** An \( i \)-balanced \((n, d)\)-sigraph need not be balanced and conversely. For example, consider the \((4, d)\)-sigraphs in Figure 2. In Figure 2(a) \( G \) is an \( i \)-balanced but not balanced, and in Figure 2(b) \( G \) is balanced but not \( i \)-balanced.
2.1 Criteria for balance

An \((n,d)\)-signed graph \(G = (V,E)\) is \(i\)-balanced if each non-identity \(n\)-tuple appears an even number of times in \(P(C)\) on any cycle of \(G\).

However, the converse is not true. For example see Figure 3(a). In Figure 3(b), the number of non-identity 4-tuples is even and hence it is balanced. But it is not \(i\)-balanced, since the 4-tuple \((+ + - -)\) (as well as \((- - + +)\)) does not appear an even number of times in \(P(C)\) of 4-tuples.

In \([10]\), the authors obtained following characterizations of balanced and \(i\)-balanced \((n,d)\)-sigraphs:

**Proposition 2.2** (E.Sampathkumar et al. [10]) An \((n,d)\)-signed graph \(G = (V,E)\) is balanced if, and only if, there exists a partition \(V_1 \cup V_2\) of \(V\) such that each identity edge joins two vertices in \(V_1\) or \(V_2\), and each non-identity edge joins a vertex of \(V_1\) and a vertex of \(V_2\).

As earlier discussed, let \(P(C)\) denote the product of the \(n\)-tuples in \(P(C)\) on any cycle \(C\) in an \((n,d)\)-sigraph \(G = (V,E)\).

**Theorem 2.3** (E.Sampathkumar et al. [10]) An \((n,d)\)-signed graph \(G = (V,E)\) is \(i\)-balanced if, and only if, for each \(k\), \(1 \leq k \leq n\), the number of \(n\)-tuples in \(P(C)\) whose \(k^{th}\) co-ordinate is \(-\) is even.

In \(H_n\), let \(S_1\) denote the set of non-identity symmetric \(n\)-tuples and \(S_2\) denote the set of non-symmetric \(n\)-tuples. The product of all \(n\)-tuples in each \(S_k\), \(1 \leq k \leq 2\) is the identity \(n\)-tuple.

![Fig.2](image1.png)

![Fig.3](image2.png)
Theorem 2.4 (E. Sampathkumar et al. [10]) An \((n,d)\)-signed graph \(G = (V,E)\) is \(i\)-balanced, if both of the following hold:

(i) In \(P(C)\), each \(n\)-tuple in \(S_1\) occurs an even number of times, or each \(n\)-tuple in \(S_1\) occurs odd number of times (the same parity, or equal mod 2).

(ii) In \(P(C)\), each \(n\)-tuple in \(S_2\) occurs an even number of times, or each \(n\)-tuple in \(S_2\) occurs an odd number of times.

In [11], the authors obtained another characterization of \(i\)-balanced \((n,d)\)-signed graphs as follows:

Theorem 2.5 (E. Sampathkumar et al. [11]) An \((n,d)\)-signed graph \(G = (V,E)\) is \(i\)-balanced if, and only if, any two vertices \(u\) and \(v\) have the property that for any two edge distinct \(u-v\) paths \(P_1 = (u = u_0, u_1, \cdots, u_m = v)\) and \(P_2 = (u = v_0, v_1, \cdots, v_n = v)\) in \(G\), \(P(P_1) = (P(P_2))^r\) and \(P(P_2) = (P(P_1))^r\).

From the above result, the following are the easy consequences:

Corollary 2.6 In an \(i\)-balanced \((n,d)\)-signed graph \(G\) if two vertices are joined by at least 3 paths then the product of \(n\) tuples on any paths joining them must be symmetric.

A graph \(G = (V,E)\) is said to be \(k\)-connected for some positive integer \(k\), if between any two vertices there exists at least \(k\) disjoint paths joining them.

Corollary 2.7 If the underlying graph of an \(i\)-balanced \((n,d)\)-signed graph is 3-connected, then all the edges in \(G\) must be labeled by a symmetric \(n\)-tuple.

Corollary 2.8 A complete \((n,d)\)-signed graph on \(p \geq 4\) is \(i\)-balanced then all the edges must be labeled by symmetric \(n\)-tuple.

2.2 Complete \((n,d)\)-Signed Graphs

In [11], the authors defined: an \((n,d)\)-sigraph is complete, if its underlying graph is complete. Based on the complete \((n,d)\)-signed graphs, the authors proved the following results: An \((n,d)\)-signed graph is complete, if its underlying graph is complete.

Proposition 2.9 (E. Sampathkumar et al. [11]) The four triangles constructed on four vertices \(\{a,b,c,d\}\) can be directed so that given any pair of vertices say \((a,b)\) the product of the edges of these 4 directed triangles is the product of the \(n\)-tuples on the arcs \(ab\) and \(ba\).

Corollary 2.10 The product of the \(n\)-tuples of the four triangles constructed on four vertices \(\{a,b,c,d\}\) is identity if at least one edge is labeled by a symmetric \(n\)-tuple.

The \(i\)-balance base with axis \(a\) of a complete \((n,d)\)-signed graph \(G = (V,E)\) consists list of the product of the \(n\)-tuples on the triangles containing \(a\) [11].

Theorem 2.11 (E. Sampathkumar et al. [11]) If the \(i\)-balance base with axis \(a\) and \(n\)-tuple of an
edge adjacent to a is known, the product of the n-tuples on all the triangles of G can be deduced from it.

In the statement of above result, it is not necessary to know the n-tuple of an edge incident at a. But it is sufficient that an edge incident at a is a symmetric n-tuple.

**Theorem 2.12** (E.Sampathkumar et al. [11]) A complete \((n, d)\)-sigraph \(G = (V, E)\) is \(i\)-balanced if, and only if, all the triangles of a base are identity.

**Theorem 2.13** (E.Sampathkumar et al. [11]) The number of \(i\)-balanced complete \((n, d)\)-sigraphs of \(m\) vertices is \(p^{m-1}\), where \(p = 2^\lfloor n/2 \rfloor\).

§ 3. Path Balance in \((n, d)\)-Signed Graphs

In [11], E.Sampathkumar et al. defined the path balance in an \((n, d)\)-signed graphs as follows:

Let \(G = (V, E)\) be an \((n, d)\)-sigraph. Then \(G\) is

1. Path \(i\)-balanced, if any two vertices \(u\) and \(v\) satisfy the property that for any \(u - v\) paths \(P_1\) and \(P_2\) from \(u\) to \(v\), \(\mathcal{P}(P_1) = \mathcal{P}(P_2)\).

2. Path balanced if any two vertices \(u\) and \(v\) satisfy the property that for any \(u - v\) paths \(P_1\) and \(P_2\) from \(u\) to \(v\) have same number of non identity n-tuples.

Clearly, the notion of path balance and balance coincides. That is an \((n, d)\)-signed graph is balanced if, and only if, \(G\) is path balanced.

If an \((n, d)\) signed graph \(G\) is \(i\)-balanced then \(G\) need not be path \(i\)-balanced and conversely.

In [11], the authors obtained the characterization path \(i\)-balanced \((n, d)\)-signed graphs as follows:

**Theorem 3.1** (Characterization of path \(i\)-balanced \((n; d)\) signed graphs) An \((n, d)\)-signed graph is path \(i\)-balanced if, and only if, any two vertices \(u\) and \(v\) satisfy the property that for any two vertex disjoint \(u - v\) paths \(P_1\) and \(P_2\) from \(u\) to \(v\), \(\mathcal{P}(\overline{P_1}) = \mathcal{P}(\overline{P_2})\).

§ 4. Local Balance in \((n, d)\)-Signed Graphs

The notion of local balance in signed graph was introduced by F. Harary [5]. A signed graph \(S = (G, \sigma)\) is locally at a vertex \(v\), or \(S\) is balanced at \(v\), if all cycles containing \(v\) are balanced. A cut point in a connected graph \(G\) is a vertex whose removal results in a disconnected graph. The following result due to Harary [5] gives interdependence of local balance and cut vertex of a signed graph.

**Theorem 4.1** (F.Harary [5]) If a connected signed graph \(S = (G, \sigma)\) is balanced at a vertex \(u\). Let \(v\) be a vertex on a cycle \(C\) passing through \(u\) which is not a cut point, then \(S\) is balanced at \(v\).
In [11], the authors extend the notion of local balance in signed graph to \((n,d)\)-signed graphs as follows: Let \(G = (V,E)\) be a \((n,d)\)-signed graph. Then for any vertices \(v \in V(G)\), \(G\) is locally \(i\)-balanced at \(v\) (locally balanced at \(v\)) if all cycles in \(G\) containing \(v\) is \(i\)-balanced (balanced).

Analogous to the above result, in [11] we have the following for an \((n,d)\) signed graphs:

**Theorem 4.2** If a connected \((n,d)\)-signed graph \(G = (V,E)\) is locally \(i\)-balanced (locally balanced) at a vertex \(u\) and \(v\) be a vertex on a cycle \(C\) passing through \(u\) which is not a cut point, then \(S\) is locally \(i\)-balanced(locally balanced) at \(v\).

§5. Symmetric Balance in \((n,d)\)-Signed Graphs

In [22], P.S.K.Reddy and U.K.Misra defined a new notion of balance called symmetric balance or \(s\)-balanced in \((n,d)\)-signed graphs as follows:

Let \(n \geq 1\) be an integer. An \(n\)-tuple \((a_1,a_2,\cdots,a_n)\) is symmetric, if \(a_k = a_{n-k+1}, 1 \leq k \leq n\). Let \(H_n = \{(a_1,a_2,\cdots,a_n) : a_k \in \{+,-\}, a_k = a_{n-k+1}, 1 \leq k \leq n\}\) be the set of all symmetric \(n\)-tuples. Note that \(H_n\) is a group under coordinate wise multiplication, and the order of \(H_n\) is \(2^n\), where \(m = \lceil n/2 \rceil\). Let \(G = (V,E)\) be an \((n,d)\)-signed graph. Then \(G\) is symmetric balanced or \(s\)-balanced if \(P(\overline{C})\) on each cycle \(C\) of \(G\) is symmetric \(n\)-tuple.

**Note:** If an \((n,d)\)-signed graph \(G = (V,E)\) is \(i\)-balanced then clearly \(G\) is \(s\)-balanced. But a \(s\)-balanced \((n,d)\)-signed graph need not be \(i\)-balanced. For example, the \((4,d)\)-signed graphs in Figure 4. \(G\) is an \(s\)-balanced but not \(i\)-balanced.

In [22], the authors obtained the following results based on symmetric balance or \(s\)-balanced in \((n,d)\)-signed graphs.

**Theorem 5.1** (P.S.K.Reddy and U.K.Mishra [22]) A \((n,d)\)-signed graph is \(s\)-balanced if and only if every cycle of \(G\) contains an even number of non-symmetric \(n\)-tuples.

The following result gives a necessary and sufficient condition for a balanced \((n,d)\)-signed graph to be \(s\)-balanced.

**Theorem 5.2** (P.S.K.Reddy and U.K.Mishra [22]) A balanced \((n,d)\) signed graph \(G = (V,E)\) is \(s\)-balanced if and only if every cycle of \(G\) contains even number of non identity symmetric \(n\)-tuples.
In [22], the authors obtained another characterization of \(s\)-balanced \((n, d)\)-signed graphs, which is analogous to the partition criteria for balance in signed graphs due to Harary [4].

**Theorem 5.3 (Characterization of \(s\)-balanced \((n, d)\)-sigraph)** A \((n, d)\)-signed graph \(G = (V, E)\) is \(s\)-balanced if and only if the vertex set \(V(G)\) of \(G\) can be partitioned into two sets \(V_1\) and \(V_2\) such that each symmetric edge joins the vertices in the same set and each non-symmetric edge joins a vertex of \(V_1\) and a vertex of \(V_2\).

An \(n\)-marking \(\mu : V(G) \to H_n\) of an \((n, d)\)-signed graph \(G = (V, E)\) is an assignment \(n\)-tuples to the vertices of \(G\). In [22], the authors given another characterization of \(s\)-balanced \((n, d)\)-signed graphs which gives a relationship between the \(n\)-marking and \(s\)-balanced \((n, d)\)-signed graphs.

**Theorem 5.4 (P.S.K. Reddy and U.K. Mishra [22])** A \((n, d)\)-signed graph \(G = (V, E)\) is \(s\)-balanced if and only if there exists an \(n\)-marking \(\mu\) of vertices of \(G\) such that if the \(n\)-tuple on any arc \(\overrightarrow{uv}\) is symmetric or nonsymmetric according as the \(n\)-tuple \(\mu(u)\mu(v)\) is.

§ 6. **Directionally 2-Signed Graphs**

In [12], E. Sampathkumar et al. proved that the directionally 2-signed graphs are equivalent to bidirected graphs, where each end of an edge has a sign. A bidirected graph implies a signed graph, where each edge has a sign. Signed graphs are the special case \(n = 1\), where directionality is trivial. Directionally 2-signed graphs (or \((2, d)\)-signed graphs) are also special, in a less obvious way. A bidirected graph \(B = (G, \beta)\) is a graph \(G = (V, E)\) in which each end \((e, u)\) of an edge \(e = uv\) has a sign \(\beta(e, u) \in \{+, -\}\). \(G\) is the underlying graph and \(\beta\) is the bidirection. (The + sign denotes an arrow on the \(u\)-end of \(e\) pointed into the vertex \(u\); a – sign denotes an arrow directed out of \(u\). Thus, in a bidirected graph each end of an edge has an independent direction. Bidirected graphs were defined by Edmonds [2].) In view of this, E. Sampathkumar et al. [12] proved the following result:

**Theorem 6.1 (E. Sampathkumar et al. [12])** Directionally 2-signed graphs are equivalent to bidirected graphs.

§ 7. **Conclusion**

In this brief survey, we have described directionally \(n\)-signed graphs (or \((n, d)\)-signed graphs) and their characterizations. Many of the characterizations are more recent. This an active area of research. We have included a set of references which have been cited in our description. These references are just a small part of the literature, but they should provide a good start for readers interested in this area.
References


