THE 57-TH SMARANDACHE’S PROBLEM II *

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Abstract For any positive integer \( n \), let \( r \) be the positive integer such that: the set \{1, 2, \ldots, \( r \)\} can be partitioned into \( n \) classes such that no class contains integers \( x, y, z \) with \( x^y = z \). In this paper, we use the elementary methods to give a sharp lower bound estimate for \( r \).

Keywords: Smarandache-type multiplicative functions; Mangoldt function; Hybrid mean value.

§1. Introduction

For any positive integer \( n \), let \( r \) be a positive integer such that: the set \{1, 2, \ldots, \( r \)\} can be partitioned into \( n \) classes such that no class contains integers \( x, y, z \) with \( x^y = z \). In [1], Schur asks us to find the maximum \( r \). About this problem, Liu Hongyan [2] obtained that \( r \geq n^{m+1} \), where \( m \) is any integer with \( m \leq n + 1 \).

In this paper, we use the elementary methods to improve Liu Hongyan’s result. That is, we shall prove the following:

**Theorem.** For sufficiently large integer \( n \), let \( r \) be a positive integer such that: the set \{1, 2, \ldots, \( r \)\} can be partitioned into \( n \) classes such that no class contains integers \( x, y, z \) with \( x^y = z \). Then we have

\[
r \geq \left( n^{n^1} + 2 \right)^{n^{n^1}+1} - 1.
\]

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§2. Proof of the Theorem

In this section, we complete the proof of the theorem.

Let \( r = \left( n^{n!} + 2 \right)^{n!} \! \! + 1 \) and partition the set \( \{1, 2, \ldots, \left( n^{n!} + 2 \right)^{n!} \! \! + 1 - 1\} \) into \( n \) classes as follows:

Class 1: \( 1, \ n^{n!} + 1, \ n^{n!} + 2, \ \ldots, \ \left( n^{n!} + 2 \right)^{n!} - 1 \).

Class 2: \( 2, \ n + 1, \ n + 2, \ \ldots, \ n^2 \).

... Class \( k \): \( k, \ n^{(k-1)!} + 1, \ n^{(k-1)!} + 2, \ \ldots, \ n^k \).

... Class \( n \): \( n, \ n^{(n-1)!} + 1, \ n^{(n-1)!} + 2, \ \ldots, \ n^n \).

It is obvious that Class \( k (k \geq 2) \) contains no integers \( x, y, z \) with \( x^y = z \).

In fact for any integers \( x, y, z \in \text{Class } k, k = 2, 3, \ldots, n \), we have
\[
x^y \geq \left( n^{(k-1)!} + 1 \right)^k > n^k \geq z.
\]

Similarly, Class 1 also contains no integers \( x, y, z \) with \( x^y = z \).
This completes the proof of the theorem.

Reference
