Smarandache U-liberal semigroup structure

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Abstract In this paper, Smarandache U-liberal semigroup structure is given. It is shown that a semigroup $S$ is Smarandache U-liberal semigroup if and only if it is a strong semilattice of some rectangular monoids. Consequently, some corresponding results on normal orthocryptou semigroups and normal orthocryptogroups are generalized and extended.

Keywords Smarandache U-liberal semigroup, $U$-semiabundant semigroups, normal band, rectangular monoid, strong semilattice.

§1. Introduction and preliminaries

In order to generalize regular semigroups, new Green’s relations, namely, the Green’s $*$-relations on a semigroup $S$ have been introduced in [1] and [2] as follows:

$$\mathcal{L}^* = \{(a, b) \in S \times S : (\forall x, y \in S^1) ax = ay \Leftrightarrow bx = by\},$$

$$\mathcal{R}^* = \{(a, b) \in S \times S : (\forall x, y \in S^1) xa = ya \Leftrightarrow xb = yb\},$$

$$\mathcal{H}^* = \mathcal{L}^* \cap \mathcal{R}^*, \quad \mathcal{D}^* = \mathcal{L}^* \lor \mathcal{R}^*.$$

In [3], Fountain investigated a class of semigroups called abundant semigroups in which each $\mathcal{L}^*$-class and each $\mathcal{R}^*$-class of $S$ contain at least an idempotent. Actually, the class of regular semigroups are properly contained in the class of abundant semigroups.

In 1980, El-Qallali generalized the Green’s $*$-relations to the Green’s $\sim$ –relations on a semigroup $S$ in [4] as follows:

$$\tilde{\mathcal{L}} = \{(a, b) \in S \times S : (\forall e \in E(S)) ae = a \Leftrightarrow be = b\},$$

$$\tilde{\mathcal{R}} = \{(a, b) \in S \times S : (\forall e \in E(S)) ea = a \Leftrightarrow eb = b\},$$

$$\tilde{\mathcal{H}} = \tilde{\mathcal{L}} \cap \tilde{\mathcal{R}}, \quad \tilde{\mathcal{D}} = \tilde{\mathcal{L}} \lor \tilde{\mathcal{R}}.$$

In his thesis, El-Qallali obtained and studied a much bigger class of semigroups, called semi-abundant semigroup.

After that, many authors study this class of semigroups, and obtain a lot of interesting conclusions and results (see [5],[6],[7] etc.).
In recent years, some scholars have observed that one can pay special attention to a subset $U$ of $E(S)$ instead of the whole set $E(S)$ of a semiabundant semigroup $S$. In particular, Lawson in [8] noticed that if $U$ is a subset of $E(S)$ of a semiabundant semigroup $S$ then $U$ is perhaps good enough to provide sufficient information for the whole semigroup $S$. The semigroup $S$ is usually denoted by $S(U)$ and the equivalence relations on $S(U)$ with respect to $U \subseteq E(S)$ can be given by

$$
\mathcal{L}^U = \{(a, b) \in S \times S | U_a^b = U_b^a\},
$$

$$
\mathcal{R}^U = \{(a, b) \in S \times S | U_a^b = U_b^a\},
$$

$$
\mathcal{H}^U = \mathcal{L}^U \cap \mathcal{R}^U,
$$

$$
\mathcal{Q}^U = \{(a, b) \in S \times S | U_a = U_b\},
$$

where $U_a^b = \{u \in U | ua = a\}$, $U_b^a = \{u \in U | au = a\}$ and $U_a = U_a^a \cap U_b^a = \{u \in U | ua = a = au\}$ for any $a \in S$.

A semigroup $S(U)$ is said to be a $U$– semiabundant semigroup if every $\mathcal{L}^U$ and every $\mathcal{R}^U$– class of $S(U)$ contain at least one element of $U$ respectively. A semigroup $S(U)$ is said to be a $U$– semi-superabundant semigroup if every $\mathcal{H}^U$ of $S(U)$ contains at least one element of $U$. In this case, the unique element in $\mathcal{H}^U \cap U$ is denoted by $a^*_U$. On the other hand, a semigroup $S(U)$ is called by He in [7] a $U$– liberal semigroup if every $\mathcal{Q}^U$– class of $S$ contains an element of $U$. It is routine to check that a $\mathcal{Q}^U$– class contains at most one element of $U$. Denote the unique element in $\mathcal{Q}^U \cap U$, if it exists, by $a^*_U$. The structure of Smarandache $U$– liberal semigroups has also been recently investigated by He in [7].

For a Smarandache $U$– liberal semigroup $S(U)$, we call the following condition the Ehresmann type condition, in brevity, the ET-condition:

$$
(\forall a, b \in S)(ab)_U^0 \mathcal{D}(U)a^0_U b^0_U,
$$

where

$$
\mathcal{D}(U) = \{(e, f) \in U \times U | (\exists g \in U) e R g L f\}.
$$

A Smarandache $U$– liberal semigroup $S(U)$ is called an orthodox $U$– liberal semigroup if $U$ is a subsemigroup of $S(U)$ and the ET-condition holds on $S(U)$.

In general, unlike the usual Green’s relations on a semigroup $S$, $\mathcal{L}^U$ is not necessarily a right congruence on $S$ and $\mathcal{R}^U$ is not necessarily a left congruence on $S$ (see [8]).

We say that a semigroup $S(U)$ satisfies the (CR) condition if $\mathcal{L}^U$ is a right congruence on $S$ and that $S(U)$ satisfies the (CL) condition if $\mathcal{R}^U$ is a left congruence on $S$. If the semigroup $S(U)$ satisfies both the (CR) and (CL) condition, then we say $S(U)$ satisfies the (C) condition.

The studies on the structures of semigroups play an important role in the research of the algebraic theories of semigroups. From [7], it is known that a $U$–semi-superabundant semigroup $S(U)$ is an orthodox $U$– liberal semigroup for some $U \subseteq E(S)$ if and only if it is a semilattice of some rectangular monoids, i.e., $S = [Y; S_\alpha(U_\alpha)]$, where $S_\alpha(U_\alpha)$ is a rectangular monoid for every $\alpha \in Y$ and $U = \cup_{\alpha \in Y} U_\alpha$ is a subsemigroup of $S$. Meanwhile, notice that a normal band is a strong semilattice of some rectangular bands. Naturally, we will quote such a question: whether will a normal orthodox $U$– liberal semigroup $S(U)$ be a strong semilattice of some rectangular monoids?
In this paper, we will consider the question quoted above. Consequently, we show that a semigroup $S(U)$ is a normal orthodox Smarandache $U$–liberal semigroup if and only if it is a strong semilattice of some rectangular monoids, i.e., $S = [Y; S_\alpha(U_\alpha); \Phi_\alpha,\beta]$, where $S_\alpha(U_\alpha)$ is a rectangular monoid for every $\alpha \in Y$ and $U = \cup_{\alpha \in Y} U_\alpha$ is a normal band of $S(U)$. Consequently, some corresponding results on normal orthocryptou semigroups and normal orthocryptogroups are generalized and extended.

For notations and terminologies not mentioned in this paper, the reader is referred to [7],[9],[10].

§2. Normal Orthodox $U$–liberal Semigroups

In this section, we will give a construction of normal orthodox $U$–liberal semigroups.

Firstly, we recall the following lemmas.

**Lemma 2.1.** [7] Let $\mathcal{F}$ be one of Green’s relations $\mathcal{L}$, $\mathcal{R}$ or $\mathcal{H}$ and $\tilde{\mathcal{F}}^U$ the corresponding Green $\sim$ relations on the semigroup $S$. Then, for any $a, b \in S$, we have

(i) $\mathcal{F} \subseteq \tilde{\mathcal{F}}^U$ and for $a, b \in \text{Reg}_U(S)$, $a, b \in \tilde{\mathcal{F}}^U$ if and only if $a, b \in \mathcal{F}$, where $\text{Reg}_U(S) = \{a \in S| \exists e, f \in U: eLaRf\}$;

(ii) $\tilde{\mathcal{H}}^U \subseteq \tilde{\mathcal{Q}}^U$ and $\tilde{\mathcal{Q}}^U_a$ contains at most one element in $U$;

(iii) If $S(U)$ is a $U$–semi-superabundant semigroup, then $S(U)$ is a Smarandache $U$–liberal semigroup with $\tilde{\mathcal{Q}}^U = \tilde{\mathcal{H}}^U$.

**Lemma 2.2.** [7] The following statements are equivalent for a semigroup $S$:

(i) $S(U)$ is a Smarandache $U$–liberal semigroup for some $U \subseteq E(S)$ and $U$ itself is a rectangular band;

(ii) $S(U)$ is an orthodox $U$–liberal semigroup such that $U$ is a rectangular band;

(iii) $S$ is isomorphic to a rectangular monoid.

**Lemma 2.3.** [7] The following statements are equivalent for a semigroup $S$:

(i) $S(U)$ is an orthodox $U$-liberal semigroup for some $U \subseteq E(S)$;

(ii) $S = [Y; S_\alpha(U_\alpha)]$, where $S_\alpha(U_\alpha)$ is a rectangular monoid for every $\alpha \in Y$ and $U = \cup_{\alpha \in Y} U_\alpha$ is a subsemigroup of $S$;

(iii) $S(U)$ is a U-semi-superabundant semigroup satisfying the (C) condition for some $U \subseteq E(S)$ and $U$ is a subsemigroup of $S$.

Now, we will give our main theorem.

**Theorem 2.4.** The following statements are equivalent for a semigroup $S$:

(i) $S(U)$ is a normal orthodox $U$-liberal semigroup for some normal band $U \subseteq E(S)$;

(ii) $S(U)$ is a strong semilattice of some rectangular monoids, i.e., $S = [Y; S_\alpha(U_\alpha); \Phi_\alpha,\beta]$, where $S_\alpha(U_\alpha)$ is a rectangular monoid for every $\alpha \in Y$ and $U = \cup_{\alpha \in Y} U_\alpha$ is a normal band of $S$;

(iii) $S(U)$ is a U-semi-superabundant semigroup satisfying the (C) condition for some $U \subseteq E(S)$ and $U$ is a normal band of $S$.

**Proof.** (i) $\Rightarrow$ (ii)
Assume that \( S(U) \) is a normal orthodox \( U \)-liberal semigroup for some normal band \( U \subseteq E(S) \). Note that \( U \subseteq E(S) \) is a normal band, we will have \( U = [Y; U_\alpha, f_{\alpha, \beta}] \) or \( U = [Y; I_\alpha \times \Lambda; \varphi_{\alpha, \beta}, \psi_{\alpha, \beta}] \), where \( (i, j)f_{\alpha, \beta} = (i\varphi_{\alpha, \beta}, j\psi_{\alpha, \beta}) \), \( (i, j) \in I_\alpha \times \Lambda_\alpha \).

For any \( \alpha \in Y \), we form the set \( S_\alpha = \{x \in S|x^0_U \in U_\alpha \} \). Since \( S(U) \) satisfies the ET-condition, for all \( x \in S_\alpha \), \( y \in S_\beta \), we have \( (xy)^0_U D(U)x^0_U y^0_U \). This leads to \( xy \in S_{\alpha \beta} \) and hence \( S(U) = [Y; S_\alpha(U_\alpha)] \).

Notice that every semigroup \( S_\alpha(U_\alpha) \) is a Smarandache \( U_\alpha \)-liberal semigroup and \( U_\alpha \) is a rectangular band. By Lemma 2.2, \( S_\alpha(U_\alpha) \) is isomorphic to a rectangular monoid. For convenience, we denote \( S_\alpha(U_\alpha) = I_\alpha \times T_{U_\alpha} \times \Lambda_\alpha \).

Now, define a mapping,

\[
\Phi_{\alpha, \beta} : S_\alpha(U_\alpha) \to S_\beta(U_\beta),
\]

\[
(i_\alpha, u_\alpha, \lambda_\alpha) \to (i_\alpha \varphi_{\alpha, \beta}, 1_{U_\beta}, \lambda_\alpha \psi_{\alpha, \beta})(i_\alpha, u_\alpha, \lambda_\alpha) = (i_\alpha \varphi_{\alpha, \beta} i_\alpha, 1_{U_\beta} u_\alpha, \lambda_\alpha \psi_{\alpha, \beta} \psi_{\alpha, \beta} \lambda_\alpha).
\]

In the following, we will prove that \( S \) is a strong semilattice of \( S_\alpha(U_\alpha) \), i.e., \( S = [Y; S_\alpha(U_\alpha); \Phi_{\alpha, \beta}] \).

Firstly, \( \Phi_{\alpha, \beta} \) is a homomorphism.

For any \( x = (i_\alpha, u_\alpha, \lambda_\alpha), y = (j_\alpha, v_\alpha, \mu_\alpha) \in S_\alpha(\forall \alpha \geq \beta) \),

\[
(xy)\Phi_{\alpha, \beta} = [(i_\alpha, u_\alpha, \lambda_\alpha)(j_\alpha, v_\alpha, \mu_\alpha)]\Phi_{\alpha, \beta}
\]

\[
= (i_\alpha j_\alpha, u_\alpha v_\alpha, \lambda_\alpha \mu_\alpha)\Phi_{\alpha, \beta}
\]

\[
= (i_\alpha, u_\alpha v_\alpha, \mu_\alpha)\Phi_{\alpha, \beta}
\]

\[
= (i_\alpha \varphi_{\alpha, \beta} i_\alpha, 1_{U_\beta} u_\alpha v_\alpha, \mu_\alpha \psi_{\alpha, \beta} \psi_{\alpha, \beta} \mu_\alpha).
\]

Thus, \( (xy)\Phi_{\alpha, \beta} = x\Phi_{\alpha, \beta} y\Phi_{\alpha, \beta} \).

Secondly, \( \Phi_{\alpha, \alpha} \) is an identity mapping.

For any \( x = (i_\alpha, u_\alpha, \lambda_\alpha) \in S_\alpha \), \( x\Phi_{\alpha, \alpha} = (i_\alpha \varphi_{\alpha, \alpha} i_\alpha, 1_{U_\alpha} u_\alpha, \lambda_\alpha \psi_{\alpha, \alpha} \lambda_\alpha) = (i_\alpha, u_\alpha, \lambda_\alpha) = x \).

Hence, \( \Phi_{\alpha, \alpha} \) is an identity mapping.

Thirdly, notice that for any \( x = (i_\alpha, 1_{U_\alpha}, \lambda_\alpha) \in E(S_\alpha) = I_\alpha \times 1_{U_\alpha} \times \Lambda_\alpha \), \( y = (i_\beta, 1_{U_\beta}, \lambda_\beta) \in E(S_\beta) = I_\beta \times 1_{U_\beta} \times \Lambda_\beta \), \( x \in E(S_{\alpha \beta}) = I_\alpha \times 1_{U_\beta} \times \Lambda_\beta \), we can get \( 1_{U_\alpha}, 1_{U_\beta} = 1_{U_{\alpha \beta}} \). Especially, when \( \alpha \geq \beta \), we have \( 1_{U_\alpha} 1_{U_\beta} = 1_{U_{\alpha \beta}} = 1_{U_{\alpha \alpha}} \). Now, for any \( \alpha, \beta, \gamma \in Y(\alpha \geq \beta \geq \gamma) \) and any \( x = (i_\alpha, u_\alpha, \lambda_\alpha) \in S_\alpha(U_\alpha) \), we will have

\[
(x\Phi_{\alpha, \beta})\Phi_{\beta, \gamma} = (i_\alpha, u_\alpha, \lambda_\alpha)\Phi_{\alpha, \beta} \Phi_{\beta, \gamma} = (i_\alpha \varphi_{\alpha, \beta} i_\alpha, 1_{U_\beta} u_\alpha, \lambda_\alpha \psi_{\alpha, \beta \beta} \lambda_\alpha) \Phi_{\beta, \gamma}
\]

\[
= (((i_\alpha \varphi_{\alpha, \beta} i_\alpha) \varphi_{\alpha, \beta} (i_\alpha \varphi_{\alpha, \beta} i_\alpha)), 1_{U_\beta} (1_{U_\beta} u_\alpha), (\lambda_\alpha \psi_{\alpha, \beta \beta} \lambda_\alpha)) \psi_{\beta, \gamma} (\lambda_\alpha \psi_{\alpha, \beta \beta} \lambda_\alpha)
\]

\[
= (((i_\alpha \varphi_{\alpha, \beta} (i_\alpha \varphi_{\alpha, \beta} i_\alpha)), \varphi_{\alpha, \beta} (i_\alpha \varphi_{\alpha, \beta} i_\alpha)), 1_{U_\beta} u_\alpha, (\lambda_\alpha \psi_{\alpha, \beta \beta} \lambda_\alpha \psi_{\alpha, \beta \beta} \lambda_\alpha)) \psi_{\beta, \gamma} (\lambda_\alpha \psi_{\alpha, \beta \beta} \lambda_\alpha)
\]

\[
= ((i_\alpha \varphi_{\alpha, \beta} i_\alpha, \varphi_{\alpha, \beta} (i_\alpha \varphi_{\alpha, \beta} i_\alpha)), 1_{U_\beta} u_\alpha, (\lambda_\alpha \psi_{\alpha, \beta \beta} \lambda_\alpha \psi_{\alpha, \beta \beta} \lambda_\alpha)) \psi_{\beta, \gamma} (\lambda_\alpha \psi_{\alpha, \beta \beta} \lambda_\alpha)
\]

\[
= (i_\alpha \varphi_{\alpha, \beta} i_\alpha, 1_{U_\beta} u_\alpha, (\lambda_\alpha \psi_{\alpha, \beta \beta} \lambda_\alpha \psi_{\alpha, \beta \beta} \lambda_\alpha)) \psi_{\beta, \gamma} (\lambda_\alpha \psi_{\alpha, \beta \beta} \lambda_\alpha)
\]

\[
= x\Phi_{\alpha, \gamma}.
\]
Finally, for any \( \alpha, \beta \in Y, a_\alpha \in S_\alpha \) and \( b_\beta \in S_\beta \), since \( a_\alpha b_\beta \in S_{\alpha \beta} \triangleq S_\gamma \), we have

\[
a_\alpha b_\beta = (i_\alpha, u_\alpha, \lambda_\alpha)(i_\beta, u_\beta, \lambda_\beta) = (i_\alpha i_\beta, u_\alpha u_\beta, \lambda_\alpha \lambda_\beta) = (i_\alpha \varphi_{\alpha, \gamma} i_\beta \varphi_{\beta, \gamma}, 1_{U}, u_\alpha 1_{U} u_\beta, \lambda_\alpha \psi_{\alpha, \gamma} \lambda_\beta \psi_{\beta, \gamma} \lambda_\beta) = (i_\alpha \varphi_{\alpha, \gamma} i_\beta, 1_{U}, u_\alpha, \lambda_\alpha \psi_{\alpha, \gamma} \lambda_\alpha)(i_\beta \varphi_{\beta, \gamma} i_\beta, 1_{U}, u_\beta, \lambda_\beta \psi_{\beta, \gamma} \lambda_\beta) = (a_\alpha \Phi_{\alpha, \gamma})(b_\beta \Phi_{\beta, \gamma}).
\]

Thus, summing up the above discussions, \( S(U) \) is isomorphic to a strong semilattice of rectangular monoids \( S_\alpha(U_\alpha) \), that is, \( S(U) = [Y; S_\alpha(U_\alpha); \Phi_{\alpha, \beta}] \).

(iii) \( \Rightarrow \) (iii) The proof is similar with the corresponding (iii) \( \Rightarrow \) (iii) of Lemma 2.3.

For any \( \alpha \in Y \), assume that \( S_\alpha(U_\alpha) = I_\alpha \times T_\alpha \times \Lambda_\alpha \). Then, it is not hard to see that for any \((i, x, \lambda) \in S_\alpha, (j, y, \mu) \in S_\beta, (i, x, \lambda)\tilde{U} (j, y, \mu)\) if and only if \( \alpha = \beta \) and \( \lambda = \mu \in \Lambda_\alpha \). On the other hand, if \((i, x, \lambda)\tilde{U} (j, y, \lambda)\) for some \( \lambda \in \Lambda_\alpha \), then for all \((k, z, \nu) \in S_\gamma(\nu \in Y)\), we have

\[
(i, 1_{T_\alpha}, \lambda)(k, z, \nu) = (k', z', \nu')(\in S_\gamma),
(i, x, \lambda)(k', 1_{T_\gamma}, \nu') = (i', x', \lambda'),
(j, y, \lambda)(k', 1_{T_\gamma}, \nu') = (j', y', \lambda''),
\]

Consequently, by using the above relations, we derive that

\[
(i, x, \lambda)(k, z, \nu) = (i, x, \lambda)(i, 1_{T_\alpha}, \lambda)(k, z, \nu) = (i, x, \lambda)(k', 1_{T_\gamma}, \nu')(k', z', \nu') = (i', x', \lambda')(k', z', \nu') = (i', x', \lambda');
(j, y, \lambda)(k, z, \nu) = (j, y, \lambda)(j, 1_{T_\gamma}, \lambda)(k, z, \nu) = (j, y, \lambda)(k', 1_{T_\gamma}, \nu')(k', z', \nu') = (j', y', \lambda'')(k', z', \nu') = (j', y', \lambda').
\]

Thereby, we obtain that \((i, x, \lambda)(k, z, \nu)\tilde{U} (j, y, \lambda)(k, z, \nu)\) so that \( \tilde{U} \) is a right congruence on \( S \).

Similarly, we can show that \((i, x, \lambda)\tilde{U} (j, y, \lambda)\) if and only if \( i = j \in I_\alpha \) for some \( \alpha \in Y \), and so \( \tilde{U} \) is a left congruence on \( S \).

Hence, together with Lemma 2.3, \( S(U) \) is a U-semi-superabundant semigroup satisfying the (C) condition for some \( U \subseteq E(S) \). Note that \( U \) is a normal band of \( S \), (iii) holds.

(iii) \( \Rightarrow \) (i) The proof is similar with the corresponding (iii) \( \Rightarrow \) (i) of Lemma 2.3.

Assume that (iii) holds. Then, by Lemma 2.1, \( S(U) \) is Smarandache \( U - \) liberal semigroup and for all \( a \in S(U) \), \( a^\alpha_U = a^\alpha_U \). Since \( S(U) \) satisfies the (C) condition, we have, for all \( a, b \in S(U) \),

\[
(ab)^\gamma_U \tilde{U} a \tilde{U} b \tilde{U} (ab)^\gamma_U \tilde{U} a \tilde{U} b \tilde{U} (ab)^\gamma_U \tilde{U} a \tilde{U} b \tilde{U}.
\]

This leads to \((ab)^\gamma_U \tilde{U} (ab)^\gamma_U \tilde{U} a \tilde{U} b \tilde{U} \). By Lemma 2.1 (i), we will get \((ab)^\gamma_U \tilde{U} (ab)^\gamma_U \tilde{U} a \tilde{U} b \tilde{U} \). Consequently, \((ab)^\gamma_U = (ab)^\gamma_U \tilde{U} a \tilde{U} b \tilde{U} \) holds. This shows that \( S(U) \) satisfies the ET-condition. Note that \( U \) is a normal band of \( S \), (i) holds.

Now, if we let \( U = E(S) \) in Theorem 2.4, then we immediately have the following corollary.

**Corollary 2.5.** The following statements are equivalent for a semigroup \( S \):

(i) \( S(U) \) is a normal orthodox \( E(S) \)-liberal semigroup;
(ii) $S(U)$ is a strong semilattice of some rectangular monoids, i.e., $S = [Y; S_\alpha(E(S_\alpha)); \Phi_{\alpha,\beta}]$, where $S_\alpha(E(S_\alpha))$ is a rectangular monoid for every $\alpha \in Y$ and $E(S)$ is a normal band of $S$.

(iii) $S(U)$ is a semi-superabundant semigroup satisfying the (C) condition, and $E(S)$ is a normal band of $S$.

In the above corollary, if we restrict the semigroup $S$ to the abundant or regular semigroups, then it is not hard for us to get

**Corollary 2.6.** The following statements are equivalent for a semigroup $S$:

(i) $S$ is a normal orthocrypto semigroup;

(ii) $S$ is a strong semilattice of rectangular cancellative monoids, i.e., $S = [Y; S_\alpha; \Phi_{\alpha,\beta}]$, where $S_\alpha = I_\alpha \times T_\alpha \times \Lambda_\alpha$, and $I_\alpha$ is a left zero band, $\Lambda_\alpha$ is a right zero band, $T_\alpha$ is a cancellative monoid for every $\alpha \in Y$.

**Corollary 2.7.** The following statements are equivalent for a semigroup $S$:

(i) $S$ is a normal orthocryptogroup;

(ii) $S$ is a strong semilattice of rectangular groups.

Hence, our main result generalizes and extends some corresponding results on normal orthocryptou semigroups and normal orthocryptogroups.

**References**


