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On the Smarandache prime part sequences and its two conjectures[1]

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Abstract For any positive integer $n \geq 2$, the Smarandache Inferior Prime Part $\{p_p(n)\}$ is defined as the largest prime less than or equal to n; The Smarandache Superior Prime Part $\{P_p(n)\}$ is defined as the smallest prime greater than or equal to n. In this paper, we defined some determinants involving the Smarandache prime part sequences, and introduced two conjectures proposed by professor Zhang Wenpeng.

Keywords Smarandache prime part sequences, Determinant, Conjecture.

§1. Introduction

For any positive integer $n \geq 2$, the Smarandache Inferior prime part $\{p_p(n)\}$ is defined as the largest prime less than or equal to n. The Smarandache Superior prime part $\{P_p(n)\}$ as the smallest prime greater than or equal to n. For example, the first few values of these sequences are $p_p(2) = 2$, $p_p(3) = 3$, $p_p(4) = 3$, $p_p(5) = 5$, $p_p(6) = 5$, $p_p(7) = 7$, $p_p(8) = 7$, $p_p(9) = 7$, $p_p(10) = 7$, $p_p(11) = 11$, $p_p(12) = 11$, $p_p(13) = 13$, $p_p(14) = 13$, $p_p(15) = 13$, $p_p(16) = 13$, $p_p(17) = 17$, $p_p(18) = 17$, $p_p(19) = 19$, $\cdots\cdots$.

$P_p(1) = 2$, $P_p(2) = 2$, $P_p(3) = 3$, $P_p(4) = 5$, $P_p(5) = 5$, $P_p(6) = 7$, $P_p(7) = 7$, $P_p(8) = 11$, $P_p(9) = 11$, $P_p(10) = 11$, $P_p(11) = 11$, $P_p(12) = 13$, $P_p(13) = 13$, $P_p(14) = 17$, $P_p(15) = 17$, $P_p(16) = 17$, $P_p(17) = 17$, $P_p(18) = 19$, $P_p(19) = 19$, $\cdots\cdots$.

From the definitions of these sequences we know that for any prime q, we have $p_p(q) = P_p(q) = q$.

In his books "Only problem, Not solutions"[1] and "Sequences of Numbers Involved in Unsolved Problem"[2], Professor F.Smarandache asked us to study the properties of these sequences. About these two problems, some authors had studied them, and obtained some interesting results, see references [3], [4], [5] and [6]. For example, Xiaoxia Yan [6] studied the the asymptotic properties of $\frac{S_n}{I_n}$, and proved that for any positive integer $n > 1$, we have the asymptotic formula

$$\frac{S_n}{I_n} = 1 + O(n^{-\frac{1}{3}}),$$

where $I_n = \{p_p(2) + p_p(3) + \cdots + p_p(n)\}/n$ and $S_n = \{P_p(2) + P_p(3) + \cdots + P_p(n)\}/n$.

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The sequences $\{p_p(n)\}$ and $\{P_p(n)\}$ are very interesting and important, because there are some close relationships between the Smarandache prime part sequences and the prime distribution problem. Now we introduced two determinants formed by the Smarandache prime part sequences. They are defined as follows: For any positive integer n, $c(n)$, $C(n)$ are $n \times n$ determinants, namely

$$c(n) = \begin{vmatrix} p_p(2) & p_p(3) & \cdots & p_p(n+1) \\ p_p(n+2) & p_p(n+3) & \cdots & p_p(2n+1) \\ \vdots & \vdots & \ddots & \vdots \\ p_p(n(n-1)+2) & p_p(n(n-1)+3) & \cdots & p_p(n^2+1) \end{vmatrix}$$

and

$$C(n) = \begin{vmatrix} P_p(1) & P_p(2) & \cdots & P_p(n) \\ P_p(n+1) & P_p(n+3) & \cdots & P_p(2n) \\ \vdots & \vdots & \ddots & \vdots \\ P_p(n(n-1)+1) & P_p(n(n-1)+2) & \cdots & P_p(n^2) \end{vmatrix}$$

For example, by definitions and calculating, we can find some values of these determinants as the following table:

n	$c(n)$	$C(n)$	n	$c(n)$	$C(n)$
2	1	4	3	14	4
4	0	188	5	-96	-1424
6	0	0	7	1.0214×10^5	37536
8	0	0	9	0	0
11	7.8299×10^7	1.1478×10^8	13	9.8338×10^8	-2.7958×10^9
17	8.2462×10^{14}	1.3164×10^{13}	19	-1.9608×10^{15}	1.629×10^{15}
23	2.4545×10^{20}	3.156×10^{18}	29	8.9308×10^{27}	-6.5008×10^{27}
31	1.1095×10^{29}	-2.2835×10^{28}	37	9.9497×10^{37}	-7.4692×10^{38}
41	-1.1502×10^{42}	3.9244×10^{45}	43	1.5152×10^{45}	-2.1671×10^{44}
47	1.1606×10^{51}	4.06×10^{50}	53	2.9359×10^{59}	5.8735×10^{59}
59	3.8402×10^{67}	-3.1043×10^{69}	61	-3.614×10^{70}	6.9858×10^{72}
67	3.2341×10^{80}	-9.5374×10^{78}	71	-2.219×10^{85}	4.7688×10^{86}
73	-3.6656×10^{85}	-3.6985×10^{89}	79	-4.2038×10^{98}	-3.8762×10^{97}
83	1.6966×10^{104}	2.1389×10^{104}	89	-1.0695×10^{113}	5.7824×10^{110}
97	$2.8.98 \times 10^{124}$	1.9968×10^{124}			

About the elementary properties of these determinants, it seems that none had studied them, at least we haven't seen any related papers before. Recently, Professor Zhang Wenpeng asked us to study the properties of these determinants, at the same time, he also proposed following two conjectures:

Conjecture 1. For any composite number $n \geq 6$, we have the identities $c(n) = 0$ and $C(n) = 0$;

Conjecture 2. For any prime q, we have $c(q) \neq 0$ and $C(q) \neq 0$.

From the above table, we believe that these two conjectures are correct. About Conjecture 1, we have solved it completely, which will be published in Pure and Applied Mathematics. But for Conjecture 2, it's still an open problem. We think that Conjecture 2 is very interesting and important, since if it is true, then we can get a new discriminant method to distinguish prime from integer numbers through calculating these kinds of determinants. So we introduce the conjectures in this place, and hope more scholars who are interested in to study it with us.

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