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The Smarandache Reverse Auto Correlated Sequences of Natural Numbers

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Abstract In this paper we give an explicit formula for the n times Smarandache reverse auto correlated sequence of natural numbers.

Keywords Smarandache reverse auto correlated sequence, natural number.

Let $A = \{a(m)\}_{m=1}^{\infty}$ be a sequence. If the sequence $B = \{b(m)\}_{m=1}^{\infty}$ satisfying

$$b(m) = \sum_{k=1}^{m} a(k)a(m-k+1), m \geq 1, \tag{1}$$

then B is called the Smarandache reverse auto correlated sequence of A, and denoted by $SRACS(A)$. Further, for any positive integer n, let $SRACS(n, A)$ denote the n times Smarandache reverse auto correlated sequence of A. Then we have $SRACS(1, A) = SRACS(A)$, $SRACS(2, A) = SRACS(SRACS(A))$ and

$$SRACS(n, A) = SRACS(SRACS(n-1, A)), n \geq 1. \tag{2}$$

Recentely, Muthy [1] proposed the following conjecture:

Conjecture. For any positive integer n, if $a(m) = m$ $(m \geq 1)$ and $SRACS(n, A) = B = \{b(m)\}_{m=1}^{\infty}$, then

$$b(m) = \begin{pmatrix} 2^{n+1} + m - 1 \\ 2^{n+1} - 1 \end{pmatrix}, m \geq 1 \tag{3}$$

In this paper we completely verify the above-mentioned conjecture as follows.

Theorem. For any positive integer n, if $a(m) = m$ $(m \geq 1)$ and $SRACS(n, A) = B = \{b(m)\}_{m=1}^{\infty}$, then $b(m)$ $(m \geq 1)$ satisfy (3).

Proof. For a fixed sequence $A = \{a(m)\}_{m=1}^{\infty}$, let

$$f(A; x) = a(1) + a(2)x + a(3)x^2 + \cdots = \sum_{m=1}^{\infty} a(m)x^{m-1}. \tag{4}$$

Further, let $B = \{b(m)\}_{m=1}^{\infty}$ be the Smarandache reverse auto correlated sequence of A, and let

$$g(A; x) = b(1) + b(2)x + b(3)x^2 + \cdots = \sum_{m=1}^{\infty} b(m)x^{m-1}. \tag{5}$$

Then, by the definition of multiplication of power series (see [2]), we see from (1), (4) and (5) that

$$g(A;x) = (f(A;x))^2. \tag{6}$$

Furthermore, for a fixed positive integer n, if $SRACS(n, A) = B = \{b(m)\}_{m=1}^{\infty}$, and

$$g(n, A; x) = b(1) + b(2)x + b(3)x^2 + \cdots = \sum_{m=1}^{\infty} b(m)x^{m-1}, \tag{7}$$

then from (2) and (6) we obtain

$$g(n, A; x) = (f(A;x))^{2^n}. \tag{8}$$

If $a(m) = m$ for $m \geq 1$, then we get

$$f(A;x) = 1 + 2x + 3x^2 + \cdots = \sum_{m=1}^{\infty} mx^{m-1} = (1-x)^{-2}, \tag{9}$$

by (4). Therefore, by (8), if $SRACS(n, A) = B = \{b(m)\}_{m=1}^{\infty}$ and $g(n, A; x)$ satisfies (7), then from (9) we obtain

$$g(n, A; x) = (1-x)^{-2^{n+1}} = \sum_{m=1}^{\infty} \binom{2^{n+1}+m-1}{2^{n+1}-1} x^{m-1}, \tag{10}$$

Thus, by (7) and (10), we get (3). The theorem is proved.

References

[1] A.Murthy, Smarandache reverse auto correlated sequences and some Fibonacci derived Smarandache sequences, Smarandache Notions J., **12**(2001), 279-282.

[2] I. Niven, Formal power series, Amer. Math. Monthly, **76**(1969), 871-889.