On finite Smarandache near-rings

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Abstract In this paper we study the Finite Smarandache-2-algebraic structure of Finite-near-ring, namely, Finite-Smarandache-near-ring, written as Finite-S-near-ring. We define Finite Smarandache near-ring with examples. We introduce some equivalent conditions for Finite S-near-ring and obtain some of its properties.

Keywords Finite-S-near-ring; Finite-Smarandache-near-ring.

§1. Introduction

In this paper, we studied Finite-Smarandache 2-algebraic structure of Finite-near-rings, namely, Finite-Smarandache-near-ring, written as Finite-S-near-ring. A Finite-Smarandache 2-algebraic structure on a Finite-set \( N \) means a weak algebraic structure \( A_0 \) on \( N \) such that there exist a proper subset \( M \) of \( N \), which is embedded with a stronger algebraic structure \( A_1 \), stronger algebraic structure means satisfying more axioms, by proper subset means a subset different from the empty set, from the unit element if any, from the whole set [5]. By a Finite-near-ring \( N \), we mean a zero-symmetric Finite-right-near-ring. For basic concept of near-ring we refer to Gunter Pilz [2].

Definition 1. A Finite-near-ring \( N \) is said to be Finite-Smarandache-near-ring. If a proper subset \( M \) of \( N \) is a Finite-field under the same induced operations in \( N \).

Example 1 [2]. Let \( N = \{0, n_1, n_2, n_3\} \) be the Finite-near-ring defined by:

Let \( M = \{0, n_1\} \subset N \) be a Finite-near-field. Defined by

Now \((N, +, \cdot, 0, 1)\) is a Finite-S-near-ring.

Example 2 [4]. Let \( N = \{0, 6, 12, 18, 24, 30, 36, 42, 48, 54\} \) (mod 60) be the Finite-near-ring since every ring is a near-ring. Now \( N \) is a Finite-near-ring, Whose proper subset \( M = \{0, 12, 24, 36, 48\} \) (mod 60) is a Finite-field. Since every field is a near-field, then \( M \) is a Finite-near-field. Therefore \( N \) is a Finite-S-near-ring.

Theorem 1. Let \( N \) be a Finite-near-ring, \( N \) is a Finite-S-near-ring if and only if there exist a proper subset \( M \) of \( N \), either \( M \cong M_\phi(z_2) \) or \( Z_p \), integers modulo \( p \), a prime number.

Proof. Part-I: We assume that \( N \) is a Finite-S-near-ring. By definition, there exist a proper subset \( M \) of \( N \) is a Finite-near-field. By Gunter Pilz Theorem (8.1)[2], either \( M \cong M_\phi(z_2) \) or \( Z_p \), integers modulo \( p \), a prime number.
In the notation of the text, \( M_c(z_2) \) or zero-symmetric. Since \( Z_p^S \) is zero-symmetric and Finite-fields implies \( Z_p \), \( S \) are zero-symmetric and Finite-near-fields because every field is a near-field. Therefore in particular \( M \) is \( Z_p \).

**Part-II:** We assume that a proper subset \( M \) of \( N \), either \( M \cong M_c(z_2) \) or \( Z_p \). Since \( M_c(z_2) \) and \( Z_p \) are Finite-near-fields. Then \( M \) is a Finite-near-field. By definition, \( N \) is a Finite-S-near-ring.

**Theorem.** Let \( N \) be a Finite-near-ring. \( N \) is a Finite-S-near-ring if and only if there exist a proper subset \( M \) of \( N \) such that every element in \( M \) satisfying the polynomial \( x^{pm} - x \).

**Proof.** **Part-I:** We assume that \( N \) is a Finite-S-near-ring. By definition, there exist a proper subset \( M \) of \( N \) is a Finite-near-field. By Gunter Pilz, Theorem (8.13)[2]. If \( M \) is a Finite-near-field, then there exist \( p \in P, \exists m \in M \) such that \( M \mid p^m \). According to I.N.Herstein[3]. If the Finite-near-field \( M \) has \( p^m \) element, then every \( a \in M \) satisfies \( a^{p^m} = a \), since every field is a near-field. Now \( M \) is a Finite-field having \( p^m \) element, every element \( a \) in \( M \) satisfies \( a^{p^m} = a \). Therefore every element in \( M \) satisfying the polynomial \( x^{p^m} - x \).

**Part-II:** We assume that there exist a proper subset \( M \) of \( N \) such that every element in \( M \) satisfying the polynomial \( x^{p^m} - x \), which implies \( M \) has \( p^m \) element. According to I.N.Herstein[3]. For every prime number \( p \) and every positive integer \( m \), there is a unique field having \( p^m \) element. Hence \( M \) is a Finite-field implies \( M \) is a Finite-near-field. By definition, \( N \) is a Finite-S-near-ring.

**Theorem 3.** Let \( N \) be a Finite-near-ring. \( N \) is a Finite-S-near-ring if and only if \( M \) has no proper left ideals and \( M_0 \neq M \). Where \( M \) is a proper sub near-ring of \( N \), in which idempotent commute and for each \( x \in M \), there exist \( y \in M \) such that \( xy \neq 0 \).

**Proof.** **Part-I:** We assume that \( N \) is a Finite-S-near-ring. By definition A proper subset \( M \) of \( N \) is a Finite-near-field. In [1] Theorem (4), it is zero-symmetric and hence every left-ideal is a M-subgroup. Let \( M_1 \neq 0 \) be a M-subgroup and \( m_1 \neq 0 \in M_1 \). Then \( m_1^{-1}m_1 = 1 \in M_1 \), therefore \( M = M_1 \). Hence \( M \) has no proper M-subgroup, which implies \( M \) has no proper left ideal.

**Part-II:** We assume that a proper sub near-ring \( M \) of \( N \) has no proper left ideals and \( M_0 \neq M \), in which idempotent commute and for each \( x \in M \) there exist \( y \in M \) such that \( xy \neq 0 \). Let \( x = 0 \) in \( M \). Let \( F(x) = \{ m \in M \mid mx = 0 \} \). Clearly \( F(x) \) is a left ideal. Since there exist \( y \in M \) such that \( xy \neq 0 \). Then \( y \notin F(x) \). Hence \( F(x) = 0 \). Let \( \phi : (M, +) \longrightarrow (Mx, +) \) given by \( \phi(m) = mx \). Then \( \phi \) is an isomorphism. Since \( M \) is finite then \( Mx = M \). Now by a theorem(2) in [1], \( M \) is a Finite-near-field. Therefore, by definition \( N \) is a Finite-S-near-ring.

We summarize what has been studied in

**Theorem 4.** Let \( N \) be a Finite-near-ring. Then the following conditions are equivalent.

1. A proper subset \( M \) of \( N \), either \( M \cong M_c(z_2) \) or \( Z_p \), integers modulo \( p \), a prime number.
2. A proper subset \( M \) of \( N \) such that every element in \( M \) satisfying the polynomial \( x^{p^m} - x \).
3. \( M \) has no proper left ideals and \( M_0 \neq M \). Where \( M \) is a proper sub near-ring of \( N \), in which idempotent commute and for each \( x \in M \), there exist \( y \in M \) such that \( xy \neq 0 \).

**Theorem 5.** Let \( N \) be a Finite-near-ring. If a proper subset \( M \), sub near-ring of \( N \), in which \( M \) has left identity and \( M \) is 0-primitive on \( M^M \). Then \( N \) is a Finite-S-near-ring.

**Proof.** By Theorem(8.3)[2], the following conditions are equivalent:
(1) $M$ is a Finite-near-field;
(2) $M$ has left identity and $M$ is 0-primitive on $M^M$.

Now Theorem is immediate.

**Theorem 6.** Let $N$ be a Finite-near-ring. If a proper subset $M$, sub near-ring of $N$, in which $M$ has left identity and $M$ is simple. Then $N$ is a Finite-$S$-near-ring.

**Proof.** By Theorem(8.3)[2], the following conditions are equivalent:
(1) $M$ is a Finite-near-field;
(2) $M$ has left identity and $M$ is simple. Now the Theorem is immediate.

**Theorem 7.** Let $N$ be a Finite-near-ring. If a proper subset $M$, sub near-ring of $N$ is a Finite-near-domain, then $N$ is a Finite-$S$-near-ring.

**Proof.** By Theorem(8.43)[2], a Finite-near-domain is a Finite-near-field. Therefore $M$ is a Finite-near-field. By definition $N$ is a Finite-$S$-near-ring.

**Theorem 8.** Let $N$ be a Finite-near-ring. If a proper subset $M$ of $N$ is a Finite-Integer-domain. Then $N$ is a Finite-$S$-near-ring.

**Proof.** By I.N.Herstein[3], every Finite-Integer-domain is a field, since every field is a near-field. Now $M$ is a Finite-near-field. By definition $N$ is a Finite-$S$-near-ring.

**Theorem 9.** Let $N$ be a Finite-near-ring. If a proper subset $M$ of $N$ is a Finite-division-ring. Then $N$ is a Finite-$S$-near-ring.

**Proof.** By Wedderburn’s Theorem(7.2.1)[3], a Finite-division-ring is a necessarily commutative field, which gives $M$ is a field, implies $M$ is a Finite-near-field. By definition $N$ is a Finite-$S$-near-ring.

**References**