Smarandache's Concurrent Lines Theorem

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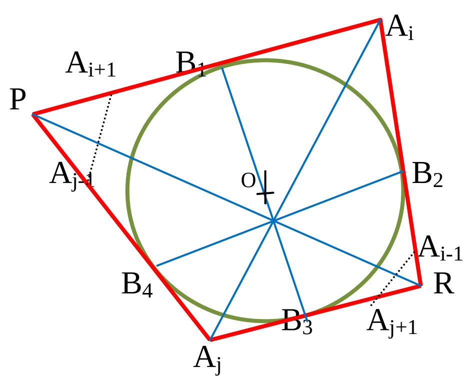
Abstract.
In this paper we present the Smarandache's Concurrent Lines Theorem in the geometry of the triangle.

Smarandache's Concurrent Lines Theorem.
Let's consider a polygon (which has at least four sides) circumscribed to a circle, and D the set of its diagonals and the lines joining the points of contact of two non-adjacent sides. Then D contains at least three concurrent lines.

Proof.
Let n be the number of sides. If $n = 4$, then the two diagonals and the two lines joining the points of contact of two adjacent sides are concurrent (according to Newton's Theorem).

The case $n > 4$ is reduced to the previous case: we consider any polygon $A_1 ... A_n$ (see the figure)



circumscribed to the circle and we choose two vertices A_i, A_j ($i \neq j$) such that

$$A_j\ A_{j-1} \cap A_i\ A_{i+1} = P$$

and

$$A_j\ A_{j+1} \cap A_i\ A_{i-1} = R.$$

Let B_h, $h \in \{1, 2, 3, 4\}$, be the contact points of the quadrilateral PA_jRA_i with the circle of center O. Because of the Newton's theorem, the lines $A_i\ A_j$, B_1B_3 and B_2B_4 are concurrent.

Open Problems related to the Smarandache Concurrent Lines Theorem.

2.1. In what conditions are there more than three concurrent lines?

2.2. What is the maximum number of concurrent lines that can exist (and in what conditions)?

2.3. What about an alternative of this problem: to consider instead of a circle an ellipse, and then a polygon *ellipsoscribed* (let's invent this word, *ellipso-scribed*, meaning a polygon whose all sides are tangent to an ellipse which inside of it): how many concurrent lines we can find among its diagonals and the lines connecting the point of contact of two non-adjacent sides?

2.4. What about generalizing this problem in a 3D-space: a sphere and a polyhedron circumscribed to it?

2.5. Or instead of a sphere to consider an ellipsoid and a polyhedron *ellipsoido-scribed* to it?

2.6. What about considering the lines that connect a vertex with a non-adjunct point of contact? Are there three or more such lines that intersect in the same point? {Consider all previous five questions.}

Comments.

Of course, we can go by construction reversely: take a point inside a circle (similarly for an ellipse, a sphere, or ellipsoid), then draw secants passing through this point that intersect the circle (ellipse, sphere, ellipsoid) into two points, and then draw tangents to the circle (or ellipse), or tangent planes to the sphere or ellipsoid) and try to construct a polygon (or polyhedron) from the intersections of the tangent lines (or of tangent planes) if possible.

For example, a regular polygon (or polyhedron) has a higher chance to have more concurrent such lines.

In the 3D space, we may consider, as alternative to this problem, the intersection of planes (instead of lines).

References:

[1] F. Smarandache, *Problèmes avec et sans… problèmes!*, Problem 5.36, p. 54, Somipress, Fés, Morocco, 1983.

[2] F. Smarandache, *Eight Solved and Eight Open Problems in Elementary Geometry*, in arXiv.org, Cornell University, NY, USA.

[3] M. Khoshnevisan, *Smarandache's Concurrent Lines*, NeuroIntelligence Center, Australia, http://www.scribd.com/doc/28325583/A-Smarandache-Concurrent-Lines-Theorem