Smarandache Φ-Theorem

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Abstract.
Fermat's and Euler's theorem on congruencies are generalized to the case when the integers a and m are not necessarily co-prime.

Smarandache Φ-Theorem.

If $a, m \in Z$ and $m \neq 0$, then

$$a^{\varphi(m_s)+s} \equiv a^s \pmod{m}$$

where φ is Euler's totient function, and m_s and s are obtained from the below

Smarandache Φ-Algorithm:

Step 1. $A := a,\ M := m,\ i := 0$

Step 2. Calculate $d = (A, M)$ and $M' = \dfrac{M}{d}$

Step 3. If $d = 1$ take $s = i,\ m_s = M'$, and stop.

If $d \neq 1$ take $A := d,\ m = M',\ i := i + 1$ and go to Step 2.

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This is a generalization of Euler's Theorem: if $a, m \in Z$ and $(a, m) = 1$, then $a^{\varphi(m)} \equiv 1 \pmod{m}$.

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Smarandache totient function is defined as:

$$S\Phi : Z^2 \to Z^2.$$

For $m \neq 0,\ S\Phi(a, m) = (m_s, s)$ such that $a^{\varphi(m_s)+s} \equiv a^s \pmod{m}$.

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Study the Smarandache Φ-Theorem, Smarandache Φ-Algorithm, and Smarandache totient function.