Commonsense local realism refutes Bell's theorem.

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Abstract: With Bell (1964) and his EPR-based mathematics contradicted by experiments, at least one step in his supposedly commonsense theorem must be false. Defining commonsense local realism as the fusion of local-causality (no causal influence propagates superluminally) and physical-realism (some physical properties change interactively), we eliminate all such contradictions and make EPR correlations intelligible by completing the quantum mechanical account in a classical way. Thus refuting the famous inequality at the heart of Bell's mathematics, we show that Bell's theorem is limited by Bell's naive realism. Endorsing the classical mantra that correlated tests on correlated things produce correlated results without mystery, we conclude that Bell's theorem and related experiments negate naive realism, not commonsense local realism.

1 Notes to the Reader

a. Pre-reading: EPR and Bell (1964), available on-line, are taken as read; EPR to the start of page 778, Bell to his equation (15). Other texts are also available via hyperlinks in References.
b. Notation: $(\vec{u}, \vec{v})$ denotes the angle between vectors $\vec{u}$ and $\vec{v}$. $\vec{u}{\cdot}\vec{v}$ is their inner product.
c. Results: Unlike Bell's theorem, all our results accord with reputable experimental findings.
d. Errors: Please report errors and typos; critical correspondence is especially welcome.
e. Key words: Dynamic equivalence class, left-to-right precedence, local realism, operator Q.

2 Introduction

Embracing commonsense local realism (CLR), the fusion of local-causality (no causal influence propagates superluminally) and physical-realism (some physical properties change interactively), we endorse EPR's (1935:777) condition of completeness: Every relevant element of the physical reality must have a counterpart in our physical theory. But we reject the naive realism in Bell (1964; 2004); and the 'nonlocality' so often linked to Bell's theorem and his impossibility proof.

> "Bell's theorem asserts [?] that if certain predictions of quantum theory are correct then our world is non-local. "Non-local" here means that there exist interactions between events that are too far apart in space and too close together in time for the events to be connected even by signals moving at the speed of light," Goldstein *et al.* (2011).
>
> NB: We accept that those 'certain predictions' are correct! "Indeed it was the explicit representation of quantum nonlocality [in de Broglie-Bohm theory] which started a new wave of investigation in this area [of local causality]. Let us hope that these analyses also may one day be illuminated, perhaps harshly, by some simple constructive model. However that may be, long may Louis de Broglie continue to inspire those who suspect that what is proved by impossibility proofs is lack of imagination," (Bell 2004:167).

In fact, a mooted 'impossibility' in Mermin (1988) led us announce CLR: a simple constructive model that counters such nonlocality in full accord with relativity and Einstein's ideas.

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On one supposition we absolutely hold fast; that of local-causality, often called Einstein-locality: "The real *factual* situation of the system S_2 is independent of what is done with the system S_1, which is spatially separated from the former," after Einstein (1949:85).

Thus, for us, natural physical variables and their local interactions alone account for the correlated results produced by correlated tests on correlated things. And for us, following Bell (2004:174): Unlike *observables,* natural physical variables are *beables*, elements of reality, things which exist, their existence – independent of *measurement* and *observation* – sometimes exposed by tests.

So, here delivering CLR, we proceed as follows: After foreshadowing our case against a wholly mathematical version of Bell's theorem, we define our terms in the context of EPRB – shorthand for the experiment in Bell (1964). We then develop operator Q to formalize the beables and equivalencies that we associate with EPRB particle/device interactions. Bell's theorem is then refuted mathematically before we discuss Q's significance and demonstrate Q's utility in Mermin's (1990a) 3-particle variant of GHZ. We end with conclusions, acknowledgments, a technical appendix and references. The busy reader might jump to the Appendix, particularly 11.3.

3 Bell's theorem and our foreshadowing

"It is a matter of indifference ... whether λ denotes a single variable or a set, or even a set of functions, and whether the variables are discrete or continuous," Bell (1964:195).

With $\langle AB\rangle$ replacing Bell's $P(\vec{a},\vec{b})$ to avoid confusion with other functions; with $p[\lambda]$ and $p'[\lambda']$ distinguishing paired-particles produced in EPRB (where $\lambda' = -\lambda$); here's a wholly mathematical version of Bell's theorem based on Bell's equations 1964:(1)-(3), (12)-(14):

$$\textbf{If } A(\vec{a},\lambda) = A^{\pm} = \pm 1;\ B(\vec{b},\lambda') = B^{\pm} = \pm 1 = -A(\vec{b},\lambda); \int d\lambda\, \rho(\lambda) = 1; \tag{1}$$

$$\textbf{then } \langle AB\rangle \equiv \int d\lambda\, \rho(\lambda) A(\vec{a},\lambda) B(\vec{b},\lambda') = -\int d\lambda\, \rho(\lambda) A(\vec{a},\lambda) A(\vec{b},\lambda) \neq -\vec{a}\cdot\vec{b}. \tag{2}$$

Introduced via the line following his 1964:(3), and based on the paragraph following his 1964:(15), the $\neq$ in (2) is the famous first inequality in a family of relations – collectively known as *Bell inequalities* – that are widely associated with the acceptance of nonlocality and the rejection of commonsense local realism. Which brings us to the development of (2) and Bell's later (2004:147) explanation:

"To explain this dénouement without mathematics I cannot do better than follow d'Espagnat (1979; 1979a)." Our paraphrase of d'Espagnat (1979:166) follows:

'One can infer that in every particle-pair [every pair of twins: to harness Bell's upbeat 'genetic' hypothesis (Bernstein 1991:84)], one particle has the property A^+ and the other has the property A^-, one has property B^+ and one B^-, ... Such conclusions require a subtle but important extension of the meaning assigned to our notation A^+. Whereas previously A^+ was merely one possible outcome of a measurement made on a particle, it is converted by this argument into an attribute of the particle itself.'

For us, preferring 'outcome of a test' to d'Espagnat's (1979:166) 'outcome of a measurement', and concluding that Bell's theorem is based on a restrictive naive realism, we reject any such limitation when working to understand EPRB. On the contrary, foreshadowing our own position:

One can infer that in any particle-pair [any pair of twins: to harness Bell's upbeat 'genetic' hypothesis (Bernstein 1991:84)], one particle has the property $[+\vec{a}\}$ and the other particle has the property $[-\vec{a}\}$ ... where $[.\}$ is not the outcome of a test — like A^+ or A^- — but the dynamic equivalence class revealed by such outcomes.

That is: We allow that A^+ reveals the previously-hidden preexisting dynamic equivalence class (DEC) $[+\vec{a}\}$ — see Appendix 11.1 — to which $p[\lambda]$ belongs: so $p[\lambda] = p[+\vec{a}\}$ reveals the previously-hidden (but related) DEC of its pristine correlate; ie, $p'[\lambda'] = p'[-\lambda] = p'[-\vec{a}\}$.

That is, in our terms: A^+ reveals that the pristine $p[\lambda]$ here is such that $\lambda \in [+\vec{a}\}$; so $p[+\vec{a}\}$ is a more complete description. Thus its twin, its pristine correlate $p'[\lambda']$, is also more completely described by $p'[-\vec{a}\}$. All in accord with Bell's 1964:(13) implication that $p'[\lambda'] = p'[-\lambda]$ in our terms. So we have here an important (because physically significant) CLR distinction. For $[+\vec{a}\}$ and $[-\vec{a}\}$ are *diametrically opposed* pristinely: making it impossible to confuse our pristine pair $p[+\vec{a}\}$ and $p'[-\vec{a}\}$ with the very different — but only candidate pair — $p[-\vec{a}\}$ and $p'[+\vec{a}\}$.

Putting it personally, but still in the context of CLR: A secret interview and painful operation on my far-off twin sister will certainly reveal that I'm a member of a particular equivalence class; yet I remain unperturbed and unbloodied by such far-off and unknown-to-me interactions. For:

> 'There are no *messages* from one system to the other. EPRB correlations do not give rise to signaling between noninteracting systems. Of course, however, *such correlations allow inferences* from events in one system (eg, A^+) to events in the other (eg, B^-),' paraphrasing Bell (2004:208) in our terms.

Thus we arrive at the key to our analysis, as we will show via operator Q: We include the CLR elements of such inferences in our mathematics. For, in the micro-physics here, allowing that there may be 'no infinitesimals by the aid of which an observation might be made without appreciable perturbation' (Heisenberg 1930:63), we also allow that preexisting pristine properties (ie, beables; such as being a member of a DEC) may be revealed by such perturbations. So, for us:

If a test on a particle reveals an associated DEC, then its pristine twin is a member of a similar class: by virtue of *the fact* that such twins are physically correlated by their birth in a spin-conserving decay. Thus do we endorse EPR's elements of physical reality, defined as follows:

> "If, without any way disturbing a system, we can predict with certainty (ie, with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality [a beable] corresponding to this physical quantity," EPR (1935:777).

Differing markedly from the analysis just given, Bell's naive realism leads to contradictions. For, subject only to our later showing *that (2) is false based on (1) alone*, Bell's 1964:(15) reads thus:

$$1 + \langle BC \rangle = 1 - \vec{b} \cdot \vec{c} \geq |\vec{a} \cdot \vec{c} - \vec{a} \cdot \vec{b}| = |\,\langle AB \rangle - \langle AC \rangle\,|; \tag{3}$$

a relation that is certainly false over the range $-\pi/2 < \phi < \pi/2$ if $(\vec{a}, \vec{b}) = (\vec{b}, \vec{c}) = \phi$ and $(\vec{a}, \vec{c}) = 2\phi$.

To derive our (3), his 1964:(15), Bell goes beyond our (1)-(2) and invokes a third unit-vector $\vec{c}$ in the unnumbered equations that follow his 1964:(14). It can be shown that Bell's theorem is limited to entities that satisfy his unnumbered equations.

So, having foreshadowed our case against Bell, and now on the way to formally refuting Bell's theorem from (1) and LHS (2) alone, we next study EPRB.

4 The EPRB context

> λ may denote "any number of hypothetical additional complementary variables needed to complete quantum mechanics in the way envisaged by EPR," Bell (2004:242).

In (1), $A^\pm$ ($B^\pm$) denotes the result that experimentalist Alice (Bob) obtains by testing a pristine spin-half particle $p[\lambda]$ ($p'[\lambda']$) with a Stern-Gerlach device SGD$\vec{a}$, denoted $\widehat{A\vec{a}}$ (SGD$\vec{b}$, denoted $\widehat{B\vec{b}}$); a device with a suitable field, detector and printer. $\vec{a}$ ($\vec{b}$), a unit-vector in 3-space ($\mathbb{R}^3$), denotes the freely-selected orientation of the principal axis; and, of course, $\vec{a}$ may equal $\vec{b}$.

Then, via the spherical symmetry associated with the pair-wise conservation of spin in EPRB, we allow that $\lambda + \lambda' = 0$ after that spin-conserving decay and prior to any test. Thus Bob's pristine particle $p'[\lambda']$ may also be represented by $p'[-\lambda]$; noting that λ and λ' are, for us, spin-related CLR beables in the form of separable hidden variables. And so, for us: $\lambda \in \Lambda \subset \mathbb{R}^3$.

Sometimes representing $\pm\vec{a}$ by $+\vec{a}$ xor $-\vec{a}$ to emphasize symmetries in our analysis, let $\widehat{A\vec{a}}\bullet p[\lambda]$ denote the local test (interaction) that transforms λ to $\pm\vec{a}$; ie, to a concluded transition (post-test orientation) denoting spin-up or spin-down with respect to $\vec{a}$. $\widehat{B\vec{b}} \bullet p'[\lambda']$ similarly. Thus, with the widely-separated $\widehat{A\vec{a}}$ and $\widehat{B\vec{b}}$ correlated by the angle $(\vec{a}, \vec{b})$, and with λ and λ' anti-correlated by their spin-conserving birth, we expect the causally-independent $A^{\pm}$ and $B^{\pm}$ to be correlated, consistent with our classical mantra.

However, in full accord with reciprocal causal independence and local-causality (ie, no causal influence propagates superluminally), a boundary condition on our analysis is this: $A^{\pm}$ is causally independent of $\widehat{B\vec{b}}, B^{\pm}, \lambda'$; but may be causally dependent on any property of λ. $B^{\pm}$ is causally independent of $\widehat{A\vec{a}}, A^{\pm}, \lambda$; but may be causally dependent on any property of λ'. That is, like Bell:

> We are "careful not to assert that there is action at a distance," Bell (1990:13).

So we move to develop a mathematics that delivers the results of local particle/device interactions *as well as their* factual *implications*. A maths that delivers the results of SGD/particle interactions and consequently updates facts re pristine correlates: ie, a mathematical IF ... THEN ... that converts the source of our inferences (*reasoning*) to physically significant properties (*what follows from that*) via the mathematical transmission of facts: independent of vague words and conjectures.

> 'Surely the big $\widehat{A\vec{a}}$ and the small $p[\lambda]$ should merge smoothly with one another? And surely in fundamental physical theory this merging should be described not just by vague words but by precise mathematics?' after Bell (2004:190).

5 Operator Q

> "The concept of 'measurement' becomes so fuzzy on reflection that it is quite surprising to have it appearing in physical theory *at the most fundamental level*. ... does not any *analysis* of measurement require concepts more *fundamental* than measurement? And should not the fundamental theory be about these more fundamental concepts? One line of development towards greater physical precision would be to have the [quantum] 'jumps' [or mergings] in the equations and not just in the talk — so it would come about as a dynamical process in dynamically defined conditions," Bell (2004:117-118).

In the context of EPRB, taking *transformation* to be a concept 'more fundamental than measurement', we accept that the local interaction $\widehat{A\vec{a}} \bullet p[\lambda]$ transforms both the particle and the device. And we allow that the transformation of the device (eg, its printing of the result A^+) will reveal a DEC $[+\vec{a}\}$. Here, that DEC is a previously-hidden preexisting property of the pristine particle that was tested. And most importantly: a pristine correlate will have a related DEC. More formally, we move to define operator Q:

For now, with g an element beyond Q's domain; with F and G physically significant functions — F associated with SGD$\vec{a}$ and $p[\lambda]$; G associated with SGD$\vec{b}$ and $p'[\lambda'] = p'[-\lambda]$ — let $Q\vec{a}(.) \equiv [. \to \vec{a}\}$ denote an operator. Informally 'operator Q', or simply Q, when the context is clear, let Q — with its left-to-right precedence over similar operators — be such that

$$[\lambda \to \vec{a}\}g \equiv g; \tag{4}$$

$$[\lambda \to \vec{a}\}F(\lambda) \equiv F(\vec{a}); \tag{5}$$

$$[\lambda \to \vec{a}\}F(\lambda)[\lambda' \to \vec{b}\}G(\lambda') \equiv [\lambda \to \vec{a}\}F(\lambda)[-\lambda \to \vec{b}\}G(-\lambda)$$
$$\equiv F(\vec{a})[-\lambda \to \vec{b}\}[\lambda \to \vec{a}\}G[-\lambda\} = F(\vec{a})[-\lambda \to \vec{b}\}G[-\vec{a}\} = F(\vec{a})G[-\vec{a}\} = F(\vec{a})G(-\vec{a}). \tag{6}$$

The argument $\lambda \to \vec{a}$ denotes Q's transformation of λ to $\vec{a}$, there being no requirement that $\lambda = \vec{a}$ prior to Q's action. (4) defines Q's operation on g. (5) shows Q's operation on a physical function: in physical terms, $[\lambda \to \vec{a}\}$ represents a local physical transformation of λ to $\vec{a}$, with a consequent local physical transformation of $F(\lambda)$ to $F(\vec{a})$. Thus, in the model we are developing here, SGD$\vec{a}$ would be triggered to print $A^+ = +1$.

In (6), we define Q's left-to-right precedence. With the left-most variable being λ, we first expose λ in all succeeding arguments to thus reveal the left-most Q's locus of operations. So, mathematically, Q here first delivers $F(\vec{a})$ — the result of the physical interaction $\widehat{A\vec{a}} \bullet p[\lambda]$ — to thus reveal A^+ and the consequent physical fact that $\lambda \in [+\vec{a}\}$.

Q next brings $\lambda \in [+\vec{a}\}$ to $G(-\lambda)$. So we have, in (6): $F(\vec{a})[-\lambda \to \vec{b}\}G[-\vec{a}\}$; which reduces to $F(\vec{a})G[-\vec{a}\}$ since $G[-\vec{a}\}$ is outside the domain of the next Q, $[-\lambda \to \vec{b}\}$ — see (4). Finally, since $[-\vec{a}\}$ and $(-\vec{a})$ are equivalent arguments under G, $F(\vec{a})G[-\vec{a}\} = F(\vec{a})G(-\vec{a})$.

To be clear: Any function of λ, with λ replaced by a constant, is a constant; in this case a constant outside the domain of the relevant operator. So $[\lambda' \to \vec{b}\}$ performs correctly in (6), per (4). However, concerning its 'apparent impotence' in (6), see Appendix 11.2.

Thus, to be clear in the context of EPRB: Sequentially, as Q's operation on functions proceeds left-to-right, the first operation represents local physical action on the related physical function, thereby triggering an output: here $A^+ = +1$; thereby identifying the DEC $[+\vec{a}\}$.

Subsequent left-to-right operations then allocate the relevant DEC to the appropriate physical variable in the twinned correlate; reflecting the now-known fact that that far-off pristine variable is related to this class.

Then, allowing that there can be no preferred reference-frame in the study of widely separated tests or disturbances, operator-precedence cannot imply operator preference here. That is — pedagogically ignoring the 'impotence' shortcut for now — (6) may be reversed to yield

$$[\lambda' \to \vec{b}\}G(\lambda')[\lambda \to \vec{a})\}F(\lambda) = [\lambda' \to \vec{b}\}G(\lambda')[-\lambda' \to \vec{a})\}F(-\lambda')$$
$$= G(\vec{b})[-\lambda' \to \vec{a}\}[\lambda' \to \vec{b}\}F(-\lambda') = G(\vec{b})[-\lambda' \to \vec{a}\}F[-\vec{b}\} = G(\vec{b})F[-\vec{b}\} = G(\vec{b})F(-\vec{b}). \quad (7)$$

Consequently, from the symmetry in EPRB, we observe that the functions $F(\lambda)$ and $G(\lambda)$ in (6) and (8) must be such that $F(\vec{a})G(-\vec{a}) = G(\vec{b})F(-\vec{b})$.

In the context of EPRB and those EPR beables, let $F(\lambda) = \vec{a} \cdot \lambda$ and $G(\lambda') = \vec{b} \cdot \lambda'$: Then, in testing $A(\vec{a}, \lambda)$, let Alice find A^+; ie, $A^+ = [\lambda \to \vec{a}\}F(\lambda) = [\lambda \to \vec{a}\}\vec{a} \cdot \lambda = \vec{a} \cdot \vec{a} = +1$. Thus, without any further disturbance anywhere, Alice can predict with certainty that $B(\vec{a}, \lambda') = B^- = -1$. For Alice has, modifying (6) and using the 'impotence' shortcut and obvious reductions,

$$[\lambda \to \vec{a}\}\vec{a} \cdot \lambda[\lambda' \to \pm\vec{a}\}\vec{a} \cdot \lambda' = [\lambda \to \vec{a}\}\vec{a} \cdot \lambda[\lambda' \to \pm\vec{a}\}\vec{a} \cdot (-\lambda) = (+1)[\lambda' \to \pm\vec{a}\}\vec{a} \cdot [-\vec{a}\} = -1. \quad (8)$$

Thus, via the equivalence class to which the λ' in this test belongs, the EPR beable in Bob's test here is $p'[-\vec{a}\}$. For our maths has properly specified the physically significant preexisting property $\lambda' \in [-\vec{a}\}$ of the pristine $p'[\lambda']$ that Bob will test. In other words: $p'[-\vec{a}\}$ — the EPR beable that corresponds to the test result B^- — allows us complement EPR with a CLR comment:

> Unsurprisingly: Without in any way disturbing particle $p'[\lambda']$ – ie, the particle $p'[-\vec{a}\}$ – we can predict with certainty the result B^- of the particle's interaction with $\widehat{B\vec{a}}$.

Of course, if you'd like to predict, with certainty, a pristine particle's interaction with $\widehat{B\vec{b}}$: be Alice and test its twin with $\widehat{A\vec{b}}$.

6 Bell's theorem refuted

> Einstein argues that 'EPR correlations can be made intelligible only by completing the quantum mechanical account in a classical way,' after Bell (2004:86). Let's see.

We have λ relating to the spin of a pristine unpolarized spin-half particle $p[\lambda]$; for "each particle, considered separately, *is* unpolarized here," Bell (2004:82). As such, λ will be perturbed by p's interaction with $\widehat{A\vec{a}}$. Representing that interaction by $\widehat{A\vec{a}}\bullet p[\lambda] \equiv [\lambda \to \pm\vec{a}\}$, λ will be transformed to $\pm\vec{a}$ equiprevalently (ie, with equal prevalence). For we allow λ to be a random parameter with a uniform distribution over 3-space. So, expanding (1) in our terms, in line with Bell's example:

$$A(\vec{a},\lambda) = A^{\pm} = [\lambda \to \pm\vec{a}\}\,\vec{a}\cdot\lambda = \vec{a}\cdot(\pm\vec{a}) = \pm 1, \tag{9}$$

$$B(\vec{b},\lambda') = B^{\pm} = -A(\vec{b},\lambda) = -[\lambda \to \pm\vec{b}\}\vec{b}\cdot\lambda = -\vec{b}\cdot(\pm\vec{b}) = \mp 1, \tag{10}$$

$$\int d\lambda\,\rho(\lambda) = \frac{1}{4\pi}\int\limits_0^{4\pi} d\Omega = 1, \tag{11}$$

where Ω is a unit of solid-angle. Then, inserting (9)-(11) into LHS (2), and recalling (6):

$$\langle AB\rangle = -\frac{1}{4\pi}\int\limits_0^{4\pi} d\Omega[\lambda \to \pm\vec{a}\}\vec{a}\cdot\lambda[\lambda \to \pm\vec{b}\}\vec{b}\cdot\lambda = -(\pm 1)\vec{b}\cdot[\pm\vec{a}\} = -\vec{b}\cdot[\vec{a}\} = -\vec{a}\cdot\vec{b}.\ \text{QED;}\ \blacksquare \tag{12}$$

Bell's theorem, as represented in (2) above consistent with Bell's formulation, is refuted. Note that, consistent with the CLR requirement for our maths, operator Q delivers the physical $\pm\vec{a}$ to the physical function $\vec{a}\cdot\lambda$ to yield ± 1. However, to the other particle p', Q attributes the fact that its DEC is, in turn, $[\pm\vec{a}\}$ *under Bell's formulation;* ie, with the opening minus-sign in (12). Note that with our formulation — see Appendix 11.3 — there would be no opening minus-sign and the physically relevant DEC $[\mp\vec{a}\}$ would be attributed to p': with the same correct outcome.

With (12) being a notable result, we next discuss Q's significance before demonstrating its utility in analyzing multiparticle experiments.

7 Q's significance

Let Z denote EPRB and let $P(X|Z)$ denote the *objective probability* of X given Z. (NB: in that our theory is devoid of subjective beliefs and non-physical entities, 'probable' and its derivatives are taken to be loaded terms here. So, to minimize confusion, we allow that P also denotes the *normalized prevalence*.) Then, taking maths to be the best logic, we now show how our beables, DECs and test outcomes relate to the mathematical/logic statement

$$P[XY|Z) = P(X|Z)\,P(Y|ZX) = P(Y|Z)\,P(X|ZY) \tag{13}$$

when X and Y are causally independent; ie, causally independent in the sense that neither exerts any direct causal influence on the other. However, like the apple and pear crop, we expect a mathematical (and hence logical) connection because of the common-cause physical correlation between them. Just as here, with our Q, we expect DECs to be related because of the physical correlations between closely-related (here, twinned) particles.

So, from (1) with the product $AB = A(\vec{a},\lambda)A(\vec{b},\lambda) = \pm 1$, we move to establish the consequential distribution of ± 1 as a function of $\vec{a}$ and $\vec{b}$. That is, we combine (9) with (10) and equate the result to a standard prevalence (objective probability) relation for their binary (± 1) outcomes:

$$AB = -[\lambda \to \pm\vec{a}\}\,\vec{a}\cdot\lambda\,[\lambda \to \pm\vec{b}\}\vec{b}\cdot\lambda = -(\pm 1)\vec{b}\cdot[\pm a\} = -\vec{a}\cdot\vec{b} \tag{14}$$

$$= (+1)P(AB = +1|Z) + (-1)[1 - P(AB = +1|Z)]. \tag{15}$$

$$\therefore P(AB = +1|Z) = (1 - \vec{a}\cdot\vec{b})/2 = \sin^2\tfrac{1}{2}(\vec{a},\vec{b});\ P(AB = -1|Z) = \cos^2\tfrac{1}{2}(\vec{a},\vec{b}). \tag{16}$$

(The $\frac{1}{2}$ in the trigonometric arguments represents the intrinsic spin $s = \frac{1}{2}$ of the spin-half particles; similar calculations on experiments with photons yield $s = 1$.) The ± 1 distributions for AB and $\langle AB \rangle$ are thus:

$$P(AB = +1|Z) = \sin^2 \tfrac{1}{2}(\vec{a}, \vec{b}) = P(\langle AB \rangle = +1|Z); \tag{17}$$

$$P(AB = -1|Z) = \cos^2 \tfrac{1}{2}(\vec{a}, \vec{b}) = P(\langle AB \rangle = -1|Z). \tag{18}$$

In this way (from first principles), we refute Bell's opinion (2004:243), and his move there from his (9) to his (10): that *causal independence* should equate to *statistical independence*, seen as a consequence of *local causality*.

> "One general issue raised by the debates over locality is to understand the connection between stochastic independence (probabilities multiply) and genuine physical independence (no mutual influence). It is the latter that is at issue in 'locality,' but it is the former that goes proxy for it in the Bell-like calculations. We need to press harder and deeper in our analysis here," Arthur Fine, in Schlosshauer (2011:45).

Derived from first principles, (17)-(18) respond to Fine's urgings and deliver: $P(A^+B^+|Z) \neq P(A^+|Z)\, P(B^+|Z)$; etc. Thus, given such physical correlations as those in EPRB, *statistical independence* does not equate to *causal independence* under *local causality*: nor with pear and apple crops. Rather, like the apple and pear crop discussed above, there is a physical correlation and hence a consequential mathematical (and therefore logical) relation between them. Just as, with our Q, we have physical correlations and consequent DECs in our maths/logic.

Comparing (14) with (12), we see that Q henceforth eliminates the need for the normalizing integral in expressions like (12); for Q is a normalized operator when, as here, its arguments are normalized. To demonstrate such normalization, and that Q's correct correlation of physical facts equates to valid and testable mathematical statements, we next expand the order of operations in (12):

$$\langle AB \rangle = P(A^+|Z)P(B^+|ZA^+) - P(A^+|Z)P(B^-|ZA^+)$$
$$- P(A^-|Z)P(B^+|ZA^-) + P(A^-|Z)P(B^-|ZA^-) \tag{19}$$

$$= \tfrac{1}{2} \sin^2 \tfrac{1}{2}(\vec{a}, \vec{b}) - \tfrac{1}{2} \cos^2 \tfrac{1}{2}(\vec{a}, \vec{b}) - \tfrac{1}{2} \cos^2 \tfrac{1}{2}(\vec{a}, \vec{b}) + \tfrac{1}{2} \sin^2 \tfrac{1}{2}(\vec{a}, \vec{b}) = -\vec{a} \cdot \vec{b}; \tag{20}$$

using (17-(18) with $P(A^+|Z) = P(A^-|Z) = 1/2$ (via equiprevalence).

Alternatively, and similarly, reversing the operations in (12) and then expanding them:

$$\langle AB \rangle = \langle BA \rangle = -[\lambda \to \pm\vec{b}\}\, \vec{b} \cdot \lambda\, [\lambda \to \pm\vec{a}\}\, \vec{a} \cdot \lambda = -(\pm 1)\vec{a} \cdot [\pm\vec{b}\} = -\vec{a} \cdot \vec{b} \tag{21}$$

$$= P(B^+|Z)P(A^+|ZB^+) - P(B^+|Z)P(A^-|ZB^+) - P(B^-|Z)P(A^+|ZB^-) + P(B^-|Z)P(A^-|ZB^-) \tag{22}$$

$$= \tfrac{1}{2} \sin^2 \tfrac{1}{2}(\vec{a}, \vec{b}) - \tfrac{1}{2} \cos^2 \tfrac{1}{2}(\vec{a}, \vec{b}) - \tfrac{1}{2} \cos^2 \tfrac{1}{2}(\vec{a}, \vec{b}) + \tfrac{1}{2} \sin^2 \tfrac{1}{2}(\vec{a}, \vec{b}) = -\vec{a} \cdot \vec{b}. \tag{23}$$

Then, since printer outputs like A^+ and B^+ identify and equate to hidden DECs: equivalent analytics for the hidden DECs themselves follow similarly; eg,

$$P(p'[+\vec{b}\}|Z,\, p[+\vec{a}\}) = \sin^2 \tfrac{1}{2}(\vec{a}, \vec{b}),\ P(p'[-\vec{b}\}|Z,\, p[+\vec{a}\}) = \sin^2 \tfrac{1}{2}(\vec{a}, -\vec{b}) = \cos^2 \tfrac{1}{2}(\vec{a}, \vec{b}). \tag{24}$$

With similar analysis delivering the correct results for GHZ (1989), GHSZ (1990), CRB (1991), we next demonstrate Q's utility via a related 3-particle experiment.

8 Q's utility

We consider Mermin's (1990; 1990a) 3-particle variant of GHZ. Respectively: Three spin-1/2 particles with spin parameters λ, μ, ν emerge from a spin-conserving decay such that

$$\lambda + \mu + \nu = \pi. \tag{25}$$

Any parameter may thus be pristinely represented in terms of the other two — eg, as (25) is used below, in the transition (29)-(30) — to allow precedent operators to supply facts re relevant variables.

The particles separate along three straight lines in the y-z plane to interact with three SGDs that are orthogonal to the related line of flight. Let a, b, c denote the azimuthal angles of each SGD's principal axis relative to the positive x-axis; let the test results be A, B, C; and let $\oplus$ denote xor. Then extending (9)-(10) and using (6)&(25) in accord with our protocol for operator and parameter reductions, we have:

$$A(a,\lambda) = A^{\pm} = [\lambda \to a \oplus a+\pi\} \cos(\lambda - a) = \pm 1, \tag{26}$$

$$B(b,\mu) = B^{\pm} = [\mu \to b \oplus b+\pi\} \cos(\mu - b) = \pm 1, \tag{27}$$

$$C(c,\nu) = C^{\pm} = [\nu \to c \oplus c+\pi\} \cos(\nu - c) = \pm 1. \tag{28}$$

$$\langle ABC \rangle = [\lambda \to a \oplus a+\pi\} \cos(\lambda - a)[\mu \to b \oplus b+\pi\} \cos(\mu - b)[\nu \to c \oplus c+\pi\} \cos(\nu - c) \tag{29}$$

$$= [\lambda \to a \oplus a+\pi\} \cos(\lambda - a)[\mu \to b \oplus b+\pi\} \cos(\mu - b)[\nu \to c \oplus c+\pi\} \cos(\pi - \lambda - \mu - c) \tag{30}$$

$$= [\mu \to b \oplus b+\pi\} \cos(\mu - b)[\nu \to c \oplus c+\pi\} \cos(\pi - [a\} - \mu - c) \oplus - \cos(-[a\} - \mu - c) \tag{31}$$

$$= [\nu \to c \oplus c+\pi\} \cos(\pi - [a\} - [b\} - c) \oplus - \cos(-[a\} - [b\} - c) \oplus - \cos(-[a\} - [b\} - c) \oplus \cos(-[a\} - [b\} - c - \pi) \tag{32}$$

$$= \cos(\pi - [a\} - [b\} - c) \oplus - \cos(-[a\} - [b\} - c) \oplus - \cos(-[a\} - [b\} - c) \oplus \cos(-[a\} - [b\} - c - \pi) \tag{33}$$

$$= \cos(\pi - a - b - c \oplus - \cos(-a - b - c) \oplus - \cos(-a - b - c) \oplus \cos(-a - b - c - \pi) \tag{34}$$

$$= -\cos(a + b + c).\ QED. \blacksquare \tag{35}$$

This is the correct result for the subject experiment; with Mermin's (1990a:733) 'crucial minus sign' properly delivered: For, from (35), $\langle ABC \rangle = -1$ when $a + b + c = 0$.

9 Conclusions

We conclude that Bell's theorem and related experiments negate naive realism, not commonsense local realism: for that famous inequality at the heart of Bell's analysis is false. Moreover, with every relevant element of each studied physical reality included in our physical theory — with no others, subjective or otherwise — we show that our classical mantra holds true: correlated tests on correlated things do produce correlated results without mystery.

We have also shown that, for us at least, mathematics is the best logic. For, though associated with hidden variables, the now discovered dynamic equivalence classes (DECs) are physically real and wholly amenable to mathematical analysis and experimental confirmation. We further note that the antipodean dichotomies associated with the DECs here are powerful discriminators.

Then, making EPR correlations intelligible by completing the quantum mechanical account in a classical way, our commonsense local realistic (CLR) theory also corrects the view — eg, Bell (2004:243) and Bell's move there from his (9) to his (10) — that *causal independence* should equate to *statistical independence*, seen as a consequence of *local causality*. For a chain of equivalence, based on physical correlations — not causal influences — links the causally independent outcomes in (1) and in (9)-(10) and in (26)-(28) to the appropriate local-realistic expectations $\langle . \rangle$.

Finally, working from first principles to show that Bell's work is limited by his naive realism, we also eliminate the source of Bell's discomfort (expressed in Bernstein 1991:84). So, thanking and honoring John Bell and Einstein, we here rephrase and reverse Bell's lament:

Perfect quantum correlations demand something like the 'genetic' hypothesis: like the triplets linked by λ, μ, ν in (25). It's so reasonable to assume that the particles carry with them programs, correlated in advance, telling them how to behave. This is so rational that when Einstein saw that, and the others refused to see it, he was the rational man. The others were burying their heads in the sand. So it's great that Einstein's idea of a classical locally-causal reality works. The reasonable thing works.

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* With Judi — the best partner in the world to me — for her enduring love and support.

11 Appendix

11.1 Dynamic equivalence classes (DECs): We define an equivalence relation $\sim$ on the set $\Lambda \subset \mathbb{R}^3$ of spin-related parameters λ as: Has the same output under operator $Q\vec{a}(.) \equiv [. \rightarrow \pm\vec{a}\} = \{\pm\vec{a} \leftarrow .]$. With (.) denoting the target, the notation is designed to be physically significant and technically convenient; eg, as in (37) below. Allowing (.) to be a random variable, it follows that the mutually-exclusive collectively-exhaustive equiprevalent outputs are $+\vec{a}$ and $-\vec{a}$ here. With $[\lambda \rightarrow \pm\vec{a}\}$ targeting λ, Λ is thus spanned by two DECs, defined as the sets

$$[+\vec{a}\} \equiv \{\lambda \in \Lambda \subset \mathbb{R}^3 | \lambda \sim +\vec{a} \in \mathbb{R}^3\},\ [-\vec{a}\} \equiv \{\lambda \in \Lambda \subset \mathbb{R}^3 | \lambda \sim -\vec{a} \in \mathbb{R}^3\}, \tag{36}$$

of elements that are related to $\pm\vec{a}$ by $\sim$. With $[\vec{a} \rightarrow \vec{a}\} = [\lambda \rightarrow \vec{a}\}$, we say that $[. \rightarrow \vec{a}\}$ is well-defined under the equivalence relation $\sim$ on Λ.

The strength of such DECs as $[\pm\vec{a}\}$ — ie, $[+\vec{a}\} \oplus [-\vec{a}\}$ — is that they are maximally discriminatory: antipodes, diametrically opposed. Hence the power of our maths resides in its identification and allocation of antipodean facts. For example, under the DECs associated with $Q\vec{a}$, Λ is partitioned dyadically so that the quotient set is a set of two extremes: $\Lambda/\sim = \{[+\vec{a}\}[-\vec{a}\}\}$.

The DECs are termed *dynamic* because there is little that Q cannot change. Thus DECs like $[\pm\vec{a}\}$ and $[\pm\vec{b}\}$ are subject to such transformations as $Q\vec{b}[\vec{a}\} \rightarrow \pm\vec{b}$ or $Q\vec{a}[\vec{b}\} \rightarrow \pm\vec{a}$.

11.2 Operator 'impotence': All our operators operate correctly in all the equations here. The right-most operator's 'apparent impotence' in our formalism — eg, the 'apparent impotence' of $[\lambda' \rightarrow \vec{b}\}$ in (6), $[\lambda' \rightarrow \pm\vec{a}\}$ in (8), $[\lambda \rightarrow \pm\vec{b}\}$ in (12), $[\nu \rightarrow c \oplus c + \pi\}$ in the transition (32)-(33) — is a consequence of the simplifications attaching to the left-to-right precedence of our operators. For the domain of each right-most operator is eliminated via the parameter reduction allowable from each pristine spin-conserving decay; see, eg, (25). We thus turn 'impotence' to our advantage; a shortcut: There is no need to modify the right-most operator under our rules of precedence; eg, $[\lambda' \rightarrow \pm\vec{a}\}$ in (8) and $[\nu \rightarrow c \oplus c + \pi\}$ in (29)-(32) allowably remain unchanged under this rule.

11.3 Bell's theorem refuted from first principles: Let $\{S\}$ denote a spin-conserving decay in EPRB; ie, $\lambda + \lambda' = 0$. Sketch the EPRB dynamics of a particle-pair triggering the SGDs thus:

$$A^{\pm} = (\pm 1) = (\vec{a} \cdot \lambda)\{\pm\vec{a} \leftarrow \lambda] \leftarrow p[\lambda] \leftarrow \{S\} \rightarrow p'[\lambda'] \rightarrow [\lambda' \rightarrow \pm\vec{b}\}(\lambda' \cdot \vec{b}) = (\pm 1) = B^{\pm}. \tag{37}$$

Then, in our terms and from the mathematical perspective of Alice — then Bob — here's (12):

$$\langle AB \rangle = [\lambda \rightarrow \pm\vec{a}\}\vec{a}\cdot\lambda[\lambda' \rightarrow \pm\vec{b}\}\vec{b}\cdot\lambda' = [\lambda \rightarrow \pm\vec{a}\}\vec{a}\cdot\lambda[\lambda' \rightarrow \pm\vec{b}\}\vec{b}\cdot(-\lambda) = (\pm 1)\vec{b}\cdot[\mp\vec{a}\} = -\vec{b}\cdot\vec{a}. \tag{38}$$

$$\langle BA \rangle = [\lambda' \rightarrow \pm\vec{b}\}\vec{b} \cdot \lambda'[\lambda \rightarrow \pm\vec{a}\}\vec{a} \cdot \lambda = [\lambda' \rightarrow \pm\vec{b}\}\vec{b} \cdot \lambda'[\lambda \rightarrow \pm\vec{a}\}\vec{a} \cdot (-\lambda') = (\pm 1)\vec{a} \cdot [\mp\vec{b}\} = -\vec{a} \cdot \vec{b}. \tag{39}$$

QED. Focussing on Alice's point of view – (38) – for the moment: Consistent with the CLR requirement for our mathematics, we see that $[\lambda \to \pm\vec{a}\}\vec{a}\cdot\lambda$ — denoting the interaction $\widehat{A\vec{a}}\bullet p[\lambda]$ — delivers the physical outcome $\pm\vec{a}$ to the physical function $\vec{a}\cdot\lambda$. The $\widehat{A\vec{a}}$ printer consequently delivers the output $A^{\pm} = \pm 1$; ie, $A^{+} = (+1) \oplus A^{-} = (-1)$ appropriately. To the particle $p'[\lambda'] = p'[-\lambda]$, $[\lambda \to \pm\vec{a}\}$ attributes a physically relevant fact via the DEC $[\mp\vec{a}\}$; and in accord with our rules of precedence – our shortcut – the right-most operator is left unmodified. We also see that $\langle AB\rangle$ is a constant of the experiment; the freely and independently chosen SGD settings $\vec{a}$ and $\vec{b}$ are definitive. Bob's point of view, (39), similarly. Bell's EPRB-based theorem is thus symmetrically refuted in CLR terms.

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