Few interesting sequences obtained by recurrence and based on Smarandache function

Marius Coman
Bucuresti, Romania
email: mariuscoman13@gmail.com

Abstract. In one of my previous papers, “An ordered set of certain seven numbers that results constantly from a recurrence formula based on Smarandache function”, combining two of my favorite topics of study, the recurrence relations and the Smarandache function, I discovered that the formula $f(n) = S(f(n - 2)) + S(f(n - 1))$, where $S$ is the Smarandache function and $f(1)$, $f(2)$ are any given different non-null positive integers, seems to lead every time to a set of seven values (i.e. 11, 17, 28, 24, 11, 15, 16) which is then repeating infinitely. In this paper I show few other interesting patterns based on recurrence and Smarandache function and I define the Smarandache-Coman constants.

Conjecture 1:

The recurrent formula $f(n) = S(f(n - 3)) + S(f(n - 2)) + S(f(n - 1))$, where $S$ is the Smarandache function, leads every time to the set of twelve consecutive values {31, 49, 52, 58, 56, 49, 50, 31, 55, 52, 55, 35}, set which is then repeating infinitely, for any given different non-null positive integers $f(1)$, $f(2)$, $f(3)$.

Verifying the conjecture for $[f(1), f(2), f(3)] = [1, 2, 3]$:

```plaintext
: f(4) = S(1) + S(2) + S(3) = 6;
: f(5) = S(2) + S(3) + S(6) = 8;
: f(6) = S(3) + S(6) + S(8) = 10;
: f(7) = S(6) + S(8) + S(10) = 12;
: f(8) = S(8) + S(10) + S(12) = 13;
: f(9) = S(10) + S(12) + S(13) = 22;
: f(10) = S(12) + S(13) + S(22) = 28;
: f(11) = S(13) + S(22) + S(28) = 31;
: f(12) = S(22) + S(28) + S(31) = 49;
: f(13) = S(28) + S(31) + S(49) = 52;
: f(14) = S(31) + S(49) + S(52) = 58;
: f(15) = S(49) + S(52) + S(58) = 56;
: f(16) = S(52) + S(58) + S(56) = 49;
: f(17) = S(58) + S(56) + S(49) = 50;
: f(18) = S(56) + S(49) + S(50) = 31;
: f(19) = S(49) + S(50) + S(31) = 55;
: f(20) = S(50) + S(31) + S(55) = 52;
: f(21) = S(31) + S(55) + S(52) = 55;
```
\[ f(22) = S(55) + S(52) + S(55) = 35; \]
\[ f(23) = S(52) + S(55) + S(35) = 31; \]
\[ f(24) = S(55) + S(35) + S(31) = 49; \]
\[ f(25) = S(35) + S(31) + S(49) = 52; \]

Note:
It can be seen that \( f(26) = f(14) \) and the sequence becomes cyclic.

Open problems:

- Is there any exception to this apparent rule?
- Is there a finite or infinite set of exceptions?
- Is there (in case that conjecture is true) a superior limit for \( n \) such that eventually \( f(n) = 31, f(n + 1) = 49 \) and \( f(n + 2) = 52 \)?

Conjecture 2:

For any \( k \) integer, \( k \geq 2 \), the function \( f(n) = S(f(n - k)) + S(f(n - k + 1)) \) + \( \ldots + S(f(n - 2)) + S(f(n - 1)) \) leads to a number of \( k \) consecutive values of \( f(n) \) from which the sequence of values of \( f(n) \) is repeating infinitely, for any given different non-null positive integers \( f(1), f(2), \ldots, f(k) \), where \( f(1) < f(2) < \ldots < f(k) \); we name these values the Smarandache-Coman constants:

- for \( k = 2 \), the set of Smarandache-Coman constants obviously contains two elements, i.e. 11 and 17;
- for \( k = 3 \), the set of Smarandache-Coman constants obviously contains three elements, i.e. 31, 49 and 52.

Open problem:

- Which is the Smarandache-Coman set of constants for \( k = 4 \)? But for \( k = 5, k = 6 \) etc.?

Conjecture 3:

The recurrent formula \( f(n) = \text{abs}\{S(f(n - 1)) - S(f(n - 2)) + S(f(n - 3)) - S(f(n - 4))\} \), where \( S \) is the Smarandache function, leads every time to \( f(m) = 0 \) for a certain \( m \), for any given different non-null positive integers \( f(1), f(2), f(3), f(4) \), where \( f(1) < f(2) < f(3) < f(4) \).

Verifying the conjecture for \( [f(1), f(2), f(3), f(4)] = [1, 2, 3, 4] \):

- \( f(5) = \text{abs}\{S(4) - S(3) + S(2) - S(1)\} = 2; \)
- \( f(6) = \text{abs}\{S(2) - S(4) + S(3) - S(2)\} = 1; \)
- \( f(7) = \text{abs}\{S(1) - S(2) + S(4) - S(3)\} = 0. \)
Verifying the conjecture for \([f(1), f(2), f(3), f(4)] = [7, 11, 125, 1729]\):

\[
\begin{align*}
\text{f(5)} &= \text{abs\{S(1729) - S(125) + S(11) - S(7)\}} = 8; \\
\text{f(6)} &= \text{abs\{S(8) - S(1729) + S(125) - S(11)\}} = 11; \\
\text{f(7)} &= \text{abs\{S(11) - S(8) + S(1729) - S(125)\}} = 7; \\
\text{f(8)} &= \text{abs\{S(7) - S(11) + S(8) - S(1729)\}} = 19; \\
\text{f(9)} &= \text{abs\{S(19) - S(7) + S(11) - S(8)\}} = 19; \\
\text{f(10)} &= \text{abs\{S(19) - S(19) + S(7) - S(11)\}} = 4; \\
\text{f(11)} &= \text{abs\{S(4) - S(19) + S(19) - S(7)\}} = 3; \\
\text{f(12)} &= \text{abs\{S(3) - S(4) + S(19) - S(19)\}} = 1; \\
\text{f(13)} &= \text{abs\{S(1) - S(3) + S(4) - S(19)\}} = 17; \\
\text{f(14)} &= \text{abs\{S(17) - S(1) + S(3) - S(4)\}} = 15; \\
\text{f(15)} &= \text{abs\{S(15) - S(17) + S(1) - S(3)\}} = 14; \\
\text{f(16)} &= \text{abs\{S(14) - S(15) + S(17) - S(1)\}} = 18; \\
\text{f(17)} &= \text{abs\{S(18) - S(14) + S(15) - S(17)\}} = 13; \\
\text{f(18)} &= \text{abs\{S(13) - S(18) + S(14) - S(15)\}} = 9; \\
\text{f(19)} &= \text{abs\{S(9) - S(13) + S(18) - S(14)\}} = 8; \\
\text{f(20)} &= \text{abs\{S(8) - S(9) + S(13) - S(18)\}} = 5; \\
\text{f(21)} &= \text{abs\{S(5) - S(8) + S(9) - S(13)\}} = 6; \\
\text{f(22)} &= \text{abs\{S(6) - S(5) + S(8) - S(9)\}} = 4; \\
\text{f(23)} &= \text{abs\{S(4) - S(6) + S(5) - S(8)\}} = 2; \\
\text{f(24)} &= \text{abs\{S(2) - S(4) + S(6) - S(5)\}} = 4; \\
\text{f(25)} &= \text{abs\{S(4) - S(2) + S(4) - S(6)\}} = 3; \\
\text{f(26)} &= \text{abs\{S(3) - S(4) + S(2) - S(4)\}} = 3; \\
\text{f(27)} &= \text{abs\{S(3) - S(3) + S(4) - S(2)\}} = 2; \\
\text{f(28)} &= \text{abs\{S(2) - S(3) + S(3) - S(4)\}} = 2; \\
\text{f(29)} &= \text{abs\{S(2) - S(2) + S(3) - S(3)\}} = 0.
\end{align*}
\]

Open problems:

: Is there any exception to this apparent rule?
: Is there a finite or infinite set of exceptions?
: Is there (in case that conjecture is true) a superior limit for \(m\) such that eventually \(f(m) = 0\)?

Reference:

Coman, Marius, An ordered set of certain seven numbers that results constantly from a recurrence formula based on Smarandache function, Vixra.