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Subconstituents of the Standard Model Particles and the Nambu–Jona-Lasinio Model

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Abstract

I propose a simple model for quarks and leptons in order to analyze what could be the building blocks of the Standard Model of particles. I start with the least number of elementary fields and generate using the Nambu–Jona-Lasinio model light masses for the subconstituent fermions. The NJL coupling constant turns out to be of the order of the gravitational coupling constant.

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1 Introduction

There are several regularities of particle properties that support the subconstituent structure of the quarks and leptons: (i) the number of elementary particles, together with the number of parameters, in presently popular theories is large, between some twenty and over one hundred, (ii) quarks and leptons occur in nature in similar structures, (iii) quarks and leptons come in three "versions" or generations that are distinguished by mass only.

All available experimental evidence indicates support for the Standard Model (SM) of particles up to about 10 TeV, perhaps even up to the Planck scale λ_{Planck}. Mathematically attractive and extensively studied ideas beyond the SM like supersymmetry or string theory lack experimental support so far.

In an attempt to study the generation question of quarks and leptons a model for the first generation quark and lepton subconstituents, or preons, was suggested in [1]. Each quark and lepton was supposed to consist of three mini black hole preons bound together by a confining interaction. In this note I re-evaluate the confinement interaction, correct the weak sector structure, replace the black holes by massless subconstituents, generate the subconstituent masses by the Nambu–Jona-Lasinio (NJL) method, and thus make a proposal to develop the raw model one step further. I find evidence that gravity may play a role in the interactions of the subconstituents.

Other subconstituent models are listed in [2]. These sources include more detailed discussions of virtues and problems of subconstituent models.

2 Quarks and Leptons

In the notation of [1] the quark and lepton constituents are a fractionally charged maxon m_i^+ with charge $+\frac{1}{3}$, and a neutral maxon m^0. This is the simplest choice if we want to build the quarks and leptons with the known charges using two maxons. No other internal quantum numbers are assigned to maxons. Their masses are zero. The first mass scale is introduced in the next section.

The first generation quark and lepton bound states are

$$u : \{(m^+m^+m^0)_r, (m^+m^0m^+)_g, (m^0m^+m^+)_b\} \tag{1}$$

$$\bar{d} : \{(m^+m^0m^0)_r, (m^0m^+m^0)_g, (m^0m^0m^+)_b\} \tag{2}$$

$$e^- : m^-m^-m^- \tag{3}$$

$$\nu_e : m^0m^0m^0 \tag{4}$$

The sum of maxon charges in (1)-(4) is zero.

The quarks and leptons are formed when three nearby maxons interact and form a stable bound state. The binding interaction can be one or more of the following:

scalar, Abelian or non-Abelian vector, tensor, four-fermion, or simply, a potential. At distances near the Planck scale the form of any potential, say inverse radius, may contain ripples due to fluctuating metric. Perhaps the safest choice at this stage is to assume eg. SU(2) color-like interaction between the maxons. The top quark mass value in particular is interesting because it is relatively near the Higgs boson mass which may not be accidental.

The weak interactions are treated, unlike in [1], traditionally as the broken symmetry gauge theory with an elementary Higgs field.

Color is introduced for quarks by a permutation of the odd maxon in (1) and (2), whereas the leptons consist of three identical maxons. The interaction between quarks is carried by the color octet of gluons.

3 Fermion Mass Generation

Let us consider the model for dynamical breaking of chiral symmetry invented by Nambu and Jona-Lasinio [3]. The NJL model was originally written down as an approximate model for the pion-nucleon system. We do not consider any of the several other features of the model except the gap generation. This mechanism is analogous to the microscopic theory of superconductivity by Bardeen, Cooper and Schrieffer [4]. We therefore believe this mechanism holds quite generally for fermionic systems of different kinds. The model is not renormalizable, and therefore a cutoff has to be defined for the fermion momenta.

The Lagrangian of the NJL model is

$$\mathcal{L} = \bar{\psi}\ i\gamma_i \partial^i \psi + G\left[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2\right]. \tag{5}$$

Its invariance under the Abelian chiral group $\mathrm{U(1)_V} \times \mathrm{U(1)_A}$ is most easily seen when the interaction is rewritten in terms of the chiral components of the Dirac field, $\mathcal{L} = \bar{\psi}\ i\gamma_i\partial^i\psi + 4G|\bar{\psi}_R\psi_L|^2$.

Spontaneous generation of mass is produced by the four-fermi interaction in $\mathcal{L}_{int} = \mathcal{L} - \mathcal{L}_{free}$ where

$$\mathcal{L}_{int} = m\bar{\psi}\psi + G\left[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2\right] \tag{6}$$

$$\mathcal{L}_{free} = \bar{\psi}(i\gamma_i\partial^i - m)\psi \tag{7}$$

The mass is required to be real and positive. The actual value of the mass m is determined by the condition of self-consistency. Instead of using $\mathcal{L}_{int}$ as perturbation Nambu and Jona-Lasinio introduced the self-energy Lagrangian $\mathcal{L}_s$ and split $\mathcal{L}$ as follows

$$\begin{aligned} \mathcal{L} &= (\mathcal{L}_0 + \mathcal{L}_s) + (\mathcal{L}_i - \mathcal{L}_s) \qquad (8) \\ &= \mathcal{L}_0' + \mathcal{L}_i' \qquad (9) \end{aligned}$$

Here it was assumed that $\mathcal{L}_0'$ leads to a linear field equation and $\mathcal{L}_s$ is of more general form, quadratic or bilinear in the fields. Next $\mathcal{L}_i'$ is treated as perturbation and $\mathcal{L}_s$ is determined from the requirement that $\mathcal{L}_i'$ does not give additional self-energy contributions. This gives rise to the gap equation

$$m = -\frac{imG}{2\pi^4} \int \frac{d^4p}{p^2 + m^2} F(p, \Lambda) \qquad (10)$$

where $F(p, \Lambda)$ is a cutoff factor. This has two solutions: either m=0 or m has the non-trivial solution

$$1 = -\frac{iG}{2\pi^4} \int \frac{d^4p}{p^2 + m^2} F(p, \Lambda) \qquad (11)$$

The latter solution will give m in terms of G and the cutoff Λ provided

$$0 < \frac{2\pi^2}{G\Lambda^2} < 1. \qquad (12)$$

In this model case I expect m to be no more than the proton mass and the cutoff Λ to be of the order 10^{19} GeV, up to which value I tentatively assume the SM to be valid. Therefore (11) is insensitive to value of m and one cannot get better estimate for m than mentioned above. However, we are at the upper limit in (12), the validity condition of the non-trivial solution, and one gets from (12) a lower limit for G in terms of Λ, in fact it is equality with high accuracy because $m^2/\Lambda^2 \sim 10^{-19} << 1$,

$$G > \frac{2\pi^2}{\Lambda^2} \sim 10^{-37} GeV^{-2} \qquad (13)$$

Putting in the cutoff Λ to Planck scale and getting out the gravitational coupling constant α_G may not prove the scheme right. I conclude, however, that the present model of maxons is consistent with the SM being valid up to λ_{Planck} and gravity playing a role in maxon interactions at energies near the Planck scale.

An interesting question to study is whether there are excited states of maxons, which would contribute to the generations of quarks and leptons.

4 Some Current Theories

Present experiments support the Standard Model well, and perhaps the SM holds up to or near the Planck scale. The present model is, by construction, consistent with the SM, but in principle with fewer number of parameters because no Yukawa terms are used for fermions.

Grand Unified Theory, like SO(10), and the present model are consistent only in representing basically one generation at a time in one multiplet. They differ significantly in the gauge and Higgs sectors. The basic difference is that the present setup goes towards smaller reps rather than larger.

5 Cosmology

The present scheme was extended to the Big Bang such that the initial singularity was smoothed into a very high but finite density banging object of maxons and antimaxons [5]. Quarks and leptons are formed from these. This should not lead to major deviations from the standard cosmological model but quantitative differences should be looked for.

For dark matter and dark energy I have nothing readily to propose. There may be, however, a connection between dark matter and scalar maxon bound state configurations.

6 Conclusions

I have discussed a model for the Standard Model particles. In principle the model does not need a Yukawa coupling for each quark and lepton but, on the other hand, the mechanism for the masses of the three generations is still unclear. It is hoped that in due time the model could be reinforced to provide an explanation for this and the hierarchy problem as well, assuming the top quark mass is related to the weak scale. To this end a new idea and detailed calculations have to be provided. It may be that gravity has a role to play here but a quantum leap is yet to be taken to uncover gravity's secrets.

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