The Smarandache-Korselt criterion, a variant of Korselt's criterion

Marius Coman
Bucuresti, Romania
email: mariuscoman13@gmail.com

Abstract. Combining two of my favourite objects of study, the Fermat pseudoprimes and the Smarandache function, I was able to formulate a criterion, inspired by Korselt's criterion for Carmichael numbers and by Smarandache function, which seems to be necessary (though not sufficient as the Korselt's criterion for absolute Fermat pseudoprimes) for a composite number (without a set of probably definable exceptions) to be a Fermat pseudoprime to base two.

Conjecture:

Any Poulet number, without a set of definable exceptions, respects either the Korselt's criterion (case in which it is a Carmichael number also) either *the Smarandache-Korselt criterion*.

Definition:

A composite odd integer $n = d_1*d_2*...*d_n$, where d_1, d_2, ..., d_n are its prime factors, is said that respects *the Smarandache-Korselt criterion* if n – 1 is divisible by $S(d_i - 1)$, where S is the Smarandache function and $1 \le i \le n$.

Note:
A Carmichael number not always respects *the Smarandache-Korselt criterion*: for instance, in the case of the number 561 = 3*11*17, 560 it is divisible by S(3 – 1) = 2 and by S(11 – 1) = 5 but is not divisible by S(17 – 1) = 6; in the case of the number 1729 = 7*13*19, 1728 it is divisible by S(6) = 3, S(12) = 4 and S(18) = 6.

Verifying the conjecture:
(for the first five Poulet numbers and for two bigger consecutive numbers which are not Carmichael numbers also):

: For P = 341 = 11*31, P – 1 = 340 is divisible by S(10) = 5 and S(30) = 5;
: For P = 645 = 3*5*43, P – 1 = 644 is divisible by S(2) = 2, S(4) = 4 and S(42) = 7;

:   For P = 1387 = 19*73, P – 1 = 1386 is divisible by S(18) = 6 and S(72) = 6;
     For P = 1905 = 3*5*127, P – 1 = 1904 is divisible by S(2) = 2, S(4) = 4 and S(42) = 7;
:   For P = 2047 = 23*89, P – 1 = 2046 is divisible by S(22) = 11 and S(88) = 11;
:   For P = 2701 = 37*73, P – 1 = 2700 is divisible by S(36) = 6 and S(72) = 6;
(...)

:   For 999855751441 = 774541*1290901, P – 1 is divisible by S(774540) = 331 and S(1290900) = 331;
:   For P = 999857310721 = 2833*11329*31153, P – 1 is divisible by S(2832) = 59 and S(11328) = 59 and S(31152 = 59).

Comment:
One exception that we met (which probably is part of a set of definable exceptions) is the Poulet number P = 999828475651 = 191*4751*1101811; indeed, P – 1 is not divisible by S(1101810) = 1933, and P is not a Carmichael number.