PREPRINT Version 1 pp.1–5 (2013)   Printed 6 February 2014   (LaTeX style file v2.2)

On the Perihelion Precession of Solar Planetary Orbits

G. G. Nyambuya[1]*
[1]*National University of Science & Technology, Faculty of Applied Sciences, School of Physics, P. O. Box AC 939, Ascot, Bulawayo, Republic of Zimbabwe.*

Monday 3 Feb. 2013

ABSTRACT
The present letter presents an improved version of the Azimuthally Symmetric Theory of Gravitation (ASTG-model) which was presented for the first time four years ago (in Nyambuya 2010). Herein, we propose a solution to the standing problem of the λ-parameters in which effort we put the ASTG-model on a clear pedestal for falsification. The perihelion precessional data of Solar planetary orbits is used to set the theory into motion. As a way of demonstrating the latent power of the new theory, we show in separate letters that – one of the most important and outstanding problems in astrophysics today – the *radiation problem*; which is thought to bedevil massive stars during their formation, may find a plausible solution in the ASTG-model. Further, from within the confines of this new theory, we also demonstrate (in a separate letter) that the *emergence of bipolar molecular outflows* may very be an azimuthal gravitational phenomenon. Furthermore, we also show (in a separate letter as-well) that the ASTG-model does, to a reasonable extent explain the *tilt of Solar planetary orbits*.

Key words: planetary orbit, perihelion shift, solar spin

1 INTRODUCTION

Four years ago (in the reading, Nyambuya 2010), we presented the Azimuthally Symmetric Theory *of* Gravitation (hereafter ASTG-model) where the azimuthal solutions of the Poison-Laplace equation are applied to the scenario of gravitation. At first glance, this theory appears as nothing more than the mundane azimuthally symmetric solutions of the well known Poison-Laplace equation, namely:

$$\nabla^2\Phi = 4\pi G\varrho, \tag{1}$$

where G is Newton's universal constant of gravitation, Φ is the gravitational potential, ϱ is the density of matter and the Laplacian operator ∇^2, when written in spherical coordinates, is given by:

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}. \tag{2}$$

The spherical coordinate setup that we assume is shown in figure (1). At its inception, our initial thoughts and feelings were that the ASTG-model is but a banal theory of gravitation only extending the gravitational theory of Sir Isaac Newton from just being a central field phenomenon to an azimuthal and polar field phenomenon. It is for this reason that we muted the comment (in Nyambuya 2010) that it [the ASTG-model] was not a new theory of gravitation. That view has since changed!

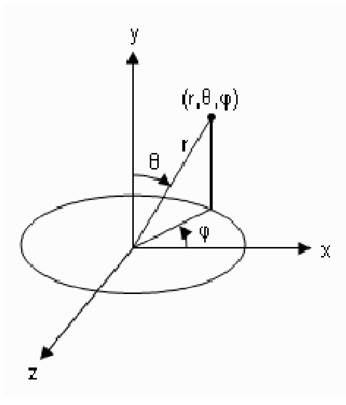

Figure (1). This figure shows a generic spherical coordinate system, with the radial coordinate denoted by r, the zenith (the angle from the North Pole; the colatitude) denoted by θ, and the azimuth (the angle in the equatorial plane; the longitude) by φ.

As will become clear as we go, the ASTG-model is a new classical theory of gravitation which makes the seemingly ambitious hypothesis that the spin of a gravitating mass has a role to play in the emergent gravitational field

* E-mail: gadzirai@gmail.com

of the spinning mass. The ASTG-model is based[1] on the solutions $\Phi = \Phi(r,\theta)$ of (1), *i.e.*:

$$\Phi(r,\theta) = -\frac{G\mathcal{M}_{\mathrm{star}}}{r}\sum_{\ell=0}^{\infty}\lambda_\ell\left(\frac{G\mathcal{M}_{\mathrm{star}}}{rc^2}\right)^{\ell}P_\ell(\cos\theta), \tag{3}$$

where $\mathcal{M}_{\mathrm{star}}$ is the mass of the central gravitating body, c is the speed of light in vacuum, r is the radial distance from this gravitating body, and $\lambda_\ell : \ell = 0, 1, 2, \ldots$ *etc* are some dynamic parameters which in the ASTG-model are assumed to be related to gravitating body in question. This property that the λ's are dynamic parameters assumed to be related to the gravitating body in question is the ingenuity and novelty of the ASTG-model and is what makes the ASTG-model a unique and new theory of gravitation where the spin of the gravitating mass enters the gravitational podium.

We are going to make a few subtle changes in the formula (3). The first change is to do with the symmetries of this potential. The resulting gravitational field needs to have an azimuthal symmetry, the meaning of which is that $\Phi(r,\theta) = \Phi(r,-\theta)$. For this to be so, then, for the Legendre polynomials $P_\ell(\cos\theta)$, we will need only take the absolute values of the odd component of this function *i.e.*, $|P_\ell(\cos\theta)|$ for $\ell = 1, 3, 5, \ldots$ *etc*. Thus, because of this, we shall redefine the Legendre polynomial to be given by $\mathcal{P}_\ell(\cos\theta)$, were:

$$\mathcal{P}_\ell(\cos\theta) = \begin{cases} P_\ell(\cos\theta), & \text{for} \quad \ell = 0, 2, 4, 6, \ldots\ etc \\ |P_\ell(\cos\theta)|\,, & \text{for} \quad \ell = 1, 3, 5, 7 \ldots\ etc \end{cases}. \tag{4}$$

As to what would warrant the above redefinition, one must realise that in its bare form, the system of polar coordinates (r, φ, θ) is such that $(r > 0)$ for $(0 < \theta < \pi)$ *i.e.*, r is positive for any point above the plane $\theta = 0$; and $(r < 0)$ for $(\pi < \theta < 2\pi)$, *i.e.*, r is negative for any point below the plane $\theta = 0$. The usual convention is that we consider r to take only positive values. With respect to the bare form of the system of polar coordinates as described above, the redefinition (4) take cognisance of this fact that in the usual convention, r is assumed to take only positive values.

The second reasoning for seeking to make subtle changes in the formula (3), has to do with the time dependent potential $\Phi(r,\theta,t)$. This problem of the time dependent potential $\Phi(r,\theta,t)$ has been tackled in the *unpublished* manuscript (see Nyambuya 2012*a*) What we will do here is to bring in the relevant portion of this reading (*i.e.*, Nyambuya 2012*a*) into the present letter. It is argued in Nyambuya (2012*a*) that in the event that the gravitational field is time dependent, the appropriate equation to describe the gravitational phenomenon is:

$$\nabla^2\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = 4\pi G\varrho, \tag{5}$$

where c is the speed of light in vacuum.

In solving (1) in the absence of a time variation of the gravitational field – we; as laid down in Nyambuya (2010), assumed separable solutions for $\Phi(r,\theta)$, that is to say, $\Phi(r,\theta)$ was set so that $\Phi(r,\theta) = \Phi(r)\Phi(\theta)$. So doing, we obtained an infinite number of solutions for $\Phi(r)$ and $\Phi(\theta)$ and labelled these with a sub-script ℓ as $\Phi_\ell(r)$ and $\Phi_\ell(\theta)$. In such a case, the resultant solution is the sum of all the products $\Phi_\ell(r)\Phi_\ell(\theta)$, *i.e.*:

$$\Phi(r,\theta) = \sum_{\ell=0}^{\infty}\Phi_\ell(r)\Phi_\ell(\theta). \tag{6}$$

Now, in the case of a time dependent gravitational field, the separable solution [*i.e.*, $\Phi(r,\theta,t) = \Phi(r)\Phi(\theta)\Phi(t)$] emerging from (5) is such that:

$$\Phi(r,\theta,t) = \Phi(t)\sum_{\ell=0}^{\infty}\Phi_\ell(r)\Phi_\ell(\theta). \tag{7}$$

Notice that the time dependence is the same for of all the gravitational poles. This means that in the event of a time varying gravitational field, the potential (3) is going to be given by:

$$\Phi(r,\theta,t) = -\overbrace{\frac{G\Phi(t)\mathcal{M}_{\mathrm{star}}}{r}}^{\textbf{Term I}}\sum_{\ell=0}^{\infty}\lambda_\ell\overbrace{\left(\frac{G\mathcal{M}_{\mathrm{star}}}{rc^2}\right)^{\ell}}^{\textbf{Term II}}\mathcal{P}_\ell(\cos\theta). \tag{8}$$

What the above implies is that if we set $G(t) = G\Phi(t)$, then we have a theory in which the gravitational constant varies with time. This is what the work presented in Nyambuya (2012*a*) proposes, that, the long held speculative assumption of a time variable Newtonian gravitational constant can be justified on the bases of of (5). In the light of the aforesaid, we can now write (8) as:

$$\Phi(r,\theta,t) = -\overbrace{\frac{G(t)\mathcal{M}_{\mathrm{star}}}{r}}^{\textbf{Term I}}\sum_{\ell=0}^{\infty}\lambda_\ell\overbrace{\left(\frac{2G\mathcal{M}_{\mathrm{star}}}{rc^2}\right)^{\ell}}^{\textbf{Term II}}\mathcal{P}_\ell(\cos\theta). \tag{9}$$

Now, the gravitational constant in Term I, is surely no-longer the same gravitational constant in Term II as is the case in (3). To take this into account, let us write (9) as:

$$\Phi(r,\theta,t) = -\frac{G(t)\mathcal{M}_{\mathrm{star}}}{r}\sum_{\ell=0}^{\infty}\lambda_\ell\left(\frac{2\mathscr{G}_\ell\mathcal{M}_{\mathrm{star}}}{rc^2}\right)^{\ell}\mathcal{P}_\ell(\cos\theta). \tag{10}$$

where the new constants $\mathscr{G}_\ell$ are now no longer the same as the traditional Newtonian gravitational constant G. The constants $\mathscr{G}_\ell$ are strictly and absolutely not depended on time, space or physical variable, they are pure constants of *Nature*.

For the purposes of simplification, since we are not so much concerned here with the time dependence of the gravitational field, it makes sense to drop this from the formula (10) and write this formula as:

$$\Phi(r,\theta) = -\frac{G\mathcal{M}_{\mathrm{star}}}{r}\sum_{\ell=0}^{\infty}\lambda_\ell\left(\frac{2\mathscr{G}_\ell\mathcal{M}_{\mathrm{star}}}{rc^2}\right)^{\ell}\mathcal{P}_\ell(\cos\theta). \tag{11}$$

The difference between (3) and (11) is found in introduction of $\mathscr{G}_\ell$. For convenience, we shall write, $\mathscr{R}_\ell = 2\mathscr{G}_\ell\mathcal{M}_{\mathrm{star}}/c^2$; so that (11) becomes:

$$\Phi(r,\theta) = -\frac{G(t)\mathcal{M}_{\mathrm{star}}}{r}\sum_{\ell=0}^{\infty}\lambda_\ell\left(\frac{\mathscr{R}_\ell}{r}\right)^{\ell}P_\ell(\cos\theta). \tag{12}$$

[1] This theory can be extended to include the polar solutions $\Phi(r,\theta,\varphi)$. Exploration of these solutions is a task we hope to look into in future readings.

The terms λ_ℓ and $\mathscr{G}_\ell$ are unknown and it is the task of the this letter to use available data on on the perihelion precision of Solar planets of make a just proposal of what these terms may be. Further, for both the interior and exterior gravitational field, the λ-parameters can not and can never dependent on the spacial coordinates (r, θ, φ), but may depend explicitly or implicitly on time, t.

In closing this section, we shall conclude by giving the synopsis of the present letter, it is as follows. In the subsequent section, we propose a form for the λ-parameters, we after we try to justify this proposal. Using the calculated values of λ_1 and λ_2 from Nyambuya (2010), we make preliminarily estimations of $\mathscr{G}_1$ and $\mathscr{G}_2$. In §(3), we write down the resulting equations of motion emerging from (12), where-after these equations of motion are applied in §(4) to the case of the perihelion precession of Solar planets. Lastly, in (5), we give a discussion and the conclusion drawn thereof.

2 λ-PARAMETERS

At the inspection of the ASTG-model, one of the first problems to be identified as an impediment to the theory is the problem of the λ-parameters. There is an infinite number of them and – to make matters worse; they are unknown. If this problem can not be solved, then, the theory is rendered obsolete. In Nyambuya (2010), we were able to deduce three of these λ-parameters *i.e.* λ_0, λ_1 and λ_2 from the available data on the perihelion precession of Solar planetary orbits. The hypothesis of the ASTG-model is that these λ's dependent on the spin of the gravitating body in questions, hence they do not have universal values but are different for different gravitating bodies depending on their spin state. However, for λ_0, this value is the same for all gravitating bodies, it is universal and is such that $\lambda_0 \equiv 1$.

In part of our *yet-to-published*[2] on-going work on the ASTG-model, we have applied the ASTG-model to the problem of the rotation curves of galaxies which has led to the darkmatter hypothesis; this we have done in-order to seek a solution *via* the ASTG-model, to the galaxy rotation problem. In-order to have the ASTG-model fit the data of the rotation curves of galaxies and other physical phenomenon to which the theory has been applied, we find that if the λ's are defined such that:

$$\lambda_\ell = (-1)^{\ell+1}\left(\frac{\mathcal{S}_{\rm star}}{\mathcal{S}_\ell}\right)^\ell = (-1)^{\ell+1}\left(\frac{\omega_{\rm star}}{\omega^*_\ell}\right)^\ell \quad \text{for } \ell > 0, \tag{13}$$

or:

$$\lambda_\ell = (-1)^{\ell+1}\left(\frac{\mathcal{S}_{\rm star}}{\mathcal{S}_\ell}\right)^\ell = (-1)^{\ell+1}\left(\frac{\mathcal{T}^*_\ell}{\mathcal{T}_{\rm star}}\right)^\ell \quad \text{for } \ell > 0, \tag{14}$$

where $\mathcal{S}_{\rm star} = \mathcal{R}^2_{\rm star}\omega_{\rm star}$ is the specific spin angular momentum of the spinning gravitating object and $\mathcal{S}_\ell = \sqrt{\mathscr{G}_\ell \mathcal{M}_{\rm star}\mathcal{R}_{\rm star}}$, is the ℓ^{th} specific spin angular momentum of the spinning gravitating where $\mathscr{G}_\ell$ is are time-independent constants with the same dimensions as Newton's gravitational G. Further, $\omega_{\rm star} = 2\pi/\mathcal{T}_{\rm star}$ is the star's spin angular frequency while $\mathcal{T}_{\rm star}$ is the star's spin period and; of the gravitating object whose spin period is $\mathcal{T}_{\rm star}$.

If we set or write $a_\ell = (\mathscr{G}_\ell/G)^{\frac{1}{2}\ell}$, then (12), can be written as:

$$\Phi(r,\theta) = -\frac{G\mathcal{M}_{\rm star}}{r}\sum_{\ell=0}^{\infty} a_\ell \left(\frac{\alpha\mathcal{R}_{\rm s}}{r}\right)^\ell \mathcal{P}_\ell(\cos\theta), \tag{15}$$

where, $\alpha = \mathcal{S}_{\rm star}/\mathcal{S}^*_{\rm star}$; such that $\mathcal{S}^*_{\rm star} = \sqrt{G\mathcal{M}_{\rm star}\mathcal{R}_{\rm star}}$. For the Sun with a mass $\mathcal{M}_\odot = 1.99\times10^{30}\,\rm kg$, radius $\mathcal{R}_\odot = 6.94\times10^8\,\rm m$ and spin period $\mathcal{T}_\odot = 25.38\,days$, we have $\mathcal{S}_\odot = 1.17\times10^{12}\,\rm m^2s^{-1}$, $\mathcal{S}^*_\odot = 2.91\times10^{14}\,\rm m^2s^{-1}$, hence $\alpha_\odot = 4.03\times10^{-3}$. For the Solar Schwarzchild radius $\mathcal{R}^\odot_{\rm s} = 2.95\,\rm km$. Hereafter, whenever we need the afore-calculated values $(\mathcal{S}_\odot, \mathcal{S}^*_\odot, \alpha_\odot, \mathcal{R}^\odot_{\rm s})$, we will assume they have been calculated here.

3 EQUATIONS OF MOTION

In spherical coordinates, the acceleration $\boldsymbol{a} = \ddot{\boldsymbol{r}}$ is given by:

$$\begin{aligned}\boldsymbol{a} = \; & (\ddot{r} - r\dot{\varphi}^2 - r\dot{\theta}^2\sin^2\varphi)\hat{\mathbf{r}} \\ & +(2\dot{r}\dot{\varphi}\sin\varphi + r\ddot{\varphi}\sin\varphi + 2r\dot{\theta}\dot{\varphi}\cos\varphi)\hat{\boldsymbol{\theta}} \; . \\ & +(2\dot{r}\dot{\varphi} + r\ddot{\varphi} + r\dot{\theta}^2\cos\varphi\sin\varphi)\hat{\boldsymbol{\varphi}}\end{aligned} \tag{18}$$

The acceleration due to gravity $\boldsymbol{g} = -\gamma\boldsymbol{\nabla}\Phi$, where γ is the ratio of the gravitational mass $(m_{\rm g})$ to the inertial mass (m_i) *i.e.* $\gamma = m_{\rm g}/m_i$. Though there may be reasons for $\gamma \neq 1$ (see *e.g.* Nyambuya and Simango 2014), for the present, we shall – as is usually assumed; take the gravitational and inertial mass of a test particle to be identical physical quantities (*i.e.*, $m_i \equiv m_{\rm g} \implies \gamma \equiv 1$), therefore, the equation of motion for such a test particle in a gravitational field is given by $\boldsymbol{a} = \boldsymbol{g}$. Comparing the different components (*i.e.*, the radial, $\hat{\boldsymbol{\theta}}$ and $\hat{\boldsymbol{\varphi}}$-components) of this equation of motion *i.e.* $\boldsymbol{a} \equiv \boldsymbol{g}$, one obtains the following equations:

$$\frac{\partial^2 r}{\partial t^2} - r\left(\frac{\partial\varphi}{\partial t}\right)^2 - r\left(\frac{\partial\theta}{\partial t}\right)^2\sin^2\varphi = -\frac{\partial\Phi}{\partial r}, \tag{19}$$

$$\frac{\partial J_\theta}{\partial t}\sin\varphi + \frac{2J_\theta J_\varphi\cos\varphi}{r^2} = -\frac{\partial\Phi}{\partial\theta}, \tag{20}$$

$$\frac{\partial J_\varphi}{\partial t} + \frac{J^2_\theta\sin\varphi\cos\varphi}{r^2} = -\frac{\partial\Phi}{\partial\varphi}, \tag{21}$$

for the $\hat{\mathbf{r}}$, $\hat{\boldsymbol{\theta}}$, and the $\hat{\boldsymbol{\varphi}}$-component respectively. In the above equations, J_φ and J_θ are the specific orbital angular momentum in the $\hat{\boldsymbol{\varphi}}$ and $\hat{\boldsymbol{\theta}}$-directions.

Now, making the substitution $u = 1/r$, the equations (19), (20) and (21) transform to:

$$\frac{\partial^2 u}{\partial\varphi^2} + \frac{\dot{J}_\varphi}{u^2 J_\varphi}\frac{\partial u}{\partial\varphi} + \left(1 + \kappa^2\sin^2\varphi\right)u = -\frac{1}{J^2_\varphi}\frac{\partial\Phi}{\partial u}, \tag{22}$$

$$\frac{\partial J_\theta}{\partial t}\sin\varphi + 2u^2 J_\theta J_\varphi\cos\varphi = -\frac{\partial\Phi}{\partial\theta}, \tag{23}$$

$$\frac{\partial J_\varphi}{\partial t} + u^2 J^2_\theta\sin\varphi\cos\varphi = -\frac{\partial\Phi}{\partial\varphi}, \tag{24}$$

respectively. In (22) $\kappa = J_\theta/J_\varphi = \mathcal{T}_\varphi/\mathcal{T}_\theta$ where $\mathcal{T}_\varphi$ and $\mathcal{T}_\varphi$ are the orbital periods of revolution in the $\hat{\boldsymbol{\theta}}$ and $\hat{\boldsymbol{\varphi}}$ directions respectively. We shall investigate these equations of motion in future readings.

[2] We hope this work will be accepted for publication in a conventional scientific journal.

Table (I). Column (1) gives the name of the planet while columns (2) and (3) give this planets distance from the Sun and the tilt of its orbital plane to the Solar equator respectively. Columns (4) and (5) give values of A_p, B_p for the give planet, while column (6) give the observed perihelion precession of the given planet and column (7) give perihelion precession of the given planet as computed from the ASTG-model.

Planet	$\mathcal{R}_{\rm orb}$ (m)	θ $(^\circ)$	A (arcsec/cy)	B (arcsec/cy)	$(\delta\varphi/\mathcal{T}_\varphi)_{\rm OBS}$ (arcsec/cy)	$(\delta\varphi/\mathcal{T}_\varphi)_{\rm ASTG}$ (arcsec/cy)
Mercury	0.38	14.00	6.40×10^{-2}	-4.54×10^{-2}	43.1000 ± 0.5000	43.1000 ± 0.3000
Venus	0.72	10.40	1.29×10^{-2}	-4.82×10^{-3}	8.0000 ± 5.0000	8.6900 ± 0.0600
Earth	1.00	7.00	5.82×10^{-3}	-1.61×10^{-3}	5.0000 ± 1.0000	3.9400 ± 0.0300
Mars	1.52	8.85	2.04×10^{-3}	-3.69×10^{-4}	1.3624 ± 0.0005	1.3840 ± 0.0090
Jupiter	5.20	8.30	9.43×10^{-5}	-4.98×10^{-6}	0.0700 ± 0.0040	0.0639 ± 0.0004
Saturn	9.54	9.49	2.06×10^{-5}	-5.91×10^{-7}	0.0140 ± 0.0020	0.0140 ± 0.0001

$$\Phi(r,\theta)=-\frac{G\mathcal{M}_{\rm star}}{r}\left[\overbrace{1}^{\textbf{Pole 0}}+\overbrace{\lambda_1\left(\frac{\mathscr{R}_1}{r}\right)|\cos\theta|}^{\textbf{Pole 1}}+\overbrace{\lambda_2\left(\frac{\mathscr{R}_2}{r}\right)^2\frac{(3\cos^2\theta-1)}{2}}^{\textbf{Pole 2}}+\ldots\right]. \tag{16}$$

$$\left(\frac{\delta\varphi}{\mathcal{T}_\varphi}\right)=\left[\overbrace{\frac{a_1\alpha\cos\theta}{3}}^{\textbf{Term I}}-\overbrace{\frac{a_2\alpha^2(3\cos^2\theta-1)}{12\pi/\mathcal{T}_\varphi}\left(\frac{\delta\varphi}{\mathcal{T}_\varphi}\right)_{\rm E}}^{\textbf{Term II}}\right]\left(\frac{\delta\varphi}{\mathcal{T}_\varphi}\right)_{\rm E}. \tag{17}$$

4 DATA ANALYSIS AND RESULTS

In the reading Nyambuya (2010), the ASTG-model's formula for calculating/predicting the perihelion precession of Solar planetary orbits was derived. In the present letter, we are not going to derive this formula again as one can – using the latest version of the ASTG-model; derive it by going through the same steps. As in the reading Nyambuya (2010), we take the ASTG-model only upto second order approximation in which case the resulting gravitational potential is given in (16), where-from, the resulting formula for calculating/predicting the perihelion precession of Solar planetary orbits is the one given in equation (17), where:

$$\left(\frac{\delta\varphi}{\mathcal{T}_\varphi}\right)_{\rm E}=\frac{6\pi G\mathcal{M}_{\rm star}}{c^2\mathcal{T}_\varphi\left(1-\epsilon^2\right)\mathcal{R}_{\rm orb}}, \tag{25}$$

where $\mathcal{T}_\varphi$ and $\mathcal{R}_{\rm orb}$ are the orbital period and radius of the planet in question respectively. Equation (25) is the formula for the precession of the perihelion of an orbit that one obtains from Professor Einstein's GTR, hence the subscript, E. Now, if we set:

$$A=\frac{\alpha\cos\theta}{3}\left(\frac{\delta\varphi}{\mathcal{T}_\varphi}\right)_{\rm E}\text{ and }B=-\frac{\alpha^2\left(3\cos^2\theta-1\right)}{12\pi/\mathcal{T}_\varphi}\left(\frac{\delta\varphi}{\mathcal{T}_\varphi}\right)^2_{\rm E}, \tag{26}$$

then, for the Solar planets, equation (17) will be given by $(\delta\varphi/\mathcal{T}_\varphi)_p=A_pa_1^\odot+B_pa_2^\odot$, where $a_1^\odot$ and $a_2^\odot$ are unknowns. We need to compute these unknowns and for this, we have five data points *i.e.*, the perihelion precession data for *Mercury, Venus, Earth, Mars, Jupiter* and *Saturn*. Ideally, any pair of these data should give us the values of $a_1^\odot$ and $a_2^\odot$. That is to say, if say $(\delta\varphi/\mathcal{T}_\varphi)_{\rm mars}=A_{\rm mars}a_1^\odot+B_{\rm mars}a_2^\odot$ and $(\delta\varphi/\mathcal{T}_\varphi)_\oplus=A_\oplus a_1^\odot+B_\oplus a_2^\odot$ are the corresponding equations for *Mars* and *Earth* respectively, we can solve these two equations simultaneously to obtain $a_1^\odot$ and $a_2^\odot$. If the theory is correct, then, logic compels that the values $a_1^\odot$ and $a_2^\odot$ thus obtained must, when applied to other Solar planets give the correct perihelion precession of that planet.

In reality, this is not what happens – though salvageable, it's a bit more complicated than that. Some of the predicted perihelion precession of that planets from the computed unknowns $a_1^\odot$ and $a_2^\odot$ do not exactly fit. What this really means is that we need to use this data to search for the best value of $a_1^\odot$ and $a_2^\odot$ which best fits all the data. The aim here would be to find the values $a_1^\odot$ and $a_2^\odot$ that best fit the all five observations and the *basis of claim* insofar as correspondence with physical and natural reality of the theory is concerned is if there exists values $a_1^\odot$ and $a_2^\odot$ that on an acceptable scale, gives a good agreement with observation for all the five observations. We find that the following value $a_1^\odot=678.00\pm4.00$ and $a_2^\odot=6.06\pm0.06$ do just that. As can be read-off from columns (6) and (7) in Table (I), these a-values ($a_1^\odot=678.00\pm4.00$ and $a_2^\odot=6.06\pm0.06$) give results that agree pretty well with those found from observations. From these values of a_1 and a_2, we are led to the following values for $\mathscr{G}_1$ and $\mathscr{G}_2$:

$$\begin{array}{l}\mathscr{G}_1=(4.52\pm0.03)\times10^{-8}\,{\rm kg^{-1}m^3s^{-2}}=(678.00\pm4.00)G\\ \mathscr{G}_2=(4.04\pm0.04)\times10^{-10}\,{\rm kg^{-1}m^3s^{-2}}=(6.06\pm0.05)G.\end{array} \tag{27}$$

With these values now having been calculated, the present ASTG-model is here put on a sure and clear pedestal for falsification.

5 DISCUSSION AND CONCLUSION

Prior to the enunciation of his triumphant, greatest and most admirable intellectual master piece in (perhaps) all history of science and human thought – *i.e.*, his universal theory of gravitation in 1687, Sir Isaac Newton noted that his theory needed a constant – a constant that we

Table (II). Column (2) of the table below gives the name of the pair of the planets which have been used to obtain the pair a-values listed in columns (3) and (4). Column (5) give the χ^2-value of the pair (a_1, a_2) as measured against the results from observations as calculated from the χ^2-*Statistics-Test*; while column and (6) gives corresponding probability of likelihood that this pair (a_1, a_2) is likely the correct pair of values.

Estimation of the Values of a_1 and a_2 from Solar Data

	Pair	$a_1^{\odot}$	$a_2^{\odot}$	χ^2	P (%)
(1)	Mer-Ven	563.99	-154.75	0.45	>95
(2)	Mer-Ear	425.65	976.15	0.96	>95
(3)	Mer-Mar	3.03	665.43	0.29	>95
(4)	Mer-Jup	-11.91	748.07	0.30	>95
(5)	Mer-Sat	6.06	678.18	0.29	>95
(6)	Ven-Ear	2404.89	1522.26	70.71	~ 0
(7)	Ven-Mar	236.06	710.30	1.94	$80-90$
(8)	Ven-Jup	375.12	762.36	3.28	$50-70$
(9)	Ven-Sat	162.22	682.66	0.23	~ 0
(10)	Ear-Mar	4566.56	1493.99	574.28	~ 0
(11)	Ear-Jup	2279.50	862.94	150.62	~ 0
(12)	Ear-Sat	1795.65	729.44	52475.53	~ 0
(13)	Mar-Jup	5526.54	1034.43	24219.68	~ 0
(14)	Mar-Sat	4225.77	799.03	1208.77	~ 0
(15)	Jup-Sat	21902.23	1305.26	1933.00	~ 0

now know as Newton's universal constant of gravitation, G. In-order to set this theory into motion – *i.e.*, by placing it on clear and sure podium where a quantitative analysis and critical scrutiny of the theory could be made; Sir Isaac Newton made an estimate of the value of this constant, G, using the available data in his day. Once an estimate of this constant was known to some reasonable degree, it meant that if Sir Isaac Newton's theory is correct, this constant had to be the same for any pair of gravitating bodies anywhere in the Universe, it did not have to depend on the bodies in question. The fact that measurements upon measurements have proved time and again that this constant is indeed the same for all gravitating bodies is but a grand triumph for Sir Isaac Newton's theory – the theory does have a meaningful correspondence with experience; the theory is correct somehow. If it had been found that this constant was different for different bodies, it would somewhat spelt the demise of Sir Isaac Newton's beautiful theory of gravitation.

On much the same pedestal, the present letter sets into motion the ASTG-model only up to second order approximation as we have calculated the two constants $\mathscr{G}_1$ and $\mathscr{G}_2$ using the available data. If the ASTG-model is correct in its current state as laid down herein, then the values $\mathscr{G}_1$ and $\mathscr{G}_2$ now leave the second-order ASTG-model as a theory with no free parameters, it (somehow) is ready to be falsified or verified.

The reader may wonder whether or not the ASTG-model is a subset of Professor Einstein's GTR since to first order approximation, the GTR reduces to this Poison-Laplace equation (1). We have argued in Nyambuya (2012*b*), that this not the case, the ASTG-model is different from Professor Einstein's GTR. Further, its a fact that Professor Einstein's GTR is currently the dominate and well accepted paradigm of gravitation (see *e.g.* Will 2006). Be that it may, it (GTR) has so far failed to explain not only the radiation problem (Bonnell et al. 1998) thought to affect the formation of massive stars in their nascences, but other problems too; such as *e.g.* the origins of bipolar molecular outflows, the Pioneer anomaly, the Earth flyby anomalies (Anderson et al. 2008), the secular recession of the Earth from the Sun (Standish 2005) and that of the Moon from the Earth (Williams and Boggs 2009), amongst other problems. In an effort to demonstrate the latent power and potency of the AST-model, we shall show in separate reading that ASTG-model gives a reasonable explain of these ponderous phenomenon.

In-conclusion, one can safely say that; in its present form, the ASTG-model is – to second order approximation; a falsifiable theory. Further, given, the fact that the computed values of a_1 and a_2 give results that agree with theory is a good sign that – to second order approximation; the theory may hold a grain or an element of truth thus making it worthy of considerations.

REFERENCES

Anderson, J. D., Campbell, J. K., Ekelund, J. E., Ellis, J. and Jordan, J. F. (2008), 'Anomalous Orbital-Energy Changes Observed during Spacecraft Flybys of Earth', *Phy. Rev. Lett.* **100**, 091102.

Bonnell, I. A., Bate, M. and Zinnecker, H. (1998), *MNRAS* **298**, 93.

Nyambuya, G. G. (2010), 'Azimuthally Symmetric Theory of Gravitation (I) – On the Perihelion Precession of Planetary Orbits', *MNRAS* **403 (3)**, 1381–1391.

Nyambuya, G. G. (2012*a*), Four Poisson-Laplace Theory of Gravitation (I). Submitted to MNRAS. **URL:** *http://vixra.org/abs/1205.0117*

Nyambuya, G. G. (2012*b*), On the Resolution of the Azimuthally Symmetric Theory of Gravitation's λ-Parameters. **URL:** *http://vixra.org/abs/1101.0054*

Nyambuya, G. G. and Simango, W. (2014), 'On the Gravitational Bending of Light', *Int. J. Astron. Astrophy.* **4**(2), 1–10. (*In Press*). **URL:** *http://vixra.org/abs/1301.0004*

Standish, E. M.: Kurtz, D. W., ed. (2005), *The Astronomical Unit Now, in Transits of Venus: New Views of the Solar System and Galaxy*, number 196 *in* 'Proceedings IAU Colloquium', IAU, Cambridge University Press, UK, Cambridge. pp.163-179.

Will, C. M. (2006), 'The Confrontation between General Relativity and Experiment', *Living Rev. Relativity* **9**, 3.

Williams, J. G. and Boggs, D. H.: Schillak, S., eds (2009), *The Astronomical Unit Now, in Transits of Venus: New Views of the Solar System and Galaxy*, number 196 *in* 'Proceedings of 16^{th} International Workshop on Laser Ranging', Space Research Centre, Polish Academy of Sciences.