On the consequences of a probabilistic space-time continuum.

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(I) Introduction:

Our understanding of gravity has been evolving since the time of Newton. Using a spherical system of coordinates, Newton described gravitational force, at a distance 'r', due to a mass 'M' by his famous equation: \( F(M,r) = GM/r^2 \), where 'G' is the universal gravitational constant. This has worked quite well for a long time and, of course, continues to work in explaining most of the phenomenon we encounter in our everyday lives, such as calculating the trajectory of a probe to a planet within the solar system and calculating the trajectory for an artificial satellite around the earth.

The next big leap in our understanding of gravitation occurred with Einstein's General Theory of Relativity (which I will designate as GTR). In the GTR, the spatial coordinates and time were considered to be on equal footing. Instead of describing an event in a three dimensional space, \((x, y, z)\), with time being considered a universal and absolute entity without any relation to the spatial coordinates, an event was described in a four dimensional space-time coordinate system. With this, if we have two events, separated in space, at \((x, y, z, t)\) and \((x', y', z', t')\), where \(x \neq x', y \neq y'\) and \(z \neq z'\), then it was not necessary that
t = t'. The GTR described all the phenomenon equally well where Newton's theory for gravitation (hereby designated as NTG) was found to be applicable. The GTR also was consistent with Bohr's correspondence principle in that it was reducible to NTG for weak gravitational fields. However, the GTR was found to be more accurate in describing phenomenon where the gravitational fields were very strong and where the NTG gave only partially correct answers, such as the precession of the planet Mercury's orbit. NTG gave an answer that was 1/2 of the actual measurement, while GTR gave an answer that agreed with the measured value almost exactly. The GTR has also been successful in describing and predicting various other phenomenon and has so far stood the test of time and experimentation. Hence, if there is to be another theory for gravitation, it will have to, as per the correspondence principle, be reducible not only to NTG, but also to GTR.

One of the limitations that have been noted very soon after the development of GTR by Einstein, was that the GTR was not applicable to the atomic and sub-atomic phenomenon. The atomic and sub-atomic phenomenon are described by the Theory of Quantum Physics (henceforth referred as TQP). In TQP probability not only plays a major role but is considered to be a characteristic of the sub-atomic world. The TQP is also consistent with the correspondence principle, as it reduces to classical physics for large masses, as it must, since classical physics has stood the test of both time and experimentation since it's formulation. The GTR does not have probability in it's description of gravitation and therefore it is unknown what phenomenon can be
explained and/or predicted if one introduces a probability coordinate into the space-time continuum (hereby designated as STC) of the GTR.

In this article, I am proposing to add probability to the STC with certain characteristics and from this make certain predictions and possibly explain some of the phenomenon that have been discovered but for which a definite explanation has so far been lacking.

(II) Methods: None

(III) The probabilistic space-time continuum:

We will start with the STC of the GTR, where there is no matter and where every point is fully described by the set of coordinates \( (x_0, x_1, x_2, x_3) \) (where \( x_0 = t, x_1 = x, x_2 = y, x_3 = z \)). We will use the shorthand \( \{x_i\} \) where \( i = 0, 1, 2, 3 \). Now, to each point \( \{x_i\} \) in this STC we add a probability coordinate, \( P_0 \), and call it the baseline probability. Hence, each point in this empty STC, is described by \( \{x_i, P_0\} \). The probability coordinate, \( P_0 \), is as much an intrinsic characteristic of the STC as any of the \( x \). This new coordinate space with probability as one of it's coordinates we will call probabilistic space-time continuum (which we will designate by PSTC).

(IV) The effect of matter on the PSTC:

According to the GTR, in the presence of matter each of the points \( \{x_i\} \) is affected in a specific way. It is found that the matter 'M' changes the geometry of the STC and this change in the geometry is
given by a specific set of equations called the "Einstein's field equations" which connects the geometrical distortion of the STC to the matter causing the distortion. This distortion of the STC geometry by a mass 'M' is taken to be the gravitational field of the mass 'M'. GTR goes into details as to how objects in this distorted STC are supposed to behave and found that their behavior is similar to the behavior of a body as described by NTG due to a mass 'M' when weak gravitational fields are considered. Just as matter affects \( \{ x, p \} \) it also has an effect on the probability coordinate, \( P_0 \). In the presence of matter the \( P_0 \) "splits" into two components, \( P_A \) and \( P_R \). \( P_A \) is the probability that an object at the point \( \{ x, p_A, p_R \} \) will have an effect that will make it move towards the mass 'M', while \( p_R \) is the probability that the same object at the same point, \( \{ x, p_A, p_R \} \), will have the effect that will make it move away from the mass 'M'. Hence, in the presence of matter a point in PSTC, \( \{ x, p_0 \} \) will change into \( \{ x, p_A, p_R \} \). This changing of \( p_0 \) into \( p_A \) and \( p_R \) we will call "splitting" of the baseline probability ' \( p_0 \) '. The \( p_0 \) has a baseline value of 1/2 (which I will derive later). Thus in empty PSTC each point is described by \( \{ x, 1/2 \} \) and in the presence of matter the \( \{ x, 1/2 \} \) "splits" into \( \{ x, p_A, p_R \} \).

(V) The characteristics of \( P_A \) and \( P_R \):

To describe the characteristics of \( P_A \) and \( P_R \), I will use a spherical coordinate system whose origin is the mass 'M' and creating the gravitational field around it.

(1) Both \( P_A \) and \( P_R \) depend only on the distance 'r' of the point \( \{ x, r \} \),
\(P_A, P_R\) from 'M' and not on the direction of that point, i.e \(P_A = P_A(r)\) and \(P_R = P_R(r)\), for all 'r'.

(2) Both \(P_A(r)\) and \(P_R(r)\) are smooth functions with respect to 'r'.

(3) Both \(P_A\) and \(P_R\) are also functions of the mass 'M', i.e \(P_A = P_A(M,r)\) and \(P_R = P_R(M,r)\). These functions are also smooth with respect to 'M'.

(4) For all 'M' and 'r' we have \(P_A(M,r) + P_R(M,r) = 1\). (As a corollary from this equation we have \(\frac{\partial P_A(M,r)}{\partial r} = -\frac{\partial P_R(M,r)}{\partial r}\) for all 'M'.)

(5) \(0 < P_A(M,r), P_R(M,r) < 1\), for all 'M' and 'r'.

(6) For a given 'M', there is an \(r = r_0\) at which \(P_A(M,r_0) = P_R(M,r_0) = 1/2\).

(7) The functions \(P_A(M,r)\) and \(P_R(M,r)\) are mirror images to each other about the line \(P = 1/2\) in a 'P' v/s 'r' coordinate frame.

(8) For \(M = 0\), there is no gravitational field and therefore for an object at \(\{x, \dot{x}, P_A, P_R\}\) the probability for it to move in any given direction is equal to the probability for it to move in the opposite direction, i.e \(P_A = P_R\). Since we have \(P_A + P_R = 1\), this implies \(P_A = P_R = 1/2\). This is precisely the baseline probability 'Po', where there is no matter in the PSTC. Hence, the value of \(Po = 1/2\), as was mentioned earlier.

(9) \(\lim_{r \to \infty} P_A(M,r) \to 1\) and \(\lim_{r \to 0} P_R(M,r) \to 0\).

(10) \(\lim_{r \to \infty} P(M,r) \to 0\) and \(\lim_{r \to \infty} P_R(M,r) \to 1\).

We can graph the functions \(P_A(r)\) and \(P_R(r)\) with respect to 'r' as follows:

\[
\begin{array}{c}
\text{Figure 1}
\end{array}
\]

\[
\begin{array}{c}
\text{For } M = 0
\end{array}
\]

\[
\begin{array}{c}
P_A(r) = P_R(r) = Po = 1/2
\end{array}
\]
For \( m \neq 0 \)

(VI) Conclusions from figure # 2:

Considering the figure # 2, we can derive the following conclusions:

1. We see that an anti-gravitational field emerges as a natural phenomenon, just as a gravitational field emerges naturally, due to the effect of matter on the probability coordinate \( P_0 \). This is due to the "splitting" of \( P \) into \( P_A \) and \( P_R \).
2. We see that there is a distance \( r_o \) from the mass 'M' where \( P_A (r_o) = P_R (r_o) \). This means that an object at \( r_o \) is equally likely to move towards 'M' as it is to move away from 'M'. To put it differently, at this distance \( r_o \), an object is in a net zero gravitational field. We will call this distance, the "point of zero gravity".
3. For \( r << r_o \), we have \( P_A (r) \approx 1 \) and \( P_R (r) \approx 0 \).
4. For \( r >> r_o \), we have \( P_A (r) \approx 0 \) and \( P_R (r) \approx 1 \).

(VII) Application of the probability idea to the Newton's law for
gravitation:

I like to apply the above probability idea to the Newton's law for gravitation which is expressed in spherical coordinates as: \( F(M, r) = \frac{GM}{r^2} \), where, \( F(M, r) \) is the gravitational force of attraction by mass 'M' at distance 'r' and 'G' is the universal gravitational constant.

First, we have for the net gravitational force due to mass 'M' at distance 'r' given by:

\[
F(M, r) = P_A(M, r) \frac{GM}{r^2} + P_R(M, r) F^*(M, r)
\]

where \( P_A(M, r) \) and \( P_R(M, r) \) have already been defined before. \( F^*(M, r) \) is the force for gravitational repulsion. In Newtonian language, we talk about attraction and repulsion when considering the effect of a force 'F' on an object.

For \( r = r_o \), we have

\[
F(M, r_o) = P_A(M, r_o) \frac{GM}{r_o^2} + P_R(M, r_o) F^*(M, r_o)
\]

But for \( r = r_o \), we have

\[
P_A(M, r_o) = P_R(M, r_o) = \frac{1}{2}
\]

and the net gravitational force \( F(M, r) = 0 \).

This means: \( 0 = \frac{1}{2} \frac{GM}{r_o^2} + \frac{1}{2} F^*(M, r_o) \).

From this we conclude, \( F^*(M, r_o) = -\frac{GM}{r_o^2} \).

As a 1° approximation we will take \( F^*(M, r) = -\frac{GM}{r^2} \), for all 'r'.

This results in the following equation for the net gravitational force for equation #1,

\[
F(M, r) = \frac{GM}{r^2} [ P_A(M, r) - P_R(M, r) ]
\]

Since \( P_A(M, r) + P_R(M, r) = 1 \), equation #2 becomes
F(M,r) = GM/r^2[2PA(M,r) - 1]  \quad \text{equation \# 3.}

This then is the modified Newton's law for gravitation in spherical coordinates.

We can immediately see that for \( r << r_0 \), \( F(M,r) \propto GM/r^2 \) (as required by the correspondence principle).

Similarly, if we derive the modified gravitational field potential, in a spherical coordinate system, due to mass 'M' and distance 'r', \( \Phi_m(r) \), we will find it to be given by the following equation:

\[
\Phi_m(r) = GM \left[ \frac{1}{r} + 2 \int \frac{PA(m,r)}{r^2} \, dr \right]
\]

(VIII) Results/Discussion/Implications:

The concept of PSTC and the effect of matter on it as has been described lead us to entertain interesting implications.

1. As we have found that the Newton's law for gravitation is a special case for \( r << r_0 \), it is also very likely that Einstein's GTR is a special case for \( r << r_0 \). This means that we will need a modification in the GTR and obtain modified Einstein's field equations that take into account \( PA \) and \( P_R \), which will then be applicable to all the distances within the universe. This in turn may provide explanation to certain phenomenon that has so far been difficult to explain. Also, the modified GTR (mGTR) may predict other phenomenon that has not been observed yet and which can than be searched for and see if the predications come true.

2. One of the phenomenon that is known at present and needs an explanation is the existence of dark energy. From the concept of PSTC and it's interaction with matter, we can imply that dark energy is the sum
total of all the anti-gravitational fields from all the matter 'M' in the universe. That is:

\[ \text{Dark Energy} = \sum_{M} \text{Anti-gravitational field.} \]

This explanation for dark energy also explains why it should have a repulsive effect on the surrounding matter.

(3) The "problem of singularity" that plagues GTR can be resolved as follows: Given that \( \lim_{r \to 0} P_A(M,r) \to 1 \) and \( P_A(M,r) < 1 \), for all 'M' and 'r', it follows that 'r' cannot be zero.

Since for a singularity \( r = 0 \), the above constraints on \( P_A(M,r) \) prevent the formation of a singularity.

(4) The GTR has predicted the existence of gravitational waves almost a century ago. Despite the best efforts so far, they have not been found. Various theories, I believe, have been put forth in the meantime to explain this. From the discussion of the effect of matter on PSTC in this article, we can put forth an explanation for the difficulty in the detection of gravitational waves. The process that produces gravitational waves is equally likely to produce anti-gravitational waves. It is very likely that the gravitational and the anti-gravitational waves cancel each other out resulting in no net detectable gravitational or anti-gravitational waves.

(5) If we consider the origin of the universe from the big bang, we can at once conclude that the universe could not have started from a singularity (i.e. \( r = 0 \)) since, as discussed above, singularities cannot exist. This means that at the time of the big bang the matter, that later gave rise to all the matter in the universe, must have had dimensions. It can be
thought of as a very small clump of some type of particles. When the big bang did occur, the particles had to go through a slow phase of expansion due to \( r < r_0 \) of each particle relative to the others. Once the particles crossed each others 'r' then they would go through a second phase consisting of a rapid and accelerating expansion. This two phase expansion of the universe, at the time of the big bang, can be clearly seen from figure # 2. Also, from figure # 2, we see that this accelerating expansion of the universe is unending due to the anti-gravitational fields produced by matter and which is now evident as the dark energy.

(6) We can hypothesize that the Cosmological constant \( \Lambda \) is an ad hoc representation of \( P_R \). This means, unlike the \( P_R \), the Cosmological constant is not a characteristic of the STC or the universe.

(7) We can express \( P_A (0,r) = P_R (0,r) = 1/2 \) as \( \lim_{M \rightarrow 0} P_A (M,r), P_R (M,r) \rightarrow \frac{1}{2} \). From this we see that the gravitational interaction between small masses is much more complicated than that given by \( \frac{GMm}{r^2} \) due to significant effects from \( P_R (M,r) \), even-though we are dealing with weak gravitational fields, due to the small masses, and small distances.

(8) There are likely many more implications of the ideas presented here, which I will leave it to the reader to consider.

(IX) **Conclusion:**

I like to conclude this article with the following:

(1) As it is said that a good theory is one that can be disproven, one can easily test the existence/non-existence of a probability space given the modern technology. If we take two small objects of masses 'm_1' and 'm_2',
with \( m_1 \gg m_2 \), and make \( m_1 \) the fixed object and \( m_2 \) the moving object, we should be able to find out if there is a distance \( r_o \) at which the net gravitational force on \( m_2 \) due to \( m_1 \) is zero. If we are not able to find the \( r_o \) for \( m_1 \), then the probability space most likely does not exist.

(2) If, however, we do find that there is an \( r_o \) for \( m_1 \) in the above experiment, then, one can find the \( r_o \) for different masses \( m \) and plot the \( r_o \) against \( m \). From this we may be able to extrapolate to get the \( r_o \) for bigger bodies, such as the moon, earth, sun, galaxy and so on. The practical applications of this information will clearly be many.

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