Pairs of primes or pseudoprimes that generate an infinity of primes or pseudoprimes via a certain recurrence relation

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Abstract. In this paper are made five conjectures about a type of pairs of primes respectively Fermat pseudoprimes which have the property to generate an infinity of primes respectively Fermat pseudoprimes via a recurrence formula that will be defined in this paper; we name the pairs with this property Coman pairs of primes respectively Coman pairs of pseudoprimes. Because it is easy to show that two given primes respectively pseudoprimes do not form such a pair and it is very difficult to prove that they form such a pair, the correct expression about two odd primes (or pseudoprimes) p, q, where p = 30*k + d and q = 30*h + d, where k, h are non-null positive integers and d has the values 1, 7, 11, 13, 17, 19, 23, 29, is that the pair (p,q) is not a Coman pair respectively that the pair (p,q) is a possible Coman pair of primes (or pseudoprimes).

Definition 1:

We call the pair of odd primes (p, q), where p = 30*k + d and q = 30*h + d, where k, h are non-null positive integers and d has the values 1, 7, 11, 13, 17, 19, 23, 29, a *[possible] Coman pair of primes* if the sequence a(n), as it will be defined below, has [possibly] an infinity of terms that are prime numbers.

The sequence a(n), where n non-null positive integer, is defined in the following way:
: a(1) = p, a(2) = q;
: a(3) is the smallest number, different from p and q, which is prime from the following three ones: p + q – d, 2*p – d and 2*q - d;
: a(n) is the smallest prime greater than a(n – 1) that can be written as a(n) = a(i) + a(j) – d, where 1 ≤ i ≤ j < n.

Note: the definition implies that the one from the numbers p + q – d, 2*p – d and 2*q - d is prime and that for any n, n ≥ 4, there exist i, j, where 1 ≤ i ≤ j < n, such that a(n) = a(i) + a(j) – d, where a(n), a(i) and a(j) are all three primes.

Examples:

The pair of primes (37, 67) is a possible *Coman pair of primes* because:
: a(3) = 37 + 67 – 7 = 97 is prime;
: a(4) = 37 + 97 – 7 = 67 + 67 = 127 is prime;
: a(5) = 37 + 127 – 7 = 67 + 97 – 7 = 157 is prime;
: a(6) = 127 + 157 – 7 = 277 is prime.
: a(7) = 37 + 277 – 7 = 157 + 157 – 7 = 307 is prime.
(...)
The pair of primes (97, 127) is not a possible *Coman pair of primes* because 2*97 – 7 = 187 is not prime, 2*127 – 7 = 247 is not prime and also 97 + 127 – 7 = 217 is not prime.

Definition 2:

We call a *Coman pair of primes* a *Coman strict pair of primes* if a(3) = p + q – d and i ≠ j, in other words a(i) ≠ a(j).

Examples:

The pair of primes (37, 67) is a possible *Coman strict pair of primes* because:
: a(3) = 37 + 67 – 7 = 97 is prime;
: a(4) = 37 + 97 – 7 = 127 is prime;
: a(5) = 37 + 127 – 7 = 67 + 97 – 7 = 157 is prime;
: a(6) = 127 + 157 – 7 = 277 is prime;
: a(7) = 37 + 277 – 7 is prime.
(...)
The pair of primes (37, 157) is not a possible *Coman strict pair of primes* because 37 + 157 – 7 = 187 is not prime, but is a possible Coman pair of primes, because a(3) = 37 + 37 – 7 = 67 is prime, a(4) = 67 + 67 – 7 = 127 is prime (...).

Definition 3:

We call the pair of odd primes (p, q) a *[possible] generalized Coman pair of primes to base b*, where b is a non-null integer, if the sequence a(n), as it will be defined below, has [possibly] an infinity of terms that are prime numbers.

The sequence a(n), where n non-null positive integer, is defined in the following way:
: a(1) = p, a(2) = q;
: a(3) is the smallest number, different from p and q, which is prime from the following three ones: p + q – b, 2*p – b and 2*q - b;
: a(n) is the smallest prime greater than a(n – 1) that can be written as a(n) = a(i) + a(j) – b, where 1 ≤ i ≤ j < n.

Note: the definition implies that the one from the numbers p + q – b, 2*p – b and 2*q - b is prime and that for any n, n ≥ 4, there exist i, j, where 1 ≤ i ≤ j < n, such that a(n) = a(i) + a(j) – b, where a(n), a(i) and a(j) are all three primes.

Examples:

The pair of primes (7, 13) is a possible *generalized Coman pair of primes to base 1* because:
: a(3) = 7 + 13 − 1 = 19 is prime;
: a(4) = 13 + 19 − 1 = 31 is prime;
: a(5) = 7 + 31 − 1 = 19 + 19 − 1 = 37 is prime;
: a(6) = 7 + 37 − 1 = 13 + 31 − 1 = 43 is prime;
: a(7) = 19 + 43 − 1 = 31 + 31 − 1 = 61 is prime.
(...)

The pair of primes (11, 17) is a possible *generalized Coman pair of primes to base -1* because:
: a(3) = 11 + 11 + 1 = 23 is prime;
: a(4) = 11 + 17 + 1 = 29 is prime;
: a(5) = 11 + 29 + 1 = 41 is prime;
: a(6) = 17 + 29 + 1 = 47 is prime;
: a(7) = 11 + 41 + 1 = 53 is prime;
: a(8) = 11 + 47 + 1 = 59 is prime.
(...)

Definition 4:

We call a *generalized Coman pair of primes* a *generalized Coman strict pair of primes* if a(3) = p + q − b and i ≠ j, in other words a(i) ≠ a(j).

Note that the term a(n) = 23 is not a term of the *generalized Coman strict pair of primes* (11, 17), but is a term of the *generalized Coman pair of primes* (11, 17).

Definition 5:

We call the pair (p, q) of odd Fermat pseudoprimes to the same base b, where p = 30*k + d and q = 30*h + d, where k, h are non-null positive integers and d has the values 1, 7, 11, 13, 17, 19, 23, 29, a *[possible] Coman pair of pseudoprimes* if the sequence a(n), as it will be defined below, has [possibly] an infinity of terms that are pseudoprimes to the same base b.

The sequence a(n), where n non-null positive integer, is defined in the following way:
: a(1) = p, a(2) = q;
: a(3) is the smallest number, different from p and q, which is prime or Fermat pseudoprime to base b from the following three ones: p + q − d, 2*p − d and 2*q - d;
: a(n) is the smallest number which is prime or pseudoprime to base b greater than a(n − 1) that can be written as a(n) = a(i) + a(j) − d, where 1 ≤ i ≤ j < n.

Note: the definition implies that one from the numbers p + q − d, 2*p − d and 2*q - d is a prime or a pseudoprime to base b and that for any n, n ≥ 4, there exist i, j, where 1 ≤ i ≤ j < n, such that a(n) = a(i) + a(j) − d, where a(n), a(i) and a(j) are primes or pseudoprimes to base b, and, finally, that there is an infinite subset of the set a(n), namely a(m), whose terms are all of them pseudoprimes to base b.

Examples:

The pair of Fermat pseudoprimes to base 2 (2701, 2821), where 2701 = 30*90 + 1 and 2821 = 30*94 + 1, is a possible *Coman pair of pseudoprimes* because:

: a(3) = 2701 + 2821 = 5521 is prime;
: a(4) = 2821 + 2821 – 1 = 5641 is prime;
: a(5) = 2701 + 5521 = 8221 is prime;
: a(6) = 5521 + 8221 = 13741 is Fermat pseudoprime to base 2.
(...)

Conjecture 1:

There is an infinity of Coman pairs of primes.

Conjecture 2:

There is an infinity of Coman strict pairs of primes.

Conjecture 3:

There is an infinity of generalized Coman pairs of primes to any base b, where b is non-null integer.

Conjecture 4:

There is an infinity of generalized Coman strict pairs of primes to any base b, where b is non-null integer.

Conjecture 5:

There exist pairs of Poulet numbers which are Coman pairs of pseudoprimes.

References:

1. Coman, Marius, *Twenty-four conjectures about "the eight essential subsets of primes"*, Vixra;
2. Coman, Marius, *Three conjectures about an infinity of subsets of integers, each with possible infinite terms primes or squares of primes*, Vixra.