Exploring Gravitational Phenomena in a Riemann-Minkowski Spacetime

PRELIMINARY DRAFT

Carsten S.P. Spanheimer

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Abstract

After the concept of a locally scale-invariant spacetime has been introduced in the companion document [3], now physical experiments on mathematical entities will be performed to find out implications of that model.

With regard to earlier solutions by others, an unbiased inspection of different gravitational scenarios under local scale-invariance in comparison with physical reality is undertaken.

This gives five results at once: A suspected locality condition, a promising gravitational ansatz, a static solution for the gravitational potential in the subjective picture together with a possible static cosmic redshift, and a set of candidate terms for governing field equations of physical spacetime.

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1 Introduction

In the companion document [3] (read that before to minimize astonishment), a scale-invariant calculus in a pseudo-Riemannian space had been introduced.

The objective here is to investigate the behaviour of a Riemann-Minkowski spacetime, without injecting arbitrary additions into it, that is, to act more as an explorer of the mathematical model than as an engineer.

On the other hand, the mathematical model alone does not tell how it shall behave. To find out the governing constrictions of actual physical spacetime, or which ‘field equation’ or set of field equations it follows, arbitrary educated choices have to be made, where engineering comes in and only comparison with physical reality can lead the way.

Of all possible separate fields which might be found in the spacetime model, gravitation shall be the simplest one, so in parallel to the previous work by Einstein [1] and Schwarzschild [2], we start an unbiased investigation of infinitesimal embedding scenarios for a static gravitational field in Riemann-Minkowski spacetime, which is somewhat like doing physical experiments on mathematical objects.

2 Provisions

2.1 Conventions

Tensors are written in index notation.

The *Einstein* summation convention is always active, unless noted otherwise.

More unusually, contraction indices may be doubled when unambiguous, like for example in $\Gamma^\alpha_{\gamma\gamma} g^{\gamma\gamma}$, since at least one of the involved tensors is symmetric in that index pair.

Index instances are printed in bold, like $(T_t, T_x, T_y, T_z)$, when in $T_a$, $a \in \{t, x, y, z\}$.

As usual, a comma before an index ($,d$) is used as a short form for the partial derivative ($\partial_d$ or $\frac{\partial}{\partial x^d}$). The partial derivative by an index instance may leave out the comma, $T_t := T_{,t} = \partial_t T$.

A dot ($\cdot$) denotes a product, but in index notation not a ‘dot product’, since tensor contractions are already signified by index notation, so $A^a_b \cdot B^b_c = B^b_c \cdot A^a_b$.

In matrices, zero elements may be left blank or replaced by a dot ($\cdot$), see

\[ 0_{ab} := \begin{bmatrix} \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \end{bmatrix}, \quad \delta_{ab} := \begin{bmatrix} 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \end{bmatrix}. \]

Where of concern, the *Minkowski* metric is chosen with signature $(-, +, +, +)$, that is,

\[ \eta_{ab} := \begin{bmatrix} -1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \end{bmatrix}. \]
2.2 Scale Invariance

Let us warm up with a one-dimensional toy model. A function
\[ g : \mathbb{R} \to \mathbb{R}; \; x \mapsto g(x) \]
be differentiable at least once,
\[ g'(x) = \frac{\partial}{\partial x} g. \]
Another function takes the first one to the exponent,
\[ f : \mathbb{R} \to \mathbb{R}^+; \; x \mapsto f(x) = e^{g(x)} , \]
and the overall derivative is proportional to the latter,
\[ f'(x) = g'(x) e^{g(x)} = g'(x)f(x) , \]
so that the derivative of the inner function is the quotient
\[ g'(x) = f'(x)/f(x) . \]
Additionally scaling \( af(x) \) by a constant factor \( a \in \mathbb{R} \), we can run through a cycle of exponentiation, derivation, division and integration,
\[ \begin{align*}
g + c & \xrightarrow{\exp} af = e^{g+c} = ae^g \quad \text{(with } a = e^c) \\
g' & \xrightarrow{1/af} f' = g'ae^g , \end{align*} \tag{1} \]
and observe, that \( g' \) does not contain the scale factor \( a = e^c \) anymore, which is as arbitrary as the integration constant \( c \). This is the notion of ‘scale invariance’ which will be employed here.

In another view, \( f \) and \( f' \) contain \( f \) to a power, or ‘\( f \)-weight’, of \( f^1 \), but in \( g' \) and \( g \) the corresponding \( f \)-weight is \( f^0 \), which is equivalent to saying that the latter expressions are scale-invariant.

Additionally, \( g' = f'/f \) is the derivative of \( \log(f) \), and lives in a ‘logarithmic space’, so multiplication in \( f \) can be represented by addition of the logarithms, \( f_1 \cdot f_2 = \exp(g_1 + g_2) \).

There is a general monomial solution, with \( a, b, c \in \mathbb{R}, \; k = e^c, \; b \neq -1 \), from \( g' = \pm ax^b \),
\[ \begin{align*}
g & = \pm a \frac{b+1}{b+1} x^{b+1} + c \xrightarrow{\exp} f = k \exp(\pm a \frac{b+1}{b+1} x^{b+1}) \\
g' & \xrightarrow{1/f} f' = \pm ax^b k \exp(\frac{a}{b+1} x^{b+1}) , \end{align*} \]
and in the case when \( b = -1 \), a special monomial solution, with \( a, c \in \mathbb{R}, \; k = e^c, \) from \( g' = \pm a/x \),
\[ \begin{align*}
g & = \pm a \ln x + c \xrightarrow{\exp} f = k x^{\pm a} \\
g' & \xrightarrow{1/f} f' = \pm a k x^{\pm a-1} , \end{align*} \]
2.3 Infinitesimal Embeddings

Now generalize from the one-dimensional function \( f \) to the Jacobi matrix of an embedding of a 4-space into another 4-space, \( J^a_b : \mathbb{R}^4 \rightarrow \mathbb{R}^4 \), and specify a ‘Jacobi logarithm’ \( \Gamma^a_b \), so that the Jacobi matrix can be seen as the matrix exponential \( J^a_b = \exp(\Gamma^a_b) \).

But to again represent matrix multiplication through addition of logarithmic matrices,

\[
1 J^a_b \cdot 2 J^a_b = \exp(\Gamma^a_b + 2 \Gamma^a_b),
\]

the Jacobi matrix has to be infinitesimally near identity, \( J^a_b \rightarrow \delta^a_b \), so that the Jacobi logarithm is near zero, \( \Gamma^a_b \rightarrow 0 \), see also [3, 2.1]. This is the notion of an ‘infinitesimal embedding’ which will be employed here.

The infinitesimal embedding is an embedding of ‘my’ local tangent spacetime into my infinitesimal neighbourhood, where my tangent spacetime is always orthonormal and my local metric tensor is the flat Minkowski metric, \( g_{ab} \rightarrow \eta_{ab} \).

Though these entities have been fixed at one position (event) in spacetime, their derivatives at that point are still allowed to have any magnitude, and it has been shown [3, 1.6], that the Christoffel symbol of the second kind is exactly the gradient of the Jacobi logarithm.

Analogous to the ‘\( f \)–weight’ of the one-dimensional case above, assign a ‘Jacobi weight’ to the tensor expressions, see [3, 2.4]. Then the Jacobi matrix has weight \( J_1 \), the metric tensor has a weight of \( J_2 \), and the logarithmic potential has weight \( J_0 \), which means scale-invariance.\(^1\)

In the picture of an infinitesimal embedding, we run through a cycle similar to (1),

\[
\begin{align*}
\text{Jacobi logarithm (} J^0 \text{)}: \quad & \quad \Gamma^a_b \mid_0 \exp \quad J^a_b \mid_{\delta^a_b} \quad \text{Jacobi matrix (} J^1 \text{)} \\
\text{Christoffel 2nd (} J^0 \text{)}: \quad & \quad \Gamma^a_{bc} \quad \frac{\partial}{\partial c} \quad J^a_{bc} \quad \text{'Hesse tensor'(} J^1 \text{)},
\end{align*}
\]

where also the metric tensor comes out of the Jacobi matrix by kind of ‘squaring’ her through multiplication with her inverse, \( g_{ab} = _{\eta_{\alpha\beta}, J^0_{\alpha\beta}} \), involving an outer metric, which may be the flat Minkowski metric, \( g_{ab} = \eta_{\eta_{\alpha\beta}, J^0_{\alpha\beta}} \).

The mapping from the Jacobi matrix to the metric tensor is not injective and thus loses information, that is at least any orthogonal part and a Lorentz boost. Then the reverse path from the metric tensor to the Jacobi matrix is not unique, like taking a square root, and requires guessing. So why not start with the Jacobi matrix in the first place?

Then the Jacobi matrix can be expressed from the Jacobi logarithm in a way, that it is always positively-definite. But the Jacobi logarithm is locally always zero, so its absolute value does not contain any information, but its derivatives do.

\(^1\)The Jacobi weight might be related to the concept of a tensor density, in that a scale-invariant entity of \( J^0 \) is a density of weight +1, and an entity of \( J^1 \) is a non-density of weight 0.
So the first derivative of the Jacobi logarithm, that is exactly the Christoffel symbol of the second kind, together with its further derivatives, shall be the most ‘fundamental tensor’ in this context.

2.4 Logarithmic Potentials and Fields

A physical potential is never absolutely defined at a particular point, but merely the integral of a conservative vector field over some path is. The Jacobi logarithm, $\Gamma^a_{\ b}$, is always zero at the local point, and any arbitrary constant scale factor in exponential space will cancel out in the scale-invariant field which still can have any magnitude.

So it comes natural to identify the Jacobi logarithm, $\Gamma^a_{\ b}$, with a local (subjective) matrix potential and its gradients, $\Gamma^a_{\ bc}$, with the constituting field strengths, which still can have any magnitude.

2.5 Local Linearity

Both subjective potentials and fields, when in their scale-invariant form (with Jacobi weight zero), are linear under derivation,

$$ (\alpha \cdot \Gamma^a_{\ b} + \beta \cdot \Gamma^a_{\ b})_{,c} = \alpha \cdot \Gamma^a_{\ bc} + \beta \cdot \Gamma^a_{\ bc}, $$

$$ (\alpha \cdot \Gamma^a_{\ bc} + \beta \cdot \Gamma^a_{\ bc})_{,d} = \alpha \cdot \Gamma^a_{\ bc,d} + \beta \cdot \Gamma^a_{\ bc,d}, $$

which, in contrast, is not possible with Christoffel 1st, $\Gamma_{\ abc}$, since its Jacobi weight is not zero.

The same linearity holds for the contracted entities, when contracted without the metric tensor between their different-variant indices,

$$ (\alpha \cdot \Gamma^a_{\ b} g^{gg} + \beta \cdot \Gamma^a_{\ b} g^{gg})_{,d} \neq \alpha \cdot \Gamma^a_{\ b,d} g^{gg} + \beta \cdot \Gamma^a_{\ b,d} g^{gg}, $$

since even though the metric tensor is Minkowski-flat, $g_{ab} \rightarrow \eta_{ab}$, it is not constant, $g_{ab,c} \neq 0$.

Contraction with the metric tensor can be done after all derivations have taken place. Then the local metric tensor may be substituted by the plain Minkowski metric, like in the total connection derivative contractions,

$$ \Gamma^\delta_{\ \delta \eta \eta} \eta^{\eta\eta}, \quad \Gamma^\delta_{\ \eta \eta \delta} \eta^{\eta\eta}. $$

The only operations remaining which are nonlinear are the connection products of the 1st and 2nd kind (see [3, 1.7]),

$$ g_{\gamma \gamma} \Gamma^\gamma_{\ ab} \Gamma^\gamma_{\ cd}, \quad \Gamma^a_{\ b\gamma} \Gamma^\gamma_{\ cd}, $$

and their respective contractions.
2.6 Routes to Christoffel 2nd

Now all the 2nd-rank entities, $\Gamma^{a}_{\ b}$, $J^{\mu}_{\ b}$ and $g_{ab}$, are locally either zero or identity and only their derivatives still contain information. Of the first derivatives, the only scale-invariant one is Christoffel 2nd, which is the interesting entity for constructing field equations.

![Figure 1: Routes to Christoffel 2nd, $\Gamma^{a}_{\ bc}$](image)

**Starting with the metric tensor,** the traditional route to Christoffel 2nd is rather cumbersome,

$$g_{ab} \xrightarrow{\partial_c} g_{ab,c} \xrightarrow{\text{muddle}} \Gamma_{abc} \xrightarrow{g^{aa} \Gamma_{abc}} \Gamma_{bc} = \frac{1}{2} g^{aa} (g_{ab,c} + g_{ac,b} - g_{bc,a})$$

and in the one-dimensional case equivalent to doing (Who wants to do that?)

$$f^2 \xrightarrow{\partial_x} (f^2)' \xrightarrow{-1/2} \frac{1}{2} (f^2)' \xrightarrow{-1/2} g' = \frac{(f^2)'}{2f^2} = \frac{2f'f}{2f^2} = \frac{f'}{f}.$$

When **starting with the Jacobi matrix,** the same journey to Christoffel 2nd takes even longer,

$$J^{\mu}_{\ b} \xrightarrow{g^{a\mu} J^{\mu}_{\ b} \eta_{\mu\mu}} g_{ab} \xrightarrow{\partial_c} g_{ab,c} \xrightarrow{\text{muddle}} \Gamma_{abc} \xrightarrow{g^{aa} \Gamma_{abc}} \Gamma_{bc},$$

but now there is a shorter alternative route by deriving the Hesse stack, then transporting it through the inverse Jacobi matrix into the inner scale-invariant logarithmic space,

$$J^{\mu}_{\ b} \xrightarrow{\partial_c} J^{\mu}_{\ bc} \xrightarrow{\Gamma^{a}_{\ bc}} = J^{a}_{\ b} J^{\mu}_{\ bc}.$$
In the case of a local infinitesimal embedding, where the Jacobi matrix is always near identity, the Jacobi logarithm, as the subjective potential, is always near zero. It should be shown, that the Jacobi matrix is typically well-formed to give a non-complex logarithm, and the exponential of a real Jacobi logarithm is always a positively-definite transform matrix.

Starting with a logarithmic potential, \( J^\mu_b \), the pathway to Christoffel 2nd is immediate,

\[
\Gamma^a_{\ b} \overset{\partial_c}{\longrightarrow} \Gamma^a_{\ bc} = \Gamma^a_{\ b,c},
\]

while the Jacobi matrix can be obtained in reverse,

\[
J^\mu_b := \exp(\Gamma^a_{\ b}).
\]

### 2.7 Subjective Path Integrals

Integrating Christoffel 2nd along a subjective path of parameter \( \rho \) with a local path velocity \( V^a(\rho) \),

\[
\Delta \Gamma^a_{\ b} = \int_{\rho_0}^{\rho_1} \Gamma^a_{\ bc}(\rho) \cdot V^c(\rho) \, d\rho,
\]

gives the difference in logarithmic potential, of which the \( \Gamma^t_t \) component is logarithmic timeshift,

\[
\sigma := \Gamma^t_t(\rho_1) - \Gamma^t_t(\rho_0),
\]

where \( \sigma > 0 \) means a redshift, and \( \sigma < 0 \) a blueshift.

The popular redshift number \( z \), with \( \lambda_1 \) the wavelength at the emitter (there) and \( \lambda_0 \) the wavelength at the receiver (here), or the respective frequencies \( f_1, f_0 \),

\[
z = \frac{\lambda_0}{\lambda_1} - 1 = \frac{f_1}{f_0} - 1,
\]

can in this context be expressed

\[
z = e^\sigma - 1, \quad \sigma = \ln(z + 1).
\]

In a spherical symmetry system, with the line of sight along the radius, \( \lambda := r \) and \( V^a(\lambda) := \hat{e}^r \),

\[
\Gamma^a_{\ b}(r) = \int \Gamma^a_{\ br}(r) \cdot \hat{e}^r \, dr, \quad \text{thus} \quad \sigma(r) = \int \Gamma^t_{\ tr}(r) \cdot \hat{e}^r \, dr.
\]

### 3 Modelling

#### 3.1 Velocity Potential

The logarithm of a symmetric matrix is itself symmetric, and so is the logarithmic special Lorentz transform, shown in a 1-D case, with \( \beta = v_x/c \), the ‘rapidity’ \( \Phi = \artanh(\beta) \),

\[
cosh(\Phi) = \gamma = 1/\sqrt{1 - \beta^2}, \quad \text{and} \quad \sinh(\Phi) = \beta \gamma,
\]

Lorentz boost
\[ \Gamma^a_b = \begin{bmatrix} 0 & \Phi & 0 \\ \Phi & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \exp \begin{bmatrix} \cosh(\Phi) & \sinh(\Phi) \\ \sinh(\Phi) & \cosh(\Phi) \\ 0 & 0 \end{bmatrix} \]

\[ J^\mu_a = \begin{bmatrix} 1 & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & 1 \end{bmatrix} \rightarrow g_{ab} = \begin{bmatrix} -1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \]

where the metric is always identically flat, which reflects the fact that inertial motion is not perceivable inside a system.

In the infinitesimal limit, where both velocity and rapidity vanish, \( v \rightarrow 0, \Phi \rightarrow 0 \), and also their time derivatives become equal, \( \partial_t v \rightarrow \partial_t \Phi \), rename \( v := \Phi \rightarrow 0 \), view it as a local ‘velocity potential’, and thus the first time derivative, \( \partial_t v = \partial_t \Phi = \vec{a} \), as the local ‘acceleration field strength’.

Making use of the full velocity 3-vector, \((v_x, v_y, v_z) = (\Phi_x, \Phi_y, \Phi_z) \rightarrow (0, 0, 0)\), the logarithmic velocity potential reads

\[ \Gamma^a_{\mu b} = \begin{bmatrix} \cdot & v_x & v_y & v_z \\ \cdot & v_x & v_y & v_z \\ \cdot & v_y & v_z & \cdot \\ \cdot & v_z & \cdot & \cdot \end{bmatrix} \rightarrow 0. \]

### 3.2 Time Dilation Potential

The component \( J^t_t = \gamma \rightarrow 1 \) of the Jacobi matrix reflects time dilation with regard to an outer space. For an infinitesimal embedding, \( J^t_t \rightarrow 1 \) and the component \( \Gamma^t_t = v_t \rightarrow 0 \) of the Jacobi logarithm is a logarithmic time dilation offset,

\[ \Gamma^a_b = \begin{bmatrix} \cdot & v_x & v_y & v_z \\ \cdot & v_x & v_y & v_z \\ \cdot & v_y & v_z & \cdot \\ \cdot & v_z & \cdot & \cdot \end{bmatrix} \rightarrow 0. \]

\[ J^\mu_a = \begin{bmatrix} 1 & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & 1 \end{bmatrix} \rightarrow g_{ab} = \begin{bmatrix} -e^{2v_t} & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}, \]

In the local frame, time flows at unit rate, \( v_t \rightarrow 0 \), but the gradient of time dilation may not vanish.

### 3.3 Acceleration Field

Defining the time derivative of rapidity to be local acceleration, \((a_t, a_x, a_y, a_z) := \frac{\partial}{\partial t}(v_t, v_x, v_y, v_z)\), the 2nd Christoffel symbols containing field strengths are, like the potential, symmetric in the first indices, and the single component \( a_t \) is the time derivative of logarithmic time dilation,

\[ \Gamma^a_{\mu bc} = \begin{bmatrix} a_t & a_x & a_y & a_z \\ a_x & v_{xx} & v_{xy} & v_{xz} \\ a_y & v_{yx} & v_{yy} & v_{yz} \\ a_z & v_{zx} & v_{zy} & v_{zz} \end{bmatrix}, \]

\[ \begin{bmatrix} a_t & v_{xt} & v_{yt} & v_{zt} \\ v_{xt} & v_{xx} & v_{xy} & v_{xz} \\ v_{yt} & v_{xy} & v_{yy} & v_{yz} \\ v_{zt} & v_{xz} & v_{yz} & v_{zz} \end{bmatrix} \rightarrow 0. \]
Where also spatial gradients of rapidity appear, \( v_{ab} := \partial_b v_a \).

With indices reordered,
\[
\Gamma^c_{ab} = \begin{bmatrix}
    a_t & a_x & a_y & a_z \\
    a_x & v_{xx} & v_{xy} & v_{xz} \\
    a_y & v_{xy} & v_{yy} & v_{yz} \\
    a_z & v_{xz} & v_{yz} & v_{zz}
\end{bmatrix},
\]

because partial derivatives commute, from the symmetry in the last indices follows that all space derivatives of the Lorentz potential must vanish identically,
\[
v_{xx} = v_{yy} = v_{zz} = v_{xy} = v_{yz} = v_{xz} = 0.
\]

Rewriting the fields accordingly,
\[
\Gamma^a_{bb} = \begin{bmatrix}
    a_t & a_x & a_y & a_z \\
    a_x & a_x & . & . \\
    a_y & . & a_y & . \\
    a_z & . & . & a_z
\end{bmatrix},
\]

we observe, that Christoffel 2nd is even symmetric in all 3 indices,
\[
\Gamma^a_{bt} = \begin{bmatrix}
    a_t & a_x & a_y & a_z \\
    a_x & . & . & . \\
    a_y & . & . & . \\
    a_z & . & . & .
\end{bmatrix},
\]

and also, that acceleration, as the time derivative of the velocity potential, equals the gradient of the logarithmic time factor, and can be seen as a field strength.

**Theorem.** Equivalence of time gradient and acceleration:
The gradient of logarithmic time dilation equals the time derivative of the velocity potential, that is acceleration,
\[
\frac{\partial}{\partial x} v_t = \frac{\partial}{\partial y} v_t = \frac{\partial}{\partial z} v_t = a_t
\]
\[
\nabla v_t = \frac{\partial}{\partial t} \mathbf{v} = \mathbf{a}.
\]
3 MODELLING

3.4 A stretch field

Assume a ‘stretch’ in a direction (x,y,z),

\[
g_{ab} = \begin{bmatrix} -f_t^2 & f_x^2 & f_y^2 \\ f_x^2 & -f_y^2 & f_z^2 \\ f_y^2 & f_z^2 & \end{bmatrix}_{\eta_{ab}}
\]

\[g_{\mu} = \begin{bmatrix} f_t \\ f_x \\ f_y \\ f_z \end{bmatrix}_{\eta_{\mu}} = \exp \begin{bmatrix} \ln f_t \\ \ln f_x \\ \ln f_y \\ \ln f_z \end{bmatrix}_{\eta_0} = g_{\mu}^a\]

\[g_{\mu} = \begin{bmatrix} f_{t,t} \\ f_{x,x} \\ f_{y,y} \\ f_{z,z} \end{bmatrix}_{\eta_{\mu}} = \begin{bmatrix} g_{t,t} \\ g_{x,x} \\ g_{y,y} \\ g_{z,z} \end{bmatrix}_{\eta_0} = g_{\mu}^a.
\]

The stretch field components live on the 3-diagonal only of Christoffel 2nd, so the Christoffel 2nd is naturally symmetric in all 3 indices,

\[g_{\mu} = \begin{bmatrix} g_{t,t} \\ \cdot \\ \cdot \\ \cdot \\ g_{x,x} \\ \cdot \\ \cdot \\ \cdot \\ g_{y,y} \\ \cdot \\ \cdot \\ \cdot \\ g_{z,z} \end{bmatrix}.
\]
3.5 Spherical Symmetry

From the metric tensor of a spherical symmetric embedding, $g_{ab}$, the Jacobi logarithm, $\Gamma_{ab}^a$, and, assuming invariance in time and polar angles, the $r$ components only of the Hesse matrix, $J_{b r}^\mu$, and of the Christoffel 2nd, $\Gamma_{br}^a$, are determined,

$$g_{ab} = \begin{bmatrix} -1 & 0 \\ 0 & r^2 \end{bmatrix}$$

$$J_{b r}^\mu = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{r} \end{bmatrix}$$

The reordered Christoffel 2nd are then

$$\Gamma_{ac}^a = \begin{bmatrix} \cdot & 1/r & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & 1/r \end{bmatrix}$$

and from the necessary symmetry in the last two indices $(a, b)$,

$$\Gamma_{ac}^a = \begin{bmatrix} 1/r & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & 1/r \end{bmatrix}$$

and again reordered back,

$$\Gamma_{bc}^a = \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & 1/r & \cdot \\ \cdot & \cdot & 1/r \end{bmatrix}$$
It looks like the Christoffel 2nd ‘wants’ to be even symmetric in all 3 indices, like those of acceleration and stretch do,

\[
\Gamma_{ab}^{a} := \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}, \\
\Gamma_{a}^{b} := \begin{bmatrix} \cdot & \cdot & \cdot \\ 1/r & \cdot & \cdot \\ 1/r & \cdot & \cdot \end{bmatrix}.
\]

The following will work either way.

4 Components of the Objective

Under the assumption, that any objective function should be expressed from the Christoffel 2nd or its further derivatives, we look to summarize the possible component terms of such ‘field equations’.

4.1 Odd-Rank Terms

The simplest terms which can be formed from Christoffel 2nd are two vector contractions, that is, the scale-invariant gradient of the logarithmic functional determinant,

\[ T_b = \Gamma^3 \delta_b \quad \text{(of Jacobi weight } J^0), \]

and a kind of coordinate divergence,

\[ T^a = \Gamma^{a \eta \eta \eta} \quad \text{(of Jacobi weight } J^{-2}). \]

4.2 Even-Rank Terms

Summarized are the possible 4-ranked terms, \( T^a_{bcd} \) (scale-invariant), or \( T_{abcd} \) (of Jacobi weight \( J^2 \)), which can be formed from the Christoffel 2nd, as well as their contractions, where \( T^a_b \) is scale-invariant, and \( T_{ab} \) as well as the total contraction \( T \) have Jacobi weight \( J^{-2} \).

Of the ‘high-level’ terms, the covariant derivative of Christoffel 2nd, with Jacobi weight \( J^0 \),

\[
\Gamma^{a}_{bcd} = \Gamma^{a}_{bc,d} = \Gamma^{a}_{bc,d} + \Gamma^{a}_{d \gamma} \Gamma^{\gamma}_{bc} - \Gamma^{a}_{\gamma \gamma} \Gamma^{\gamma}_{bd} - \Gamma^{a}_{b \gamma} \Gamma^{\gamma}_{cd},
\]

decomposes as a linear combination into a partial derivative, \( \Gamma^{a}_{b \gamma} \Gamma^{\gamma}_{cd} \), and three products of the 2nd kind, which are the ‘daisy chain’ contraction between a last index of a Christoffel 2nd with the first index of another Christoffel 2nd, \( \Gamma^{a}_{b \gamma} \Gamma^{\gamma}_{cd} \).

Also a ‘high-level’ construct, the Riemann curvature tensor, when formulated in weight \( J^0 \),

\[
T^a_{bcd} = R^a_{bcd} = (\Gamma^a_{bd,c} - \Gamma^a_{bc,d}) + (\Gamma^a_{c \gamma} \Gamma^{\gamma}_{bd} - \Gamma^a_{d \gamma} \Gamma^{\gamma}_{be}),
\]

contains the same types of partial connection derivatives and connection products of the 2nd kind.
Those ‘high-level’ expressions are themselves composed of ‘low-level’ terms. Summarizing the partial derivatives, 
\[ T_{abcd}^a = \Gamma_{bc,d}^a \quad \text{(of Jacobi weight } J^0) \],
and the connection product ‘of the 2nd kind’, 
\[ T_{abcd}^a = \Gamma_{b\gamma}^a \Gamma_{\gamma cd}^\gamma \quad \text{(of Jacobi weight } J^0) \],
and for completeness, the connection product ‘of the 1st kind’, 
\[ T_{abcd} = \Gamma_{ab}^\eta \Gamma_{cd}^\eta \eta_{\eta \eta} \quad \text{(of Jacobi weight } J^{-2}) \].

All terms of these with exactly one contravariant index, \( T_{abcd}^a \), are scale-invariant and the all-covariant terms, \( T_{abcd} \), have Jacobi weight \( J^{-2} \).

But a 4-ranked tensor equated to fixed values would make the model immobile. Contracting an expression to rank 2 or even fully to a scalar loosens restrictions, so that the model is allowed to move in some ways.

### 4.3 Even-Rank Contractions

Contracting the connection derivatives, \( \Gamma_{bc,d}^a \), to a ‘field divergence’, in an ‘even’ way,
\[ T_{ab} = \Gamma_{\delta a,b}^\delta \quad (J^0), \quad T_{a}^b = \Gamma_{b\eta,\eta}^a \eta^{\eta \eta} \quad (J^{-2}), \quad T = \Gamma_{\delta \eta,\eta}^\delta \eta^{\eta \eta} \quad (J^{-2}), \]
or cross-wise (odd),
\[ T_{ab} = \Gamma_{ab,\delta}^\delta \quad (J^0), \quad T_{a}^b = \Gamma_{\eta \eta,b}^a \eta^{\eta \eta} \quad (J^{-2}), \quad T = \Gamma_{\eta \eta,\delta}^\eta \eta^{\eta \eta} \quad (J^{-2}), \]

Contracting the connection product of the 2nd kind, \( \Gamma_{b\gamma}^a \Gamma_{\gamma cd}^\gamma \), evenly,
\[ T_{ab} = \Gamma_{\delta \gamma, a\gamma}^\delta \Gamma_{\gamma ab} \quad (J^0), \quad T_{a}^b = \Gamma_{\gamma \eta,\eta}^a \eta^{\eta \eta} \quad (J^{-2}), \quad T = \Gamma_{\delta \eta,\eta}^\delta \Gamma_{\gamma \eta \eta} \eta^{\eta \eta} \quad (J^{-2}), \]
or cross-wise (odd),
\[ T_{ab} = \Gamma_{a\gamma, b\delta}^\gamma \Gamma_{\gamma b\delta} \quad (J^0), \quad T_{a}^b = \Gamma_{\gamma \eta, \eta \eta}^a \eta^{\eta \eta} \quad (J^{-2}), \quad T = \Gamma_{\delta \eta, \eta \eta}^\delta \Gamma_{\gamma \eta \eta \eta} \eta^{\eta \eta} \quad (J^{-2}), \]
and for completeness, the connection product of the 1st kind, \( \eta_{\gamma \gamma}^a \Gamma_{ab}^\gamma \Gamma_{\gamma cd}^\gamma \), even,
\[ T_{ab} = \eta_{\gamma \gamma}^\gamma \Gamma_{\gamma \gamma}^a \Gamma_{\gamma \eta \eta} \eta^{\eta \eta} \quad (J^0), \quad T = \eta_{\gamma \gamma}^\gamma \Gamma_{\gamma \eta \eta \eta} \Gamma_{\mu \eta \mu \mu} \eta^{\eta \eta} \quad (J^{-2}), \]
or cross-wise (odd),
\[ T_{ab} = \eta_{\gamma \gamma}^\gamma \Gamma_{\gamma \gamma}^a \Gamma_{\gamma \eta \eta} \eta^{\eta \eta} \quad (J^0), \quad T = \eta_{\gamma \gamma}^\gamma \Gamma_{\gamma \eta \eta \eta} \Gamma_{\mu \eta \mu \mu} \eta^{\eta \eta} \quad (J^{-2}). \]

Of the first contraction, the all-covariant terms, \( T_{ab} \), are scale-invariant, and those with one contravariant index, \( T_{a}^b \), have Jacobi weight \( J^{-2} \). The total contractions, \( T \), are all of Jacobi weight \( J^{-2} \).
4.4 Linear Combinations

Of the matrices and the scalars which, when equated to zero, give a particular solution, any linear combination again yields matrices and scalars respectively which give the same solution, so there is some freedom of composition,

\[
a \cdot \Gamma^{\delta_{ab}} + b \cdot \Gamma^{\delta_{r\gamma}} = 0,
\]

\[
c \cdot \Gamma^{\delta_{\eta\eta}} \eta^{\eta} + d \cdot \Gamma^{\delta_{r\gamma}} \eta^{\eta} = 0.
\]

5 Obtaining Differential Equations and Solutions

Two kinds of resulting equations are investigated, that are scalars and 4x4-matrices which are equated to zero (or something else constant). For the matrices, which can yield up to 16 different equations, the separate equations are understood in conjunction – they all have to be true. The matrix may be filled up with always true equations like \(0 = 0\), but false equations like \(1 = 0\) will render the whole result to ‘false’, and some, like \(r \to \infty\) where locally \(r = 1\), to ‘not applicable’.

In the investigation following below, mainly two types of differential equations will show up, so their solutions shall be discussed before.

5.1 A ‘Newton’ function

From the function

\[
f(r) = \pm \frac{k}{r} \quad \Rightarrow \quad f,r = \mp \frac{k}{r^2} \quad \Rightarrow \quad f,rr = \pm \frac{2k}{r^3},
\]

we get some scale-invariant derivative terms where sign and cofactor disappear,

\[
g,r = \frac{f,r}{f} = -\frac{1}{r}, \quad g,r = \frac{f,r}{rf} = -\frac{1}{r^2}, \quad g,r^2 = \left(\frac{f,r}{f}\right)^2 = \frac{1}{r^2}, \quad f,rr = \frac{2}{r^2},
\]

\[
g,rr = \left(\frac{f,r}{f}\right)_{,r} = \frac{f,rr}{f} - \left(\frac{f,r}{f}\right)^2 = \frac{1}{r^2}.
\]

Possible differential equations which accept (4) as the solution include

\[
g,r + \frac{1}{r} = 0, \quad g,r + \frac{1}{r^2} = 0, \quad g,r^2 - \frac{1}{r^2} = 0, \quad g,r^2 + g,r = 0,
\]

\[
g,rr + \frac{g,r}{r} = 0, \quad g,rr - \frac{1}{r^2} = 0.
\]

In the following, matrices and scalars, which when equated to zero, admit the Newton function as a solution, shall be just said to be ‘Newton’.
5.2 An ‘Inverse Newton function’

From the function
\[ f(r) = \pm \frac{r}{k} \Rightarrow f_r = \pm \frac{1}{k} \Rightarrow f_{rr} = 0, \]
we get some scale-invariant derivative terms where sign and cofactor disappear,
\[ g_r = f_r f = \frac{1}{r}, \quad g_{rr} = f_r f = \frac{1}{r^2}, \quad g_r^2 = \left( \frac{f_r}{f} \right)^2 = \frac{1}{r^2}. \]

Possible differential equations which accept (5) as the solution include
\[ g_r - \frac{1}{r} = 0, \quad g_{rr} - \frac{1}{r^2} = 0, \quad g_r^2 - \frac{1}{r^2} = 0, \quad g_r^2 - g_{rr} = 0. \]

In the following, matrices and scalars, which when equated to zero, admit the inverse Newton function as a solution, shall be said to be ‘inverse–Newton’.

6 A Dual Ansatz for the Static Gravitational Solution

According to Einstein’s equivalence of gravity and acceleration, the stretch field and acceleration field should cancel out in some identity equation, which would be a linear (matrix or scalar) expression of some terms equated to zero.

Starting with the metric tensor, we assume, like Schwarzschild[2] did, that it is purely diagonal, and choose a 1-dimensional system in radial direction of a spherical symmetry embedding,
\[ g_{aa} = \text{diag}[g_{tt} \quad g_{rr} \quad g_{\varphi\varphi} \quad g_{\vartheta\vartheta}]. \]

From \( g_{aa} := J_{\mu}^a J_{\nu}^a \eta_{\mu\nu} \) the corresponding Jacobi matrix should be
\[ J_{\mu}^a = \text{diag}[\sqrt{|g_{tt}|} \quad \sqrt{|g_{rr}|} \quad \sqrt{|g_{\varphi\varphi}|} \quad \sqrt{|g_{\vartheta\vartheta}|}]. \]

Schwarzschild[2] chose a metric tensor similar to
\[ g_{ab} = \text{diag}[-f(r)^2 \quad f(r)^{-2} \quad 1/r^2 \quad 1/r^2], \]
of which a corresponding Jacobi matrix is
\[ J_{\mu}^a = \text{diag}[f(r) \quad f(r)^{-1} \quad 1/r \quad 1/r], \]
with \( g_{rr} \) reciprocal to \( g_{tt} = 1/g_{rr} \), to fulfil Einstein’s condition[1], that the functional determinant, \( \sqrt{|\det(g_{ab})|} = \det(J_{\mu}^a) \), be a constant 1, so that even her derivatives vanish.

In our case of an infinitesimal local embedding, the subjective Jacobi matrix is the identity, so the functional determinant is locally 1 anyway, but their derivatives may not vanish.

\[ \text{Ansatz} \]
Taking the radial speed of light as \( c_0 = \frac{J^r}{J^t} = \sqrt{\frac{g_{rr}}{g_{tt}}} \), an alternative restriction is imposed on the gravitational model, namely that \( c_0 \) should be constant when integrating over paths, which requires to set \( g_{rr} = g_{tt} \).

In the following, both variants are exercised, a (+) and a (−) case, with

\[
J^\mu_a = \text{diag} \left[ f(r)^1 \ f(r)^{\pm1} \ 1/r \ 1/r \right] \delta^\mu_b,
\]

and

\[
g_{ab} = \text{diag} \left[ -f(r)^2 \ f(r)^{\pm2} \ 1/r^2 \ 1/r^2 \right] \eta_{ab},
\]

from which the corresponding Jacobi logarithm, \( \Gamma^a_{\ b} \), is calculated and, assuming invariance in time and polar angles, the \( r \) components of the Hesse matrix, \( J^\mu_{br} \), and of the Christoffel 2nd, \( \Gamma^a_{\ br} \),

\[
g_{ab} = \begin{bmatrix} -f^2 & f^{\pm2} & f^2 & f^{\pm2} \\ f^{\pm2} & f^2 & f^{\pm2} & f^2 \\ f^2 & f^{\pm2} & f^2 & f^{\pm2} \\ f^{\pm2} & f^2 & f^{\pm2} & f^2 \end{bmatrix}_{(J^2)} \]

\[
J^\mu_b = \begin{bmatrix} f & f^{\pm1} & f^{\pm1} & f^{\pm1} \\ f^{\pm1} & f & f^{\pm1} & f^{\pm1} \\ f^{\pm1} & f & f^{\pm1} & f^{\pm1} \\ f^{\pm1} & f & f^{\pm1} & f^{\pm1} \end{bmatrix}_{(J^1)} \exp \begin{bmatrix} \ln f & \pm \ln f & \ln r & \pm \ln r \\ \ln f & \pm \ln f & \ln r & \pm \ln r \end{bmatrix}_{(J^0)} = \Gamma^a_{\ b}
\]

\[
J^\mu_{b,r} = \begin{bmatrix} f, r & +f, r & -f, r/f^2 & -f, r/f^2 \\ +f, r & f, r & -f, r/f^2 & -f, r/f^2 \\ -f, r/f^2 & -f, r/f^2 & 1 & 1 \\ -f, r/f^2 & -f, r/f^2 & 1 & 1 \end{bmatrix}_{(J^1)} \frac{J^{a\ b}, J^{a\ r}}{\pm g, r, \ 1/r, \ \pm g, r, \ 1/r} = \Gamma^a_{\ br},
\]

defining \( g, r := f, r/f \).

The reordered Christoffel 2nd is then

\[
\]

and expanding him to be symmetric in all 3 indices,

\[
\Gamma^a_{\ ab} = \begin{bmatrix} . & g, r & . & . \\ g, r & . & . & . \\ . & . & . & 1/r \\ . & . & 1/r & . \end{bmatrix}
\]

\[
\Gamma^a_{\ ba} = \begin{bmatrix} . & g, r & . & . \\ g, r & . & . & . \\ . & . & . & 1/r \\ . & . & 1/r & . \end{bmatrix}
\]

\[
\Gamma^a_{\ aa} = \begin{bmatrix} . & g, r & . & . \\ g, r & . & . & . \\ . & . & . & 1/r \\ . & . & 1/r & . \end{bmatrix}
\]

\[
\Gamma^a_{\ bb} = \begin{bmatrix} . & g, r & . & . \\ g, r & . & . & . \\ . & . & . & 1/r \\ . & . & 1/r & . \end{bmatrix}
\]
which is the sum of one stretch (gravitational) component, $\tilde{g}_{rr} = \pm g_{,r}$, three of radial acceleration, $a_{r} = \tilde{\Gamma}^{t}_{tr} = \tilde{\Gamma}^{r}_{tt} = g_{,r}$, and a spherical embedding, $\tilde{\Gamma}^{a}_{bc}$.

The following rather lengthy calculations are given for reference. You might want to skip forward to the discussion of gravitational solutions in section 7 (p.24).

6.1 Following the (+) Ansatz

One contraction of the connection is

$$\Gamma^{\delta}_{\delta a} = 2 \begin{bmatrix} g_{,r} + \frac{1}{r} \\ . \\ . \end{bmatrix} \text{ (of weight } J^{0}) ,$$

which can be seen as the 4-volume gradient of the logarithmic functional determinant. Note, that equating this to zero, $\Gamma^{\delta}_{\delta a} = 0 \iff \frac{2x}{r} + \frac{1}{r} = 0$, already accepts $g_{,r} = -\frac{1}{r}$ and yields the Newton solution $f = \pm \frac{k}{r}$.

The other contraction is

$$\Gamma^{a}_{\eta \eta} \eta^{\eta} = 2 \begin{bmatrix} 1/r \\ . \\ . \end{bmatrix} \text{ (of weight } J^{-2}) ,$$

which can be seen as the divergence of the Jacobi logarithm.

Observing, that the latter contains components of the coordinate embedding only, and that equating it to zero, $1/r = 0$, gives $r \to \pm \infty$, which contradicts the condition $r = 1$ of the logarithmic spherical embedding and thus says ‘not applicable’ to the result, it is not a candidate for a field equation.

Partial Divergences

The (+) even partial divergences are

$$\Gamma^{\delta}_{\delta a,b} = 2 \begin{bmatrix} g_{,rr} - \frac{1}{r^{2}} \\ . \end{bmatrix} \text{ (weight } J^{0}, g_{,r} = -\frac{1}{r}, f = \pm \frac{k}{r}) ,$$

which is Newton, and

$$\Gamma^{a}_{\eta \eta} \eta^{\eta} = \begin{bmatrix} g_{,rr} & g_{,rr} \\ g_{,rr} & -1/r^{2} \end{bmatrix} \text{ (weight } J^{-2}) ,$$
which is not, and the even total contraction

\[ \Gamma_{\delta\eta\eta}^{\eta\eta} = 2 \left( g_{rr} - \frac{1}{r^2} \right) (\text{weight } J^{-2}, g_{rr} = -\frac{1}{r}, f = \pm \frac{k}{r}) , \tag{9} \]

which is also Newton.

The (+) odd (cross-wise contracted) partial divergences,

\[ \Gamma_{\delta\eta\eta}^{\eta\eta} = \left[ \begin{array}{c} g_{rr} \\
-1/r^2 \\
-1/r^2 \end{array} \right] (\text{weight } J^0) , \tag{10} \]

\[ \Gamma_{\eta\eta\eta}^{\eta\eta} = \left[ \begin{array}{c} -2/r^2 \\
. \\
. \end{array} \right] (\text{weight } J^{-2}) , \]

\[ \Gamma_{\delta\eta\eta}^{\eta\eta} = -2/r^2 (\text{weight } J^{-2}) , \]

all say ‘not applicable’ to the result, because of \( 1/r^2 = 0 \Rightarrow r \rightarrow \pm \infty \).

**Connection Products**

The (+) even second connection products,

\[ \Gamma_{\delta\gamma\gamma}^{\alpha\beta\delta} = 2 \left[ \begin{array}{c} g_{rr}^2 + \frac{g_{rr}}{r} \\
g_{rr}^2 + \frac{2g_{rr}}{r} \\
g_{rr}^2 + \frac{g_{rr}}{r} + \frac{1}{r^2} \end{array} \right] (\text{weight } J^0, g_{rr} = -\frac{1}{r}, f = \pm \frac{k}{r}) , \tag{11} \]

which is Newton, and

\[ \Gamma_{\beta\gamma\gamma}^{\alpha\beta\delta} = 2 \left[ \begin{array}{c} \frac{g_{rr}}{r} \\
\frac{g_{rr}}{r} + \frac{1}{r^2} \end{array} \right] (\text{weight } J^{-2}) , \]

which is not, and the total contraction,

\[ \Gamma_{\delta\gamma\gamma}^{\gamma\eta\eta} = 4 \left( \frac{g_{rr}}{r} + \frac{1}{r^2} \right) (\text{weight } J^{-2}, g_{rr} = -\frac{1}{r}, f = \pm \frac{k}{r}) , \tag{12} \]

which is also Newton.

The (+) odd (cross-wise contracted) second connection products are
6 A DUAL ANSATZ FOR THE STATIC GRAVITATIONAL SOLUTION

\[ \Gamma^\gamma_{\alpha\gamma} \Gamma^\gamma_{\beta\delta} = 2 \begin{bmatrix} \frac{g_{r^2}}{2} & \frac{g_{r^2} + \frac{1}{r^2}}{2} & \frac{1}{r^2} & \frac{1}{r^2} \\ \frac{1}{r^2} & \frac{1}{r^2} & \frac{1}{r^2} & \frac{1}{r^2} \end{bmatrix} \text{ (weight } J^0) \],

\[ \Gamma^\gamma_{\eta\gamma} \Gamma^\gamma_{\eta\eta} \Gamma^{\eta\eta} = 2 \begin{bmatrix} \frac{1}{r^2} & \frac{1}{r^2} & \frac{1}{r^2} & \frac{1}{r^2} \end{bmatrix} \text{ (weight } J^{-2}) \],

\[ \Gamma^\delta_{\eta\gamma} \Gamma^\gamma_{\eta\delta} \Gamma^{\eta\eta} = \frac{6}{r^2} \text{ (weight } J^{-2}) \],

all say ‘not applicable’ to the result, because of \( \frac{1}{r^2} = 0 \Rightarrow r \to \pm \infty \).

Of the (+) first connection products and their total contractions the even ones,

\[ \eta_{\gamma\gamma} \Gamma^\gamma_{ab} \Gamma^\gamma_{\eta\eta} \Gamma^{\eta\eta} = 2 \begin{bmatrix} \frac{g_{r^2}}{r} & \frac{g_{r^2}}{r} & \frac{1}{r^2} & \frac{1}{r^2} \end{bmatrix} \text{ (weight } J^0) \],

\[ \eta_{\gamma\gamma} \Gamma^\gamma_{aa} \Gamma^\gamma_{\eta\eta} \eta^{\eta\eta} = \frac{6}{r^2} \text{ (weight } J^{-2}) \],

and the odd,

\[ \eta_{\gamma\gamma} \Gamma^\gamma_{an} \Gamma^\gamma_{bn} \eta^{\eta\eta} = 2 \begin{bmatrix} \frac{g_{r^2}}{r} & \frac{g_{r^2}}{r} & \frac{1}{r^2} & \frac{1}{r^2} \end{bmatrix} \text{ (weight } J^0) \],

\[ \eta_{\gamma\gamma} \Gamma^\gamma_{an} \Gamma^\gamma_{\eta\eta} \eta^{\eta\eta} = \frac{4}{r^2} \text{ (weight } J^{-2}) \],

also say ‘not applicable’ to the result, because of \( \frac{1}{r^2} = 0 \Rightarrow r \to \pm \infty \).

6.2 Findings from the (+) Ansatz

Of the terms from the (+) ansatz,

\[ g_{ab} = \text{diag}[-f(r)^2, f(r)^2, 1/r^2, 1/r^2] \],

the single vector expression

\[ \Gamma^\delta_{\delta a} = 0 \],

and as matrices, the even connection divergence and the even second connection product,

\[ \Gamma^\delta_{\delta a,b} = 0 \],

\[ \Gamma^\delta_{\delta a} \Gamma^\gamma_{ab} = 0 \],

and as scalars, the full contractions of those,

\[ \Gamma^\delta_{\delta \eta, \eta} \eta^{\eta\eta} = 0 \],

\[ \Gamma^\delta_{\delta \gamma} \Gamma^\gamma_{\eta \eta} \eta^{\eta\eta} = 0 \],
when equated to zero, all lead to the Newton solution, \( g_r = -\frac{1}{r}, \quad f = \pm \frac{k}{r} \).

All connection products of the first kind give a ‘false’ or ‘not applicable’ result.

Note that (12) is also the scalar product of the volume gradient (6) with the divergence (7),

\[
\Gamma^\delta_{\delta a} \cdot \Gamma^{\alpha \eta \eta} = 2 \begin{bmatrix}
\cdot \\
\cdot \\
\cdot
\end{bmatrix} \cdot 2 \begin{bmatrix}
\cdot \\
\cdot \\
\cdot
\end{bmatrix} = 4 \left( \frac{g_r}{r} + \frac{1}{r^2} \right),
\]

where the latter, (7), is non-zero only because of the spherical embedding. In cartesian coordinates it will vanish and the whole expression vanishes, so that \( \Gamma^\delta_{\delta \gamma} \Gamma^{\eta \eta} \eta^\eta = 0 \) gives a tautology. Thus this term alone can not suffice to form a field equation.

### 6.3 Following the (-) Ansatz

In the (-)-case the possible contractions of the connection are

\[
\Gamma^\delta_{\delta a} = 2 \begin{bmatrix}
\cdot \\
\cdot \\
\cdot
\end{bmatrix} \text{ (of weight } J^0\text{)}
\]

(15)

and

\[
\Gamma^{\alpha \eta \eta} \eta^\eta = 2 \begin{bmatrix}
\cdot \\
\cdot \\
\cdot
\end{bmatrix} \text{ (of weight } J^{-2}\text{)}
\]

(16)

where the latter, \( \Gamma^{\alpha \eta \eta} \eta^\eta = 0 \) \( \Leftrightarrow g_r - \frac{1}{r} = 0 \), accepts \( g_r = +\frac{1}{r} \) and yields the solution \( f = \pm \frac{r}{k} \).

### Partial Divergences

The (−) even partial divergences are

\[
\Gamma^\delta_{\delta a, b} = 2 \begin{bmatrix}
\cdot \\
\cdot \\
\cdot
\end{bmatrix} \text{ (of weight } J^0\text{)}
\]

(17)

\[
\Gamma^{a \eta \eta} \eta^\eta = \begin{bmatrix}
+g_{rr} \\
-g_{rr} \\
-1/r^2 \\
-1/r^2
\end{bmatrix} \text{ (of weight } J^{-2}\text{)}
\]

(18)

and the even total contraction

\[
\Gamma^\delta_{\delta \eta, \eta} \eta^\eta = -2/r^2 \text{ (of weight } J^{-2}\text{)}
\]
which all say ‘not applicable’ to the result, because of \(1/r^2 = 0 \Rightarrow r \rightarrow \pm \infty\).

The \((-)\) odd (cross-wise contracted) partial divergences are
\[
\Gamma_{ab,\delta} = \begin{bmatrix}
+g,rr & -g,rr \\
-g,rr & -1/r^2 \\
1/r^2 & 1/r^2
\end{bmatrix}
\] (of weight \(J^0\)),
\[(19)\]

which is voted ‘not applicable’, and
\[
\Gamma_{\eta,\eta,\eta,\eta} = 2\begin{bmatrix}
\cdots \\
-g,rr - \frac{1}{r^2} \\
\cdots
\end{bmatrix}
\] (weight \(J^{-2}\), \(g,rr = +\frac{1}{r}, f = \pm \frac{r}{K}\)),
\[(20)\]
\[
\Gamma_{\delta,\eta,\delta,\eta} = 2\left(-g,rr - \frac{1}{r^2}\right)
\] (weight \(J^{-2}\), \(g,rr = +\frac{1}{r}, f = \pm \frac{r}{K}\)),
\[(21)\]

which are both inverse-Newton.

Connection Products

The \((-)\) even second connection products are
\[
\Gamma^a_{\delta,\gamma} \Gamma^\gamma_{\delta,\eta} = 2\begin{bmatrix}
g,rr & -g,rr \\
1/r^2 & 1/r^2
\end{bmatrix}
\] (of weight \(J^0\)),
\[(22)\]

which just votes ‘not applicable’ to \(g\) being constant, and
\[
\Gamma^a_{b,\gamma} \Gamma^\gamma_{\eta,\eta} = 2\begin{bmatrix}
-g,rr + \frac{g,rr}{r} & +g,rr - \frac{g,rr}{r} \\
1/r^2 - \frac{g,rr}{r} & 1/r^2 - \frac{g,rr}{r}
\end{bmatrix}
\] (of weight \(J^{-2}\)),
\[(23)\]
\[
\Gamma^\delta_{\delta,\gamma} \Gamma^\gamma_{\eta,\eta} = 4\left(\frac{1}{r^2} - \frac{g,rr}{r}\right)
\] (weight \(J^{-2}\), \(g,rr = +\frac{1}{r}, f = \pm \frac{r}{K}\)),
\[(24)\]

which are both inverse-Newton.

The \((-)\) odd (cross-wise contracted) second connection products
\[
\Gamma^\delta_{a,\gamma} \Gamma^\gamma_{b,\delta} , \quad \Gamma^a_{\eta,\gamma} \Gamma^\gamma_{\eta,\eta} , \quad \Gamma^\delta_{\eta,\eta} \Gamma^\gamma_{\eta,\eta} ,
\] are identical to the \((+)\) odd second products, and thus vote ‘not applicable’.

Of the \((-)\) first connection products and their total contractions are the \textbf{even} ones
\[
\]
\[ \eta_{\gamma\gamma} \Gamma^\gamma_{ab} \eta^{\eta\eta} = 2 \left[ \begin{array}{c} -g_{r}^2 + \frac{2g_{r}}{r} \\ +g_{r}^2 - \frac{g_{r}}{r^2} \\ \frac{1}{r^2} \end{array} \right] \text{ (of weight } J^0 \text{)}, \]

\[ \eta_{\gamma\gamma} \Gamma^\gamma_{aa} \eta^{\eta\eta} = 4 \left( g_{r}^2 - \frac{g_{r}}{r} + \frac{1}{r^2} \right) \text{ (of weight } J^{-2} \text{)}, \]

where the matrix says 'not applicable', and the scalar... TODO

The odd (-) first connection products are

\[ \eta_{\gamma\gamma} \Gamma^\gamma_{an} \eta^{\eta\eta} = 2 \left[ \begin{array}{c} \frac{g_{r}}{r} \\ \frac{g_{r}}{r} \\ \frac{1}{r^2} \end{array} \right] \text{ (of weight } J^0 \text{)}, \]

\[ \eta_{\gamma\gamma} \Gamma^\gamma_{an} \eta^{\eta\eta} = \frac{4}{r^2} \text{ (of weight } J^{-2} \text{)}, \]

which are all 'not applicable'.

6.4 Findings from the (-) Ansatz

Of the terms from the (-) ansatz,

\[ g_{ab} = \text{diag} \left[ -f(r)^2 \quad f(r)^{-2} \quad 1/r^2 \quad 1/r^2 \right], \]

the vector expression

\[ \Gamma^a_{\eta\eta} \eta^{\eta\eta} = 0, \text{ see (16)}, \]

and as matrices, the odd connection divergence and the even second connection product,

\[ \Gamma^a_{\eta\eta, b} \eta^{\eta\eta} = 0, \text{ see (20)}, \quad \Gamma^a_{b\gamma} \Gamma^\gamma_{\eta\eta} \eta^{\eta\eta} = 0, \text{ see (23)}, \]

and as scalars, the respective full contractions of those,

\[ \Gamma^\delta_{\eta\eta, \delta} \eta^{\eta\eta} = 0, \text{ see (21)}, \quad \Gamma^\delta_{\delta\gamma} \Gamma^\gamma_{\eta\eta} \eta^{\eta\eta} = 0, \text{ see (24)}, \]
when equated to zero, give the inverse Newton solution as the result, with $g, r = \pm \frac{1}{r}$ and $f = \pm \frac{r}{k}$, which is not meaningful as a description of physical gravitation.

\[
g_{ab} = \begin{bmatrix} -r^2 & r^2 \\ r^2 & r^2 \end{bmatrix}_{(J^2)}
\]

\[
\Gamma^a_{\delta a} \cdot \Gamma^\eta_\eta_\eta = 2 \begin{bmatrix} 1/r \\ 1/r \end{bmatrix} \cdot 2 \begin{bmatrix} 1/r - g/r \\ 1/r \end{bmatrix} = 4 \left( \frac{1}{r^2} \right) \left( \frac{1}{r^2} - \frac{g/r}{r} \right), \tag{25}
\]

where now the former, (7), is zero except for the spherical embedding, which fulfils Einstein’s condition, in that the gradient of the functional determinant vanishes. This product term alone also can not suffice to form a field equation.

Inverting both exponents,

\[
g_{ab} = \text{diag} \left[ -f(r)^{-2} \quad f(r)^2 \quad 1/r^2 \quad 1/r^2 \right],
\]

would again give a Newton solution, but with gravitational blueshift instead of the redshift which is observed. So this can also be ruled out.

7 Gravitational Solutions

7.1 An Exponential Non-Solution

In Newton’s gravity, the potential follows $\pm 1/r$, and acceleration $a_r = -1/r^2$ everywhere. Assuming that in a general relativistic spacetime the same law holds true in the subjective view of
local logarithmic space, when integrated along a line of local infinitesimal embeddings, then in a the \textit{Jacobi} matrix an exponential gravitational law shows up,

\[
g_{ab} = \begin{bmatrix}
-k^2 e^{2/r} & k^2 e^{2/r} \\
k^2 e^{2/r} & r^2
\end{bmatrix}
\]

\[
J^\mu_b = \begin{bmatrix}
ke^{1/r} & k e^{1/r} \\
r & r
\end{bmatrix}
\]

\[
J^\mu_{br} = \begin{bmatrix}
-k e^{1/r} & -k e^{1/r} \\
1 & 1
\end{bmatrix}
\]

where \(\Gamma^r_r = 1/r + c\) is the inverse-law potential, and \(\Gamma^r_{rr} = -1/r^2\) is the inverse-square-law acceleration towards the center.

Fortunately, for \(r \to \infty\), the \textit{Jacobi} matrix approaches identity and the metric tensor approaches the \textit{Minowski} metric, to fulfil another condition of \textit{Einstein}'s.

In a flat universe with only one gravitational source, when looking from a gravitational well to the outside, this function suggests to perceive a redshift in the direction \(r \to 0\), and a limited blueshift even at infinite distances in the direction \(r \to \infty\), which corresponds to the finite potential at the observer’s distance from the source.
In an empty flat universe without any gravitational sources the volume element would shrink to a finite neutral value, even though in an empty universe there would not be any object to measure its size or frequency.

This intuitively conforms to the observations in our physical reality. Unfortunately, this is not a solution that shows up in these calculations.

### 7.2 A Newton Potential Solution

The Newton solution of the (+) ansatz suggests, that Newton’s law holds true in the Jacobi matrix, when integrated along a line of local infinitesimal embeddings, but in the subjective view the logarithmic potential is logarithmic,

\[
g_{ab} = \begin{pmatrix}
-\frac{\alpha^2}{r^2} & \frac{\alpha^2}{r^2} \\
\frac{\alpha^2}{r^2} & \frac{r^2}{r^2} \\
\end{pmatrix}^{(J^2)}
\]

\[
J^\mu_b = \begin{pmatrix}
\alpha/r & \alpha/r \\
\alpha/r & r \\
\end{pmatrix}^{(J^1)}
\]

\[
\Gamma^a_{bc} = \exp \left[ -\ln r + c \right] = \Gamma^a_{bc}
\]

\[
J^\mu_{br} = \begin{pmatrix}
-\frac{\alpha^2}{r^2} & -\frac{\alpha^2}{r^2} \\
-\frac{\alpha^2}{r^2} & 1 \\
\end{pmatrix}^{(J^1)}
\]

\[
\Gamma^a_{br} = \begin{pmatrix}
-1/r & -1/r \\
1/r & 1/r \\
\end{pmatrix}^{(J^0)}
\]

where \( c = \ln \alpha \) and \( J^\mu_r = \alpha/r \) is the inverse-law potential, and \( J^\mu_{rr} = -\alpha/r^2 \) is the inverse-square-law acceleration towards the center, so that the logarithmic potential is

\[
\Gamma^t_t = \Gamma^r_r = -\ln \left( \frac{\alpha}{r} \right)
\]

Unfortunately, this way the Jacobi matrix is not forced to approach identity at infinity \( r \to \infty \).

But incidentally, in the local view the Jacobi matrix is always at identity and the metric tensor is Minkowski-flat, so that locally this condition of Einstein’s is already fulfilled.

What that condition reduces to is that merely the derivatives, \( J^\mu_{bc} \) and \( \Gamma^a_{bc} \), approach zero at infinity, which is fulfilled more easily.

Unfortunately, the solution suggests to perceive an unlimited blueshift at infinite distances in the direction \( r \to \infty \), when looking from a gravitational well to the outside. This is not observed in physical reality.
In an empty flat universe without any gravitational sources the volume element would in the limit even shrink to zero, which is unacceptable. But fortunately, our actual universe is not empty and also a cosmic (Hubble) redshift of whatever origin is observed, which would overcome the blueshift at large enough distances, so this solution is still considerable and possibly leads to testable predictions.

7.3 A Second-Order Corollary

The logarithmic potential has already been forced to be zero, $\Gamma^a_{b} \rightarrow 0$, so that only its derivatives still contain information. Now the first derivatives, that is Christoffel 2nd, are also forced to vanish, $\Gamma^a_{bc} \rightarrow 0$, with only the second derivatives remaining, $\Gamma^a_{bc,d}$.

This still allows the even fully contracted divergence of Christoffel 2nd from the (+) ansatz to be a nonzero constant,

$$\frac{g}{1} \Gamma_{\delta \eta \eta}^{\eta \eta} = 2\Lambda.$$ 

In an isotropic spherically symmetric coordinate system with $R := \sqrt{x^2 + y^2 + z^2} \rightarrow 0$,

$$\frac{g}{1} \Gamma_{\delta}^{\delta}(R) = \Lambda R^2$$

might provide a static solution for the observed cosmic (Hubble) redshift, which is commonly associated with a ‘cosmic expansion’. Now the ‘cosmic expansion’ with all its implications is not the only sufficient explanation for cosmic redshift, but a ‘steady-state’ solution of a ‘closed bubble’ universe will also suffice.

7.4 Trace Reversal

In Einstein’s field equation, see (27), the Einstein tensor is taken from the Ricci tensor by the peculiar operation

$$G_{ab} := R_{ab} - \frac{1}{2}R g^{\eta \eta} g_{ab},$$

which can be generalized to a function $\tilde{G}$ that can now conveniently be applied to an arbitrary doubly-covariant matrix $T_{ab}$, (of our scale-invariant matrices of weight $J^0$) and in the infinitesimal limit operates on a flat spacetime, $g_{ab} \rightarrow \eta_{ab}$,

$$\tilde{G} : T_{ab} \rightarrow \tilde{G}(T_{ab}) := T_{ab} - \frac{1}{2}T_{\eta \eta} g^{\eta \eta} g_{ab},$$

that can now conveniently be applied to an arbitrary doubly-covariant matrix $T_{ab}$, (of our scale-invariant matrices of weight $J^0$) and in the infinitesimal limit operates on a flat spacetime, $g_{ab} \rightarrow \eta_{ab}$,

$$\tilde{G} : T_{ab} \rightarrow \tilde{G}(T_{ab}) := T_{ab} - \frac{1}{2}T_{\eta \eta} g^{\eta \eta} g_{ab}. \quad (26)$$

If the trace-reversal function (26) is applied to a matrix, which when equated to zero, gives differential equations with a particular result, as well as its trace does, so the trace-reversal will also give that same result.
For example, the trace reversal of (8),

\[
\eta G(\Gamma^\delta_{\delta a,b}) = \begin{bmatrix}
g_{rr} - \frac{1}{r^2} & g_{rr} - \frac{1}{r^2} \\
\frac{1}{r^2} - g_{rr} & \frac{1}{r^2} - g_{rr}
\end{bmatrix}
\]

(weight \( J^0, g_{rr} = -\frac{1}{r^2}, f = \pm \frac{2}{r} \)),

is Newton, since both (8) and (9) are, or the trace reversal of (11),

\[
\eta G(\Gamma^\delta_{\delta \gamma a,b}) = 2 \begin{bmatrix}
g_{rr}^2 + 2 \frac{2g_{rr}}{r} + \frac{1}{r^2} & g_{rr}^2 - \frac{1}{r^2} \\
\frac{1}{r^2} - g_{rr}
\end{bmatrix}
\]

(weight \( J^0, g_{rr} = -\frac{1}{r^2}, f = \pm \frac{2}{r} \)),

is also Newton, since both (11) and (7) are.

Yet it might be of interest if any of the broken matrices with a ‘false’ or ‘not applicable’ result will give a valid result under this operation. So far there has not been found one.

### 7.5 Einstein’s vacuum equation

The popular Einstein vacuum equation,

\[
R_{ab} = \frac{2}{\eta} G(\Gamma^\delta_{ab,\delta} - \Gamma^\delta_{\delta a,b} + \Gamma^\delta_{\delta \gamma a} \Gamma^\gamma_{ab} - \Gamma^\delta_{a \gamma} \Gamma^\gamma_{b \delta}) = 0,
\]

is composed of terms investigated above and utilizes trace reversal.

**In the \((+)\) ansatz**, the Ricci tensor composes of (10), (8), (11) and (13),

\[
\text{\((+)
\) Ricci scalar:} \quad R_{\eta \eta} = -2 \left( g_{rr} - \frac{2g_{rr}}{r} + \frac{1}{r^2} \right) \overset{!}{=} 0 \quad \text{(accepts } g_{rr} = \frac{1}{3r})
\]

yields a solution \( f = k \sqrt{r} \).
In the \((-\) ansatz\) similar to Schwarzschild’s, the Ricci tensor composes of (19), (17), (22) and (13), and
\[
_{(-)}R_{ab} = \begin{bmatrix} g_{,rr} + \frac{2g_{,r}}{r} - 2g_{,r}^2 & -g_{,rr} - \frac{2g_{,r}}{r} - 2g_{,r}^2 & \frac{1}{r^2} & \frac{1}{r^2} \\ -g_{,rr} - \frac{2g_{,r}}{r} - 2g_{,r}^2 & g_{,rr} + \frac{2g_{,r}}{r} & \frac{1}{r^2} & \frac{1}{r^2} \end{bmatrix} = 0
\]
says ‘not applicable’, and its trace reversal becomes
\[
\eta_{(-)}G(R_{ab}) = \begin{bmatrix} -2g_{,r}^2 - \frac{1}{r^2} & -2g_{,r}^2 + \frac{1}{r^2} \\ -2g_{,r}^2 + \frac{1}{r^2} & g_{,rr} + \frac{2g_{,r}}{r} \end{bmatrix} = 0,
\]
where the second line accepts the Newton solution but the other lines contradict it.
Only the \((-\) Ricci scalar,
\[
_{(-)}R_{\eta\eta}\eta^{\eta} = -2\left(g_{,rr} + \frac{2g_{,r}}{r} + \frac{1}{r^2}\right) = 0 \text{ (accepts } g_{,r} = -\frac{1}{r} \text{)}
\]
yields the Newton solution, \(f = \pm \frac{k}{r}\).

8 Conclusions

8.1 Results

After identifying fields of acceleration and gravity in the metric connection, we experimented with a dual ansatz for a gravitational solution and investigated several separate terms of which a governing field equation could consist. This way we got five results at once:

1. A suspected auxiliary condition for the gravitational ansatz, to be applied before a coordinate embedding, which says ‘the divergence of the Jacobi logarithm shall vanish locally’,
   \[\Gamma^\eta_{\eta\eta}\eta^{\eta} = 0,\]

2. A promising static gravitational ansatz, different from Schwarzschild’s,
   \[g_{ab} = \text{diag}\left[-f(r)^2, f(r)^2, 1/r^2, 1/r^2\right],\]
   where \(\log(f(r))\) is the gravitational potential in the logarithmic Jacobi matrix, which locally always vanishes, \(\Gamma^a_{b\eta} = 0\), but its derivatives do not in general,

3. A corresponding static solution for the gravitational potential, which is proportional to Newton’s gravity over a subjective path, and does not introduce any ‘event horizon’,
   \[f(r) \propto \frac{1}{r}, \quad f(r = 1) = 1,\]
4. A possible static solution for the observed cosmic redshift, without the need to assume a cosmic expansion,

\[ \Gamma^\delta_\delta(r) = \lambda r^2, \quad \Gamma^\delta_\delta(r = 0) = 0, \]

5. A set of candidate terms for possible field equations, which can be linearly combined to equate a matrix to something,

\[ \Gamma^\delta_\delta_{a,b} + \alpha \cdot \Gamma^\delta_\delta \gamma_{ab} \nonumber \]

or a single scalar to a constant,

\[ \Gamma^\delta_\delta_{\eta,\eta} \eta^{\eta} + \alpha \cdot \Gamma^\delta_\delta \gamma_{\eta,\eta} \eta^{\eta} \nonumber = 2\Lambda, \]

where with \( \alpha = 1 \) the fully contracted covariant divergence be constant,

\[ \Gamma^\delta_\delta_{\eta,\eta} \eta^{\eta} + \Gamma^\delta_\delta \gamma_{\eta,\eta} \eta^{\eta} = \Gamma^\delta_\delta \gamma_{\eta,\eta} \eta^{\eta} \nonumber = 2\Lambda. \]

In contrast, Einstein’s vacuum equation does not lead to any viable solution in this context, and the Schwarzschild solution does not show up anywhere in these experiments, instead the presented gravitational solution does not lead to any ‘event horizon’ or ‘black hole’.

8.2 What’s Different Here

- More fundamental than the metric tensor, \( g_{\mu\nu} \), is the Jacobi matrix, \( J^\mu_b \), from which the metric results.
- For an infinitesimal embedding, the logarithm of the Jacobi matrix, the ‘logarithmic potential’ \( \Gamma^a_{b} \), is yet more fundamental.
- Since in an infinitesimal embedding that logarithm vanishes in the instant, \( \Gamma^a_{b} = 0 \), and only its derivatives contain information, thus the Christoffel symbol of the second kind, \( \Gamma^a_{bc} \), is the most fundamental entity.
- Instead of requiring the functional determinant to be at unity everywhere, \( \det \sqrt{-g_{\mu\nu}} = \det J^\mu_b = 1 \), here the Jacobi matrix is locally completely flat, \( J^\mu_b = \delta^\mu_b \), but the speed of light is required to be constant everywhere, \( \Gamma^a_{\eta,\eta} \eta^{\eta} = 0 \).
- In the far limit of the static gravitational solution, \( r \rightarrow \infty \), the Christoffel 2nd vanishes, \( \Gamma^a_{bc} \rightarrow 0 \). This is a more relaxed condition than the metric tensor approaching flatness, \( g_{\mu\nu} \rightarrow \eta^{\mu\nu} \).
- In the static gravitational solution, the gravitational potential as described by the Jacobi matrix, \( J^r_r = \pm k/r \), resembles Newton’s law exactly, not just in the far limit.
8.3 Outlook

It shall be fruitful to search for more, possibly non-gravitational, phenomena by more ‘doing physical experiments on the math’ and, comparing the results with real-world experimental evidence, to further constrain the governing equation on a Riemann–Minkowski spacetime and possibly approach a first-principle description of physical reality.

References


Contact

carsten (dot) spanheimer (at) student (dot) uni-tuebingen (dot) de