The Gravitational Mass of the Millisecond Pulsars

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In this work it is theoretically shown that a millisecond pulsar spinning with angular velocity close to 1000 rotations per second (or more) has its gravitational mass reduced below its inertial mass, i.e., under these circumstances, the gravitational and the inertial masses of the millisecond pulsar are not equivalents. This can easily be experimentally checked, and it would seem to be an ideal test to the equivalence principle of general relativity.

Key words: Gravity, Gravitation, Equivalence Principle, Pulsars, Millisecond Pulsars.

1. Introduction

Millisecond pulsars are neutron stars with *radius* in the range of $9.5-14km$ [1] and rotational period in the range of milliseconds. Thus, they rotate hundreds of times per second. They are the product of an extended period of mass and angular momentum transfer to a neutron star from an evolving companion star [2, 3, 4, 5, 6, 7, 8]. Millisecond pulsars are the fastest spinning stars in the Universe. The fastest known millisecond pulsar rotates *716 times per second* [9]. Current theories of neutron star structure and evolution predict that pulsars would break apart if they reach about of ~1500 rotations per second [10, 11] and that at 1000 rotations per second they would lose energy by gravitational radiation faster than the accretion process would speed them up [12]. However, in 2007 it was discovered a neutron star XTE J1739-285 rotating at 1122 times per second.

We show in this paper that a millisecond pulsar spinning with angular velocity close to 1000 rotations per second (or more) has its gravitational mass significantly reduced below its inertial mass, showing therefore, that the gravitational mass is not equivalent to the inertial mass as claims the equivalence principle of general relativity .

2. Theory

The physical property of mass has two distinct aspects, *gravitational mass* m_g and *inertial mass* m_i. The gravitational mass produces and responds to gravitational fields. It supplies the mass factors in Newton's famous inverse-square law of gravity ($F=GM_g m_g /r^2$). The inertial mass is the mass factor in Newton's 2nd Law of Motion ($F=m_i a$).

Einstein's *Equivalence Principle* asserts that a experiment performed in a uniformly accelerating reference frame with acceleration a are undistinguishable from the same experiment performed in a non-accelerating reference frame in a gravitational field where the acceleration of gravity is $g = -a$. One way of stating this fundamental principle of general relativity theory is to say that gravitational mass is equivalent to inertial mass.

However, the quantization of gravity shows that that the *gravitational mass* m_g and *inertial mass* m_i are correlated by means of the following factor [13]:

$$\chi = \frac{m_g}{m_{i0}} = \left\{1 - 2\left[\sqrt{1+\left(\frac{\Delta p}{m_{i0}c}\right)^2} - 1\right]\right\} \qquad (1)$$

where m_{i0} is the *rest* inertial mass of the particle and Δp is the variation in the particle's *kinetic momentum*; c is the speed of light.

Equation (1) shows that *only* for $\Delta p = 0$ *the gravitational mass is equal to the inertial mass.*

In general, the *momentum* variation Δp is expressed by $\Delta p = F\Delta t$ where F is the applied force during a time interval Δt. Note that there is no restriction concerning the *nature* of the force F, i.e., it can be mechanical, electromagnetic, etc.

For example, we can look on the *momentum* variation *Δp* as due to absorption or emission of *electromagnetic energy*. In this case, we can write that

$$\Delta p = n\hbar k_r = n\hbar\omega/(\omega/k_r) = \Delta E/(dz/dt) =$$

$$= \Delta E/v = \Delta E/v(c/c) = \Delta E n_r/c \qquad (2)$$

where k_r is the real part of the *propagation vector* $\vec{k}$; $k = |\vec{k}| = k_r + ik_i$; ΔE is *the electromagnetic energy absorbed or emitted by the particle;* n_r is the index of refraction of the medium and v is the *phase* velocity of the electromagnetic waves, given by:

$$v = \frac{dz}{dt} = \frac{\omega}{\kappa_r} = \frac{c}{\sqrt{\frac{\varepsilon_r\mu_r}{2}\left(\sqrt{1+(\sigma/\omega\varepsilon)^2}+1\right)}} \qquad (3)$$

ε , μ and σ, are the electromagnetic characteristics of the particle ($\varepsilon = \varepsilon_r\varepsilon_0$ where ε_r is the *relative electric permittivity* and $\varepsilon_0 = 8.854\times10^{-12}F/m$; $\mu = \mu_r\mu_0$ where μ_r is the *relative magnetic permeability* and $\mu_0 = 4\pi\times10^{-7}H/m$).

Thus, substitution of Eq. (2) into Eq. (1), gives

$$\chi = \frac{m_g}{m_{i0}} = \left\{1-2\left[\sqrt{1+\left(\frac{\Delta E}{m_{i0}c^2}n_r\right)^2}-1\right]\right\} \qquad (4)$$

If the particle is *also* rotating, with an angular speed ω around its central axis, then it acquires an additional energy equal to its rotational energy $\left(E_k = \tfrac{1}{2}I\omega^2\right)$. Since this is an increase in the internal energy of the particle, and this energy is basically electromagnetic, we can assume that E_k, such as ΔE, corresponds to an amount of electromagnetic energy absorbed (or emitted) by the particle. Thus, we can consider E_k as an increase $\Delta U = E_k$ in the electromagnetic energy ΔE absorbed (or emitted) by the particle. Consequently, in this case, we must replace ΔE in Eq. (4) for $(\Delta E + \Delta U)$. *In the case of a millisecond pulsar,* we can assume $\Delta E << \Delta U$. Thus, Eq. (4) reduces to

$$m_g \cong \left\{1-2\left[\sqrt{1+\left(\frac{I\omega^2 n_r}{2m_{i0}c^2}\right)^2}-1\right]\right\}m_{i0} \qquad (5)$$

where I is the moment of inertia of the pulsar in respect to its rotation axis; n_r is the index of refraction of the pulsar $(n_r \cong 1)$; m_{i0} is the *rest* inertial mass of the pulsar and c is the speed of light.

Since a pulsar is a rigid sphere then we can assume $I = \frac{2}{5}m_{i0}R^2$, where R is the pulsar radius. In this case, Eq. (5) can be rewritten as follows

$$m_g \cong \left\{1-2\left[\sqrt{1+\left(\frac{R\omega}{\sqrt{5}c}\right)^4}-1\right]\right\}m_{i0} \qquad (6)$$

In the case of millisecond pulsars, we can take $R \cong 10km$ (There are various models predicting radii on the order of 10 km[14]). Therefore, if the pulsar is spinning with angular velocity close to 1000 rotations per second $(\omega \cong 6{,}300\,rad/s)$ then Eq. (6) shows a decreasing of about 0.01% in the *gravitational mass* of the millisecond pulsar, in respect to the inertial mass of the pulsar.

However, the shortest possible period T of a pulsar can be estimated starting from the assumption that the speed v at the pulsar's surface cannot exceed the speed of light

$$v = c = \frac{2\pi R}{T} \quad \Rightarrow \quad T = \frac{2\pi R}{c} \qquad (7)$$

For a pulsar of period $T = 0.001s$ the radius is $R = cT/2\pi = 47km$.

Equation (6) shows that millisecond pulsars with radius of about 30km, spinning with angular velocity close to 1000 rotations per second, have their gravitational masses decreased of about 1% in respect to the inertial mass of the pulsar.

This should provide an interesting new test for equivalence principle of general relativity. As show the article "*Rare Celestial Trio to Put Einstein's Theory to the Test*", published in *Science* [15], recent astrophysical discoveries about millisecond pulsars are also pointing to this possibility.

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