Proving Goldbach's Conjecture by Two Number Axes' Positive Half Lines which Reverse from Each Other's Directions

Zhang Tianshu

Nanhai west oil corporation,China offshore Petroleum, Zhanjiang city, Guangdong province, P.R.China

Email: tianshu_zhang507@aliyun.com;

Abstract

We know that every positive even number 2n(n≥3) can express in a sum which 3 plus an odd number 2k+1(k≥1) makes. And then, for any odd point 2k+1 (k≥1)at the number axis, if 2k+1 is an odd prime point, of course even number 3+(2k+1) is equal to the sum which odd prime number 2k+1 plus odd prime number 3 makes; If 2k+1 is an odd composite point, then let 3<B<2k+1, where B is an odd prime point, and enable line segment B(2k+1) to equal line segment 3C. If C is an odd prime point, then even number 3+(2k+1) is equal to the sum which odd prime number B plus odd prime number C makes. So the proof for Goldbach's Conjecture is converted to prove there be certainly such an odd prime point B at the number axis's a line segment which take odd point 3 and odd point 2k+1 as ends, so as to prove the conjecture by such a method indirectly.

Keywords

Number theory, Goldbach's Conjecture, Even number, Odd prime number, Mathematical induction, Two number axes' positive half lines which reverse from each other's direction, OD, PL, CL, and RPL.

Basic Concepts

Goldbach's conjecture states that every even number 2N is a sum of two prime numbers, and every odd number 2N+3 is a sum of three prime numbers, where N≥2.

We shall prove the Goldbach's conjecture thereinafter by odd points at two number axes' positive half lines which reverse from each other's directions and which begin with odd point 3.

First we must understand listed below basic concepts before the proof of the conjecture, in order to apply them in the proof.

Axiom . Each and every even number 2n (n≥3) can express in a sum which 3 plus each odd number 2k+1 (k≥1) makes.

Definition 1 . A line segment which takes two odd points as two ends at the number axis's positive half line which begins with odd point 3 is called an odd distance. "OD" is abbreviated from "odd distance".

The OD between odd point N and odd point N+2t is written as OD N(N+2t), where N≥3, and t≥1.

A integer which the length of OD between two consecutive odd points expresses is 2.

A length of OD between odd point 3 and each odd point is unique.

Definition 2 . An OD between odd point 3 and each odd prime point at the number axis's positive half line which begins with odd point 3, otherwise called a prime length. "PL" is abbreviated from "prime length", and "PLS" denotes the plural of PL.

An integer which each length of from small to large PL expresses be successively 2, 4, 8, 10, 14, 16, 20, 26. . .

Definition 3 . An OD between odd point 3 and each odd composite point at the number axis's positive half line which begins with odd point 3, otherwise called a composite length. "CL" is abbreviated from "composite length".

An integer which each length of from small to large CL expresses be successively 6, 12, 18, 22, 24, 30, 32. . .

We know that positive integers and positive integers' points at the number axis's positive half line are one-to-one correspondence, namely each integer's point at the number axis's positive half line represents only a positive integer. The value of a positive integer expresses the length of the line segment between point 0 and the positive integer's point here. When the line segment is longer, it can express in a sum of some shorter line segments; correspondingly the positive integer can also express in a sum of some smaller integers.

Since each and every line segment between two consecutive integer's points and the line segment between point 0 and point 1 have an identical length, hence when use the length as a unit to measure a line segment between two integer's points or between point 0 and any integer's point, the line segment has some such unit length, then the integer which the line segment expresses is exactly some.

Since the proof for the conjecture relate merely to positive integers which are not less than 3, hence we take only the number axis's positive half line which begins with odd point 3. However we stipulate that an integer which each integer's point represents expresses yet the length of the line segment between the integer's point and point 0. For example, an odd prime value which the right end's point of any PL represents expresses yet the length of the line segment between the odd prime point and point 0 really.

We can prove next three theorems easier according to above-mentioned some relations among line segments, integers' points and integers.

Theorem 1 . If the OD which takes odd point F and odd prime point P_S as two ends is equal to a PL, then even number 3+F can express in a sum of two odd prime numbers, where $F>P_S$.

Proof . Odd prime point P_S represents odd prime number P_S, it expresses the length of the line segment from odd prime point P_S to point 0.

Though lack the line segment from odd point 3 to point 0 at the number axis's positive half line which begins with odd point 3, but odd prime point P_S represents yet odd prime P_S according to above-mentioned stipulation;

Let OD P_SF=PL $3P_b$, odd prime point P_b represents odd prime number P_b, it expresses the length of the line segment from odd prime point P_b to point 0.

Since PL $3P_b$ lack the segment from odd point 3 to point 0, therefore the integer which the length of PL $3P_b$ expresses is even number P_b-3, namely the integer which the length of OD P_SF expresses is even number P_b-3.

Consequently there is $F=P_S+(P_b-3)$, i.e. 3+F=odd prime P_S + odd prime P_b .

Theorem 2 . If even number 3+F can express in a sun of two odd prime numbers, then the OD which takes odd point 3 and odd point F as ends can express in a sun of two PLS, where F is an odd number which is more than 3.

Proof . Suppose the two odd prime numbers are P_b and P_d, then there be $3+F= P_b+P_d$.

It is obvious that there be OD $3F$=PL $3P_b$ + OD P_bF at the number axis's positive half line which begins with odd point 3.

Odd prime point P_b represents odd prime number P_b according to above-mentioned stipulation, then the length of line segment $P_b(3+F)$ is precisely P_d, nevertheless P_d expresses also the length of the line segment from odd prime point P_d to point 0. Thereupon cut down 3 unit lengths of line segment $P_b(3+F)$, we obtain OD P_bF; again cut down 3 unit lengths of the line segment from odd prime point P_d to point 0, we obtain PL $3P_d$, then there be OD P_bF=PL $3P_d$.

Consequently there be OD $3F$=PL $3P_b$ + PL $3P_d$.

Theorem 3 . If the OD between odd point F and odd point 3 can express in a sum of two PLS, then even number 3+F can express in a sum of two odd prime numbers, where F is an odd number which is more than 3.

Proof . Suppose one of the two PLS is PL $3P_S$, then there be $F > P_S$, and the OD between odd point F and odd prime point P_S is another PL. Consequently even number 3+F can express in a sum of two odd prime numbers according to theorem 1.

The Proof

First let us give ordinal number K to from small to large each and every odd number $2k+1$, where $k\geq 1$, then from small to large each and every even number which is not less than 6 is equal to 3+(2k+1).

We shall prove this conjecture by the mathematical induction thereinafter.

1 . When k=1, 2, 3 and 4, we getting even number be orderly 3+(2*1+1)=6=3+3, 3+(2*2+1)=8=3+5, 3+(2*3+1)=10=3+7 and 3+(2*4+1)=12=5+7. This shows that each of them can express in a sum of two odd prime numbers.

2 . Suppose k=m, the even number which 3 plus №m odd number makes, i.e. 3+(2m+1) can express in a sum of two odd prime numbers, where m≥4.

3 . Prove that when k=m+1, the even number which 3 plus №(m+1) odd number makes, i.e. 3+(2m+3) can also express in a sum of two odd prime numbers.

Proof . In case 2m+3 is an odd prime number, naturally even number 3+（2m+3）is the sum of odd prime number 3 plus odd prime number 2m+3 makes.

When 2m+3 is an odd composite number, suppose that the greatest odd prime number which is less than 2m+3 is P_m, then the OD between odd prime point P_m and odd composite point 2m+3 is either a PL or a CL.

When the OD between odd prime point P_m and odd composite point 2m+3 is a PL, the even number 3+（2m+3）can express in a sum of two odd prime numbers according to theorem 1.

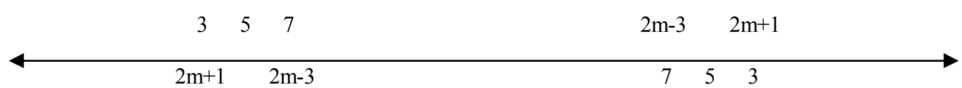
If the OD between odd prime point P_m and odd composite point 2m+3 is a

CL, then we need to prove that OD 3（2m+3）can express in a sum of two PLS, on purpose to use the theorem 3.

When OD $P_m(2m+3)$ is a CL, from small to large odd composite number 2m+3 be successively 95, 119, 125, 145. . .

First let us adopt two number axes' positive half lines which reverse from each other's directions and which begin with odd point 3.

At first, enable end point 3 of either half line to coincide with odd point 2m+1 of another half line. Please, see first illustration:



First Illustration

Such a coincident line segment can shorten or elongate, namely end point 3 of either half line can coincide with any odd point of another half line.

This proof will perform at some such coincident line segments. And for certain of odd points at such a coincident line segment, we use usually names which mark at the rightward direction's half line.

We call PLS which belong both in the leftward direction's half line and in a coincident line segment "reverse PLS". "RPLS" is abbreviated from "reverse PLS", and "RPL" denotes the singular of RPLS.

The RPLS whereby odd point 2k+1 at the rightward direction's half line acts as the common right endmost point are written as $RPLS_{2k+1}$, and RPL_{2k+1} denotes the singular, where k>1.

This is known that each and every OD at a line segment which takes odd point 2m+1 and odd point 3 as two ends can express in a sum of a PL and a

RPL according to preceding theorem 2 and the supposition of №2 step of the mathematical induction.

We consider a PL and the RPL_{2k+1} wherewith to express together the length of OD 3(2k+1) as a pair of PLS, where k >1. One of the pair's PLS is a PL which takes odd point 3 as the left endmost point, and another is a RPL_{2k+1} which takes odd point 2k+1 as the right endmost point. We consider the RPL_{2k+1} and another RPL_{2k+1} which equals the PL as twin $RPLS_{2k+1}$.
For a pair of PLS, the PL is either unequal or equal to the RPL_{2k+1}. If the PL is unequal to the RPL_{2k+1}, then longer one is more than a half of OD 3(2k+1), yet another is less than the half. If the PL is equal to the RPL_{2k+1}, then either is equal to the half. A pair of PLS has a common end's point.

Since each of $RPLS_{2k-1}$ is equal to a RPL_{2k+1}, and their both left endmost points are consecutive odd points, and their both right endmost points are consecutive odd points too. So seriatim leftwards move $RPLS_{2k+1}$ to become $RPLS_{2k-y}$, then part left endmost points of $RPLS_{2k+1}$ plus $RPLS_{2k-y}$ coincide monogamously with part odd prime points at OD 3(2k+1), where y=1, 3, 5, ...

Thus let us begin with odd point 2m+1, leftward take seriatim each odd point 2m-y+2 as a common right endmost point of $RPLS_{2m-y+2}$, where y= 1, 3, 5, 7 ... ỹ ...
Suppose that y increases orderly to odd number ỹ, and ∑ part left endmost points of $RPLS_{2m-y+2}$ ($1 \le y \le \tilde{y}+2$) coincide just right with all odd prime points

at OD 3(2m+1) monogamously, then there are altogether (ỹ+3)/2 odd points at OD (2m-ỹ)(2m+1), and let μ=(ỹ+3)/2.

Let us separate seriatim OD 3(2m-y+2) (y=1, 3, 5,…) from each coincident line segment of two such half lines, and arrange them from top to bottom orderly. After that, put an odd prime number which each odd prime point at the rightward direction's half line expresses to on the odd prime point, and put another odd prime number which each left endmost point of $RPLS_{2m-y+2}$ at the leftward direction's half line expresses to beneath the odd prime point.

For example, when 2m+3=95, 2m+1=93, 2m-1=91 and 2m-ỹ =89 , μ=3.
For the distributer of odd prime points which coincide monogamously with left endmost points of $RPLS_{95}$, $RPLS_{93}$, $RPLS_{91}$, $RPLS_{89}$ and $RPLS_{87}$, please see second illustration:

OD 3(95)						19			31	37						61	67			79		
						79			67	61						37	31			19		
OD 3(93)		7		13	17		23	29		37		43		53	59		67		73	79	83	89
		89		83	79		73	67		59		53		43	37		29		23	17	13	7
OD 3(91)	5		11				23				41		47	53				71			83	89
	89		83				71				53		47	41				23			11	5
OD 3(89)				13		19			31							61			73	79		
				79		73			61							31			19	13		
OD 3(87)		7	11		17	19	23	29	31	37		43	47	53	59	61	67	71	73	79	83	
		83	79		73	71	67	61	59	53		47	43	37	31	29	23	19	17	11	7	

Second Illustration

Two left endmost points of twin $RPLS_{2m-y+2}$ at OD 3(2m-y+2) coincide monogamously with two odd prime points, they assume always bilateral symmetry whereby the centric point of OD 3(2m-y+2) acts as symmetric centric. If the centric point is an odd prime point, then it is both the left endmost point of RPL_{2m-y+2} and the odd prime point which coincides with the left endmost point, e.g. centric point 47 of OD 3(91) in above-cited that example.

We consider each odd prime point which coincides with a left endmost point of $RPLS_{2m-\tilde{y}}$ alone as a characteristic odd prime point, at OD 3(2m+1).
Thus it can seen, there is at least one characteristic odd prime point at OD 3(2m-ỹ) according to aforesaid the way of making things, e.g. odd prime points 19, 31 and 61 at OD 3(89) in above-cited that example.
Whereas there is not any such characteristic odd prime point in odd prime points which coincide monogamously with left endmost points of $RPLS_{2m+1}$ plus $RPLS_{2m-1}$ ... plus $RPLS_{2m-\tilde{y}+2.}$
In other words, every characteristic odd prime point is not any left endmost point of $RPLS_{2m+1}$ plus $RPLS_{2m-1}$ ... plus $RPLS_{2k-\tilde{y}+2}$.

Moreover left endmost points of $RPLS_{2m-y}$ are №1 odd points on the lefts of left endmost points of $RPLS_{2m-y+2}$ monogamously, where y is an odd number≥1.

Consequently, №1 odd point on the left of each and every characteristic odd prime point isn't any left endmost point of $RPLS_{2m-1}$ plus $RPLS_{2m-3}$ …plus $RPLS_{2m-\tilde{y}}$. — {1}

Since each RPL_{2m-y+2} is equal to a PL at OD 3(2m-y+2).
In addition, odd prime points which coincide monogamously with left

endmost points of $RPLS_{2m+1}$ plus $RPLS_{2m-1}$ ... plus $RPLS_{2m-\tilde{y}}$ are all odd prime points at OD 3(2m+1).

Hence considering length, at OD 3(2m+1) RPLS whose left endmost points coincide monogamously with all odd prime points are all RPLS at OD 3(2m+1), irrespective of the frequency of RPLS on identical length.

Evidently the longest RPL at OD 3(2m+1) is equal to PL $3P_m$.

When OD $P_m(2m+3)$ is a CL, let us review aforesaid the way of making thing once again, namely begin with odd point 2m+1, leftward take seriatim each odd point 2m-y+2 as a common right endmost point of $RPLS_{2m-y+2}$, and $\sum$ part left endmost points of $RPLS_{2m-y+2}$ $(1 \le y \le \tilde{y}+2)$ coincide just right with all odd prime points at OD 3(2m+1) monogamously.

Which one of left endmost points of $RPLS_{2m-y+2}$ $(1 \le y \le \tilde{y}+2)$ coincides first with odd prime point 3? Naturally it can only be the left endmost point of the longest RPL_{P_m} whereby odd prime point P_m acts as the right endmost point.

Besides all coincidences for odd prime points at OD 3(2m+1) begin with left endmost points of $RPLS_{2m+1}$, whereas left endmost points of $RPLS_{2m-\tilde{y}}$ are final one series in the event that all odd prime points at OD 3(2m+1) are coincided just right by left endmost points of RPLS.

Therefore odd point 2m-ỹ as the common right endmost point of $RPLS_{2m-\tilde{y}}$ cannot lie on the right of odd prime point P_m, then odd point 2m-ỹ-2 can only lie on the left of odd prime point P_m. This shows that every $RPL_{2m-\tilde{y}-2}$ at OD 3(2m-ỹ-2) is shorter than PL 3 P_m.

In addition, №1 odd point on the left of a left endmost point of each and every $RPL_{2k-\tilde{y}}$ is a left endmost point of $RPLS_{2k-\tilde{y}-2.}$

Therefore each and every $RPL_{2m-\tilde{y}-2}$ can extend contrary into at least one $RPL_{2m-y+2,}$ where y is a positive odd number $\leq\tilde{y}+2$.

That is to say, every left endmost point of $RPLS_{2m-\tilde{y}-2}$ is surely at least one left endmost point of $RPLS_{2k-y+2}$.

Since left endmost points of $RPLS_{2m-\tilde{y}-2}$ lie monogamously at №1 odd point on the left of left endmost points of $RPLS_{2m-\tilde{y}}$ including characteristic odd prime points.

Consequently, №1 odd point on the left of each and every characteristic odd prime point is surely a left endmost point of $RPLS_{2m-y+2}$, where $1\leq y\leq\tilde{y}+2$.—— {2}

So we draw inevitably such a conclusion that №1 odd point on the left of each and every characteristic odd prime point can only be a left endmost point of $RPLS_{2m+1}$ under these qualifications which satisfy both above-reached conclusion {1} and above-reached conclusion {2}.

Such being the case, let us rightwards move a RPL_{2m+1} whose left endmost point lies at №1 odd point on the left of any characteristic odd prime point to adjacent odd points, then the RPL_{2m+1} is moved into a $RPL_{2m+3.}$

Evidently the left endmost point of the RPL_{2m+3} is the characteristic odd prime point, and its right endmost point is odd point 2m+3.

So OD 3(2m+3) can express in a sum of two PLS, and the common endmost point of the two PLS is exactly the characteristic odd prime point.

Thus far we have proven that even if OD $P_m(2m+3)$ is a CL, likewise OD 3(2m+3) can also express in a sum of two PLS.

Consequently even number which 3 plus №(m+1) odd number makes, i.e. 3+(2m+3) can also express in a sum of two odd prime numbers according to aforementioned theorem 3.

Proceed from a proven conclusion to prove a larger even number for each once, then via infinite many an once, namely let k to equal each and every natural number, we reach exactly a conclusion that every even number 3+(2k+1) can express in a sum of two odd prime numbers, where $k \geq 1$.
To wit every even number 2N can express in a sum of two odd prime numbers, where $N > 2$.

In addition let N =2, get 2N=4=even prime number 2+even prime number 2. Consequently every even number 2N can express in a sum of two prime numbers, where $N \geq 2$.

Since every odd number 2N+3 can express in a sum which a prime number plus the even number makes, consequently every odd number 2N+3 can express in a sum of three prime numbers, where $N \geq 2$.

To sum up, we have proven that two propositions of the Goldbach's conjecture are tenable, thus Goldbach's conjecture holds water.