Two conjectures which generalize the conjecture on the infinity of Sophie Germain primes

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Abstract. In a previous paper ("Five conjectures on Sophie Germain primes and Smarandache function and the notion of Smarandache-Germain primes") I defined two notions: the Smarandache-Germain pairs of primes and the Coman-Germain primes of the first and second degree. The few conjectures that I made on these particular types of primes inspired me to make two other conjectures regarding two sets of primes that are generalizations of the set of Sophie Germain primes. And, based on the observation of the first few primes from these two possible infinite sets of primes, I also made a conjecture regarding the primes q of the form q = p*2^n + 31 = r*2^m + 3, where p, r are primes an m, n are non-null positive integers.

Conjecture 1:

There exist an infinity of primes q of the form q = p_1*p_2*...*p_m*2^n + 1, where p_1, p_2, ..., p_m are odd distinct primes, for any n non-null natural integer respectively for any m non-null natural integer. We call this type of primes *Coman-Germain primes of the first kind*.

Note:

For [n, m] = [1, 1] the conjecture is the same with the conjecture on the infinity of Sophie Germain primes, *i.e.* the primes of the form q = 2*p + 1.

The first three primes of this form for few values of [n, m]:

1. For [n, m] = [2, 1] the primes q are of the form 4*p + 1; the sequence of these primes is: 13, 29, 43, ...;
2. For [n, m] = [3, 1] the primes q are of the form 8*p + 1; the sequence of these primes is: 41, 89, 137, ...;
3. For [n, m] = [1, 2] the primes q are of the form 2*p_1*p_2 + 1; the sequence of these primes is: 31, 43, 67, ...;
4. For [n, m] = [2, 2] the primes q are of the form 4*p_1*p_2 + 1; the sequence of these primes is: 61, 157, 229, ...;
5. For [n, m] = [3, 2] the primes q are of the form 8*p_1*p_2 + 1; the sequence of these primes is: 281, 409, 457, ...;
6. For [n, m] = [1, 3] the primes q are of the form 2*p_1*p_2*p_3 + 1; the sequence of these primes is: 211, 331, 571 (...).

Conjecture 2:

There exist an infinity of primes r of the form $r = 2*(p_1*p_2*...*p_m*2^n + 1) + 1$, where p_1, p_2, ..., p_m are odd distinct primes, for any n non-null natural integer respectively for any m non-null natural integer. We call this type of primes *Coman-Germain primes of the second kind*.

The first three primes of this form for few values of [n, m]:

1. For [n, m] = [1, 1] the primes q are of the form 2*(2*p + 1) + 1 = 4*p + 3;
   the sequence of these primes is: 23, 31, 47, ...;
2. For [n, m] = [2, 1] the primes q are of the form 2*(4*p + 1) + 1 = 8*p + 3;
   the sequence of these primes is: 43, 59, 107, ...;
3. For [n, m] = [3, 1] the primes q are of the form 2*(8*p + 1) + 1 = 16*p + 3;
   the sequence of these primes is: 83, 179, 211, ...;
4. For [n, m] = [1, 2] the primes q are of the form $2*(2*p_1*p_2 + 1) + 1 = 4*p_1*p_2 + 3$;
   the sequence of these primes is: 223, 263, 311, ...;
5. For [n, m] = [2, 2] the primes q are of the form $2*(4*p_1*p_2 + 1) + 1 = 8*p_1*p_2 + 3$;
   the sequence of these primes is: 283, 443, 523 (...).

Conjecture 3:

There exist an infinity of primes q of the form q = p*2^n + 31 that can be also written as q = r*2^m + 3, where n, m are non-null positive integers and p, r odd primes.

The first three primes of this form for few values of [n, m]:

1. For [n, m] = [1, 1] we have q = 2*p + 31 = 2*r + 3:
   : q = 37 = 2*3 + 31 = 2*17 + 3, so [p, r] = [3, 17];
   : q = 41 = 2*5 + 31 = 2*19 + 3, so [p, r] = [5, 19];
   : q = 89 = 2*29 + 31 = 2*43 + 3, so [p, r] = [29, 43].
2. For [n, m] = [2, 3] we have q = 4*p + 31 = 8*r + 3:
   : q = 43 = 4*3 + 31 = 8*5 + 3, so [p, r] = [3, 5];
   : q = 59 = 4*7 + 31 = 8*7 + 3, so [p, r] = [7, 7];
   : q = 107 = 4*19 + 31 = 8*13 + 3, so [p, r] = [19, 13].
3. For [n, m] = [2, 4] we have q = 4*p + 31 = 16*r + 3:
   : q = 83 = 4*13 + 31 = 16*5 + 3, so [p, r] = [13, 5];
   : q = 179 = 4*37 + 31 = 16*11 + 3, so [p, r] = [37, 11];
   : q = 467 = 4*109 + 31 = 16*29 + 3, so [p, r] = [109, 29].

Annex

The two conjectures from my previous paper mentioned in Abstract where I defined the *Smarandache-Germain pairs of primes* and the *Coman-Germain primes of the first and second degree*:

Conjecture 1:

For any pair of Sophie Germain primes $[p_1, p_2]$ with the property that $S(p_1 - 1)$ is prime, where S is the Smarandache function, we have a corresponding pair of primes $[S(p_1 - 1), S(p_2 - 1)]$, which we named it Smarandache-Germain pair of primes, with the property that between the primes $q_1 = S(p_1 - 1)$ and $q_2 = S(p_2 - 1)$ there exist the following relation: $q_2 = n*q_1 + 1$, where n is non-null positive integer.

Conjecture 2:

For any p Sophie Germain prime with the property that S(p – 1) is prime, where S is the Smarandache function, one of the following two statements is true:

1. there exist m non-null positive integer such that (p – 1)/(2^m) = q, where q is prime, q ≥ 5;
2. there exist n prime and m non-null positive integer such that (p – 1)/(n*2^m) = q, where q is prime, q ≥ 5.

Note: we call the primes q from the first statement *Coman-Germain primes of the first degree*; we call the primes q from the second statement *Coman-Germain primes of the second degree*.