Five conjectures on Sophie Germain primes and Smarandache function and the notion of Smarandache-Germain primes

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Abstract. In this paper I define a new type of pairs of primes, id est the Smarandache-Germain pairs of primes, notion related to Sophie Germain primes and also to Smarandache function, and I conjecture that for all pairs of Sophie Germain primes but a definable set of them there exist corespondent pairs of Smarandache-Germain primes. I also make a conjecture that attributes to the set of Sophie Germain primes but a definable subset of them a corespondent set of smaller primes, id est Coman-Germain primes.

Conjecture 1:

For any pair of Sophie Germain primes $[p_1, p_2]$ with the property that $S(p_1 - 1)$ is prime, where S is the Smarandache function, we have a corresponding pair of primes $[S(p_1 - 1), S(p_2 - 1)]$, which we named it Smarandache-Germain pair of primes, with the property that between the primes $q_1 = S(p_1 - 1)$ and $q_2 = S(p_2 - 1)$ there exist the following relation: $q_2 = n*q_1 + 1$, where n is non-null positive integer.

Note:

For a list of Sophie Germain primes see the sequence A005384 in OEIS. For the values of Smarandache function see the sequence A002034 in OEIS.

Verifying the Conjecture 1:
(for the first 26 pairs of Sophie Germain primes)

: For [2, 5] we have S(2 – 1) = 1, not prime;
: For [3, 7] we have S(3 – 1) = 2, not odd prime;
: For [5, 11] we have S(5 – 1) = 4, not prime;
: For [11, 23] we have [S(10), S(22)] = [5, 11]
and 5*2 + 1 = 11;
: For [23, 47] we have [S(22), S(46)] = [11, 23]
and 11*2 + 1 = 23;
: For [29, 59] we have [S(28), S(58)] = [7, 29]
and 7*4 + 1 = 29;
: For [41, 83] we have [S(40), S(82)] = [5, 41]
and 5*8 + 1 = 11;
: For [53, 107] we have [S(52), S(106)] = [13, 53]
and 13*4 + 1 = 53;
: For [83, 167] we have [S(82), S(166)] = [41, 83]
and 41*2 + 1 = 83;

: For [89, 179] we have [S(88), S(178)] = [11, 89]
    and 11*8 + 1 = 89;
: For [113, 227] we have [S(112), S(226)] = [7, 113]
    and 7*16 + 1 = 113;
: For [131, 263] we have [S(130), S(262)] = [13, 131]
    and 13*10 + 1 = 131;
: For [173, 347] we have [S(172), S(346)] = [43, 173]
    and 43*4 + 1 = 173;
: For [179, 359] we have [S(178), S(358)] = [89, 179]
    and 89*2 + 1 = 179;
: For [191, 383] we have [S(190), S(382)] = [19, 191]
    and 19*10 + 1 = 191;
: For [233, 467] we have [S(232), S(466)] = [29, 233]
    and 29*8 + 1 = 233;
: For [239, 479] we have [S(238), S(478)] = [17, 239]
    and 17*14 + 1 = 239;
: For [251, 503] we have S(250 – 1) = 15, not prime;
: For [281, 563] we have [S(280), S(562)] = [7, 281]
    and 7*40 + 1 = 281;
: For [293, 587] we have [S(292), S(586)] = [73, 293]
    and 73*4 + 1 = 293;
: For [359, 719] we have [S(358), S(718)] = [179, 359]
    and 179*2 + 1 = 359;
: For [419, 839] we have [S(418), S(838)] = [19, 419]
    and 19*22 + 1 = 419;
: For [431, 863] we have [S(430), S(862)] = [43, 431]
    and 43*10 + 1 = 431;
: For [443, 887] we have [S(442), S(886)] = [17, 443]
    and 17*26 + 1 = 443;
: For [491, 983] we have S(491 – 1) = 14, not prime;
: For [509, 1019] we have [S(508), S(1018)] = [127, 509]
    and 127*4 + 1 = 509.

Conjecture 2:

There exist an infinity of Smarandache-Germain pairs of primes.

Note:

It can be seen that $q_2 = S(p_2 - 1) = p_1$ and also n is often a power of the number 2, so I make a new conjecture:

Conjecture 3:

For any p Sophie Germain prime with the property that S(p – 1) is prime, where S is the Smarandache function, one of the following two statements is true:

1. there exist m non-null positive integer such that (p – 1)/(2^m) = q, where q is prime, q ≥ 5;
2. there exist n prime and m non-null positive integer such that (p – 1)/(n*2^m) = q, where q is prime, q ≥ 5.

Note: we call the primes q from the first statement Coman-Germain primes of the first degree; we call the primes q from the second statement Coman-Germain primes of the second degree.

Verifying the Conjecture 3:
(for the first 21 Sophie Germain primes with the property showed)

The first statement:
: For p = 11, 23, 83, 179 we have m = 1
and q = 5, 11, 41, 89;
: For p = 29, 53, 173, 293, 509 we have m = 2
and q = 7, 13, 43, 73, 127;
: For p = 41, 89, 233 we have m = 3
and q = 5, 11, 29;
: For p = 113 we have m = 4
and q = 7.

The second statement:
: For p = 131, 191, 431 we have (m, n) = (1, 5)
and q = 13, 19, 43;
: For p = 239 we have (m, n) = (1, 7)
and q = 17;
: For p = 281 we have (m, n) = (3, 5)
and q = 7;
: For p = 419 we have (m, n) = (1, 11)
and q = 19;
: For p = 443 we have (m, n) = (1, 13)
and q = 17.

Conjecture 4:
There exist an infinity of Coman-Germain primes of the first degree.

Conjecture 5:
There exist an infinity of Coman-Germain primes of the second degree.

Notes:
We have the following sequence of Smarandache-Germain pairs of primes:
[5, 11], [11, 23], [7, 29], [5, 41], [13, 53], [41, 83], [11, 89], [7, 113], [13, 131], [43, 173], [89, 179], [19, 191], [29, 233], [17, 239], [7, 281], [73, 293], [179, 359], [19, 419], [43, 431], [17, 443], [127, 509] (...).

We have the following sequence of Coman-Germain primes of the first degree:
5, 11, 7, 5, 13, 41, 11, 7, 13, 43, 89, 29, 73, 179, 127 (...).

We have the following sequence of Coman-Germain primes of the second degree:
13, 19, 17, 7, 19, 43, 17 (...).