Exotic R4 and E8 Physics

Frank Dodd Tony Smith Jr - 2014 - viXra 1401.0070

E8 Physics (see viXra 1312.0036 and 1310.0182) is based on E8 which lives in the Clifford Algebra Cl(16) and models fermions by using spinor structures. As spinor structures are fundamentally related to Exotic Spheres, E8 Physics fits nicely with Exotic Sphere structures.

Exotic R4 is also relevant to physics as Carl Brans, Torsten Asselmeyer-Maluga, and their co-workers have shown in increasing detail. This paper is an attempt to describe how the physics of Exotic R4 might be related to E8 Physics.

As to fermions: Exotic R4 gives a mass term $\mu \cdot vol(S^1 \times S^3)$ for fermions with the constant $\mu = \bar{\mu}$ representing the curvature of $S^1 \times S^3$ with fermions being represented by hyperbolic knots.
Torsten Asselmeyer-Maluga et al say "... at the moment we have no idea how to generate realistic masses from this idea ...". E8 Physics shows how to "generate realistic masses" using geometric volumes in a way that may be equivalent to the Exotic R4 approach (see viXra 1311.0088 which unites Schwinger's Source Theory, the geometry of L. K. Hua, and the work of Armand Wyler).

As to gauge bosons: Exotic R4 gets the Standard Model U(1)xSU(2)xSU(3) from the structure of connecting tubes that have "similarity with ... brane theory: n parallel branes ... described by ... U(n) gauge theory". E8 Physics when formulated as 26D String Theory with Strings as World-Lines (see viXra 1210.0072) also gets gauge bosons for the Standard Model and MacDowell-Mansouri Conformal Gravity in terms of brane-to-brane connecting links.

As to a hyperfinite von Neumann factor Algebraic Quantum Field Theory (AQFT):
Exotic R4 constructs a hyperfinite von Neumann factor algebra by foliations of S3 of an Exotic R4 Akbulut Cork with Casson handle but lacks an explicit AQFT.
E8 Physics has such an AQFT from a generalization of the II1 factor, constructed according to Periodicity of Real Clifford Algebras by completing the union of all tensor products of the Clifford Algebra Cl(16) that contains E8 = D8 + half-spinor of D8.
Figure-eight 4_1 knot geometry may show how E8 Physics AQFT applies to Exotic R4.

Here on the following pages are some details about Exotic Structures.

Corresponding details about E8 Physics are given in the viXra references set out above.

Thanks to Daniel Rocha for suggesting that I study the interesting Exotic R4 work of Carl Brans, Torsten Asselmeyer-Maluga, and their co-workers.

There are **two kinds of exotic differentiable structures**:

The first kind is **Exotic Spheres**,
such as Smooth=Differentiable manifolds that are Combinatorial=Piecewise Linear equivalent to the n-sphere Sn. Some examples:

S1 - 1
S2 - 1
S3 - 1 (but S3 is a subset of any Exotic R4 and R x S3 is part of Exotic R4 Theory)
S4 - 1
S5 - 1
S6 - 1
S7 - 28
S8 - 2
S9 - 8
S10 - 6
S11 - 992
S12 - 1
S13 - 3
S14 - 2
S15 - 16,256
S16 - 2

Note that

S7 = Spin(8) / Spin(7) and Spin(8) half-spinors are 8-dim and 28 = 8/\8 = 8x7/2

S11 = Spin(12) / Spin(11) and Spin 12 half-spinors are 32-dim and 992 = 32x31

S15 = Spin(16) / Spin(15) and Spin 16 half-spinors are 128-dim and 16,256 = 128x127

It turns out that Exotic Sphere Structures are directly due to Spinor Structures
and
are accounted for in E8 Physics by the Clifford Algebra foundation of E8 Physics.

The second kind is **Exotic R4.**

Exotic R4 corresponds to 3 fundamental components of E8 Physics:
1 - fermionic fields 2 - bosonic fields 3 - hyperfinite factor AQFT

Torsten Asselmeyer-Maluga and Helge Rose in arXiv 1006.2230 said:
"... start... with a smooth 4-manifold M
admitting an exotic smoothness structure MK ... constructed by using knot surgery. ...
consider... the Einstein-Hilbert action on MK and the decomposition

$$M = (M \setminus N(T^2)) \cup_{T^2} (S^1 \times (S^3 \setminus N(K)))$$

... with $N(T^2) = D^2 \times T^2$ ...

Because of the diffeomorphism invariance of the action, one can split the Einstein-Hilbert action like

$$S_{EH}(M_K) = S_{EH}(M \setminus N(T^2)) + \int\limits_{S^1 \times (S^3 \setminus N(K))} R_K \sqrt{g_K}\, d^4x$$

...

$$S_{exotic} = \mu \cdot vol(S^1 \times S^3) - \int\limits_{S^1 \times \partial(N(K))} H_\theta \sqrt{h}\, d\theta d^2x - \lambda \cdot vol(D^2)$$

with the constant $\mu = \bar{\mu}$ representing the curvature of $S^1 \times S^3$ which is identical to the curvature of S^3. with H_θ as *mean curvature* of $\partial(N(K))$
The manifold $S^1 \times \partial N(K)$ is a knotted 3-torus $T^3(K) = K \times S^1 \times S^1$.

... Exotic Smoothness generates Fermionic and Bosonic fields ...

... define an action over the knot complement to identify two contributions:
knotted tori and connecting tubes between two tori.

1. a knotted solid torus can be described by a spinor
so that the mean curvature is the Dirac action of this spinor. ...

2. connecting tube ... as cobordism between two tori ... we obtained the Yang-Mills action ... The three possible types of torus bundles were identified with three interactions to get the gauge group

$$U(1) \times SU(2) \times SU(3)$$

... 1 ... Exotic Smoothness generates Fermionic ... fields ...

The action $\int_{S^1\times\partial N(K_-)} H_{\partial N(K)}\sqrt{g}\, d\theta d^2x$ is completely determined by the knotted torus $\partial N(K) = K \times S^1$ and its mean curvature $H_{\partial N(K)}$.

This knotted torus is an immersion of a torus $S^1 \times S^1$ into $\mathbb{R}^3$.

Spin representation of a surface gives back an expression for $H_{\partial N(K)}$ and the Dirac equation as geometric condition on the immersion of the surface.

The action can be interpreted as Dirac action of a spinor field.

$$\int\limits_M (R + \bar{\Phi} D^M \Phi)\sqrt{g} d^4x$$

... ...

The action is the usual Einstein-Hilbert action for a Dirac field Φ as source.

What about the mass term? In our scheme there is one possible way to do it: using the constant length $|\Phi|^2 = const.$ of the spinor, we can introduce the scalar curvature R_Γ of an additional 3-manifold Γ with constant curvature coupled to the spinor. Then we obtain

$$\int\limits_M \bar{\Phi}(D^M - m)\Phi\sqrt{g}\, d^4x$$

with $m = -R_\Gamma$ and $\Gamma \subset M$. But we already have a natural choice for this manifold, the 3-sphere $\Gamma = S^3$ as the embedding space for the knotted torus $\partial N(K) = K \times S^1$. Then the knotted 3-torus $T^3(K) = K \times S^1 \times S^1$ is given by an embedding of the 3-torus T^3 into $S^1 \times S^3$. Therefore as a conjecture the term $\mu \cdot vol(S^1 \times S^3)$ can be interpreted as mass term for the fermions.

Especially we obtain

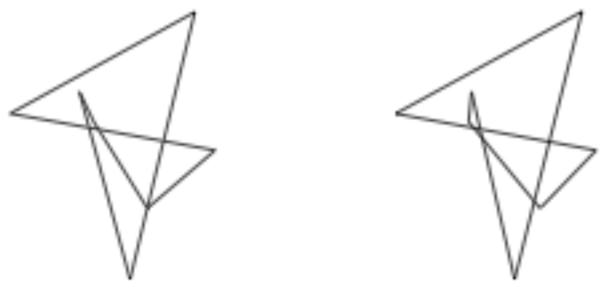
$$\int\limits_M m\bar{\Phi}\Phi\sqrt{g}\, d^4x = \mu \cdot vol(S^1 \times S^3)$$

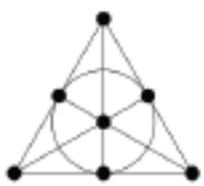
having the correct sign in the action But at the moment we have no idea how to generate realistic masses from this idea.

Let K be a hyperbolic knot with its hyperbolic complement C(K). Hyperbolic 3-manifolds are subject to … Mostow rigidity … a property which we should expect for fermions ...".

According to Wikipedia:

"… … the figure-eight knot ...  ... can be considered to be the simplest hyperbolic knot …[it has]... Stick no. 7 … the stick number is the smallest number of edges of a polygonal path equivalent to … a knot ...".

I conjecture that the 7 Sticks  of the 8 knot (image from Robert Glenn Scharein 1998 U. British Columbia Ph.D. thesis) correspond to the

7 lines of the Fano Plane  (Wikipedia image) and the 7 Imaginary Octonions and the 7 first-generation Fermion types that carry charge (electric and/or color)

electron
red up quark
green up quark
blue up quark
red down quark
green down quark
blue down quark

The neutrino, carrying no charge of either kind, would correspond to the unknot 0_1 .

... 2 ... Exotic Smoothness generates ... Bosonic fields ...

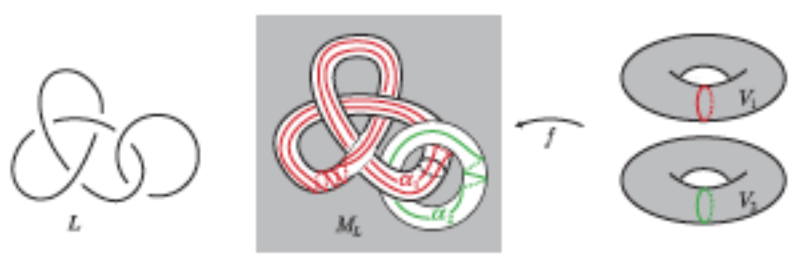
Spacetime in which the Fermions propagate is 10-dimensional in E8 Physics when formulated as 26D String Theory with Strings as World-Lines (see viXra 1210.0072) with 6 dimensions representing 6-dim Spin(2,4) Conformal spacetime that includes 4-dim M4 Spin(1,3) Minkowski spacetime of 4+4 = 8 dim M4 x CP2 Kaluza-Klein
and
with 4 dimensions representing 4-dim CP2 = SU(3) / U(2) Internal Symmetry Space of 4+4 = 8 dim M4 x CP2 Kaluza-Klein .

According to Wikipedia:
"… The figure-eight knot and the (-2,3,7) pretzel knot are the only two hyperbolic knots known to have more than 6 exceptional surgeries, Dehn surgeries resulting in a non-hyperbolic manifold ...
The figure-eight knot … ha[s] … 10 … exceptional surgeries …
the largest possible number of exceptional surgeries of any hyperbolic knot …".

Sangyop Lee said in "Dehn Fillings Yielding Klein Bottles" (Int. Math. Res. Not. 2006 (ID 24253) 1-34):
"... We give a complete list of hyperbolic 3-manifolds admitting two Dehn fillings at distance 4, each of which yields a Klein bottle ... the only hyperbolic knots in S3 admitting two distinct Klein bottle surgeries are the figure-eight knot ... and ... the (-2,3,7)-pretzel knot and its mirror image ... Such fillings are exceptional because a 3-manifold containing a Klein bottle is not hyperbolic ...".

Sangyop Lee said in "Dehn Fillings on 3-manifolds" (slides 27-28 January 2008):
"...

Dehn surgeries = deleting + filling



$L(\alpha_1, \alpha_2) = M_L \cup_f (V_1 \cup V_2)$

K : a knot in S^3, $E(K) = S^3 - \text{int}N(K)$
μ, λ : meridian and longitude $\subset \partial E(K)$
α : an essential simple closed curve in $\partial E(K)$
$\alpha \sim m\mu + l\lambda$ for some coprime integers m, l
$\{\text{slopes}\} \leftrightarrow \mathbb{Q} \cup \{1/0\}$
$\alpha \leftrightarrow m/l$

... Thurston's ... Geometrization Conjecture ... A closed 3-manifold is not hyperbolic if and only if it is reducible, toroidal, or a small Seifert fiber space ...

The figure-eight knot exterior has 10 exceptional slopes ...

Let M be a hyperbolic 3-manifold with a torus boundary component T. Define $\mathcal{E}(M;T) = \mathcal{E}(M) = \{\alpha \subset T \mid M(\alpha) \text{ is not hyperbolic } \}$

Let M be the exterior of the figure-8 knot.

Then $\mathcal{E}(M) = \{-4, -3, -2, -1, 0, 1, 2, 3, 4, \infty \}$

Since the figure-8 knot is amphicheiral, $M(r) \cong M(-r)$.

boundary slope 4 boundary slope −4 $\Delta(4,-4)=8$

...".

The slope oo case corresponds to the empty Dehn Filling and to S3 itself.
The slope 0 case corresponds to S1xS2 (Rolfsen, "Knots and Links")
Masakazu Teragaito (Hiroshima University) said in "Toroidal Surgery on Hyperbolic Knots":
"... the figure-eight knot ... has exactly three integral toroidal slopes 0 , -4 , and 4.
... +/-4 surgery gives a graph manifold which is the union of two Seifert fibered manifolds over the disk with two exceptional fibers. For the figure-eight knot ...[
-4 and 4 give Klein bottles and]...
+/-1 , +/-2 , +/-3 yield small Seifert fibered manifolds ...".
Masakazu Teragaito (Hiroshima University) said in arXiv 0705.3715:
"... Except for the figure-eight knot with six Seifert surgeries,
a hyperbolic knot seems to admit at most three Seifert surgeries ...".

In E8 Physics as 26-dim World-Line String Theory with 8-dim M4 x CP2 Kaluza-Klein 10-dim spacetime = 4-dim CP2 Internal Symmetry Space + 6-dim Conformal Spacetime where 6-dim Conformal Spacetime contains 4-dim M4 Minkowski Physical Spacetime.

The 4 slopes -3 , -2 , -1 , oo correspond to 4-dim CP2 Internal Symmetry Space.
The 6 slopes -4 , 0 , 1 , 2 , 3 , 4 correspond to 6-dim Conformal Spacetime
and 0 , 1 , 2 , 3 correspond to 4-dim Minkowski Physical Spacetime.

The 4_1 figure-eight knot is the unique hyperbolic knot
with 10 exceptional surgeries that could correspond to such a 10-dim spacetime.
(Many other hyperbolic knots have 6 exceptional Dehn surgeries that could represent Conformal spacetime but would not give the 4-dim CP2 internal symmetry space of 4+4 dim Kaluza-Klein.
The only other hyperbolic knot known to have over 6 exceptional surgeries, the (-2,3,7) pretzel knot with 6+1 = 7 , could represent 4+1 = 5 dim Kaluza-Klein with 1-dim U(1) Internal Symmetry Space.)
In E8 Physics as 26D String Theory with Strings as World-Lines spacetime goes from 26 dim down to 10 dim by orbifolding 8+8 + 16 dim of first-generation Fermions and then goes down from 10 = 6+4 to 8 dim 4+4 Kaluza-Klein by reduction of Conformal Spin(2,4) = SU(2,2) 6-dim spacetime to Minkowski Spin(1,3) 4-dim.

The 8-dim spacetime structures correspond to D8 branes in E8 as String Theory.
Connections between stacked D8 branes give Standard Model U(1)xSU(2)xSU(3)
in a way similar to

the process of Exotic R4 Physics

described by Torsten Asselmeyer-Maluga and Helge Rose in arXiv 1006.2230 :

Torsten Asselmeyer-Maluga and Helge Rose in arXiv 1006.2230 said: "...

consider the integral $$\int_{(S^3\setminus N(K))} R_{(3)}\sqrt{h}N\, d^3x$$

Let $C(K) = S^3 \setminus N(K)$ be the knot complement for the knot K and assume for K a sum

$$K = K_1 \# K_2$$

of prime knots K_1, K_2. Then the knot complements admits a splitting

$$C(K) = C(K_1) \cup_{T^2} T(K_1, K_2) \cup_{T^2} C(K_2)\,.$$

We call $T(K_1, K_2)$ the *connecting tube* between the knot complements $C(K_1)$ and $C(K_2)$. The connecting tube $T(K_1, K_2)$ has a boundary consisting of three disjoint tori $\partial T(K_1, K_2) = T_1^2 \sqcup T_2^2 \sqcup T_3^2$ (we ignore the orientation) where one of these tori T_3^2 is the boundary $\partial C(K) = T_3^2$ of $C(K)$. If we ignore this boundary (by closing it with a solid torus $T(K_1, K_2) \cup_{T_3^2} (D^2 \times S^1)$) then we have a trivial torus bundle $T^2 \times [0, 1]$ between T_1^2 and T_2^2.

...

we obtain for the action $S_{EH}(S^1 \times T(K_1, K_2))$

$$\int_{S^1\times T(K_1,K_2)} R_K\sqrt{g_K}\, d^4x = L_{S^1} \cdot L \cdot CS(T(K_1, K_2), A)$$

with respect to the (Levi-Civita) connection A and the length L and the Chern-Simons action $CS(T(K_1, K_2), A)$. For the 3-manifold $T(K_1, K_2)$, there is a 4-manifold M_T with $\partial M_T = T(K_1, K_2)$ (take for instance $M_T = T(K_1, K_2) \times [0, 1) \subset T(K_1, K_2) \times S^1$). By using the Stokes theorem we obtain

$$S_{EH}(M_T) = \int_{M_T} tr(F \wedge F)$$

with the curvature $F = DA$

...

We contract $T(K_1, K_2)$ to thin tubes connecting the thick parts. Conversely one also finds a scaling so that the thin part becomes large (but the thick part has the same size). Thus we can interpret the curvature $\bar{F}$ of the thin part as field located between the thick part. The thick part can be interpreted as fermions the action integral of the bosons can be written as

$$\int\limits_{M \setminus vol(fermion)} tr(\bar{F} \wedge *\bar{F})$$

considered over $M \setminus vol(fermions)$

...

In the action we have two constant terms $\mu \cdot vol(S^1 \times S^3)$ and $\lambda \cdot vol(D^2)$
The first constant was interpreted as a mass term of the fermion.
the second constant $\lambda \cdot vol(D^2)$ as the *cosmological constant* Λ

$$\Lambda = \frac{\lambda \cdot vol(D^2)}{vol(M)} .$$

showing a combined Dirac-gauge-field coupled to the Einstein-Hilbert action

$$S(M) = \int\limits_M \left(R - \Lambda + \sum_n (\bar{\Phi}(D^M - m)\Phi)_n \right) \sqrt{g}\, d^4x + \int\limits_M tr(\bar{F} \wedge *\bar{F})$$

... the connecting tube T(K1,K2) is ... a torus bundle ... which can always be decomposed into three elementary pieces ...

finite order (orders 2,3,4,6): the tangent bundle is 3-dimensional
... 2 isotopy classes (= no/even twist or odd twist) ...
no twist must be the photon ... even twist or odd twist ... should be ... the Z0 boson ...

Dehn-twist (left/right twist): the tangent bundle is a sum of a 2-dim and a 1-dim bundle
... 2 isotopy classes (= left or right Dehn twists) ...
are the W+/- bosons ...

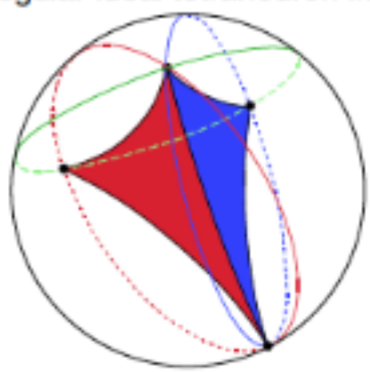
Anosov: the tangent bundle is a sum of three 1-dim bundles.
... 8 isotopy classes (= ... orientations of the three line bundles ..)
correspond to the 8 gluons ...

... We remark the similarity with ... brane theory:
n parallel branes ... are described by an U(n) gauge theory ...".

The Brane Model construction of the Standard Model U(1)xSU(2)xS(3)
is consistent with the Two-Tetrahedra structure of the figure-eight 4_1 knot:

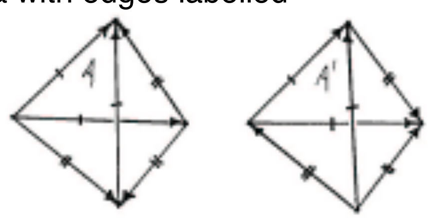
According to Wikipedia:
"… The figure-eight complement is a double-cover of the Gieseking manifold, which has the smallest volume among non-compact hyperbolic 3-manifolds. … The … double cover … underlying compact manifold has boundary a Klein bottle … The Gieseking manifold can be constructed by removing the vertices from a tetrahedron, then gluing the faces together in pairs …". So the figure-eight knot 4_1 complement is the minimal two ideal tetrahedra and 4_1 is the simplest hyperbolic knot (image from Craig Hodgson "Hyperbolic Structures from Ideal Triangulations"):

a regular *ideal* tetrahedron in $\mathbb{H}^3$



with vertices on the sphere at infinity

William P. Thurston in "The Geometry and Topology of Three-Manifolds" (web version) said:
"... Begin with two tetrahedra with edges labelled



There is a unique way to glue the faces of one tetrahedron to the other so that arrows are matched. For instance, A is matched with A_0. All the /-> arrows are identified and all the //-> arrows are identified, so the resulting complex has 2 tetrahedra, 4 triangles, 2 edges and 1 vertex. Its Euler characteristic is +1, and ... a neighborhood of the vertex is the cone on a torus. Let M be the manifold obtained by removing the vertex ... this manifold is homeomorphic with the complement of a figure-eight knot. ... the fundamental group of the complement of the figure-eight knot is isomorphic to a subgroup of index 12 in PSL_2(Z[w]) , where w is a primitive cube root of unity ...".

The order 12 of the fundamental group of the figure-eight 4_1 knot
is the same as the order of the Tetrahedral Symmetry Group of a Tetrahedron.
The Tetrahedron-Double-Cover (Two-Tetrahedra) structure of 4_1 has symmetry
of the 24-element Binary Tetrahedral Group which in turn corresponds to
the 24 Unit Integral Quaternions = vertices of the 24-cell =
= Root Vector Vertices of the 28-dim D4 Lie Algebra,
where 16 vertices correspond to U(2,2) = U(1)xSpin(2,4) of Conformal Gravity
and 12 vertices correspond to U(1)xSU(2)xSU(3) of the Standard Model.

3 - Exotic Smoothness generates Hyperfinite Factor AQFT

John Baez in his week 175 said:

"... a von Neumann algebra is a ... *-algebra of operators that is closed in the weak topology. Every von Neumann algebra can be built from ... "simple" ones as ... a "direct integral" ... People call simple von Neumann algebras "factors" ...

The first step in classifying factors was done by von Neumann and Murray, who divided them into types I, II, and III. ...

We say a factor is type I if it admits a nonzero trace for which the trace of a projection lies in the set $\{0,1,2,...,+\infty\}$. We say it's type I_n if we can normalize the trace so we get the values $\{0,1,...,n\}$. Otherwise, we say it's type I_∞, and we can normalize the trace to get all the values $\{0,1,2,...,+\infty\}$.

It turn out that every type I_n factor is isomorphic to the algebra of n x n matrices. ...

a factor is type II_1 if it admits a trace whose values on projections are all the numbers in the unit interval $[0,1]$. We say it is type II_∞ if it admits a trace whose value on projections is everything in $[0,+\infty]$. Playing with type II factors amounts to letting dimension be a continuous rather than discrete parameter!

...

to construct a type II_1 factor ... Start with the algebra of 1 x 1 matrices, and stuff it into the algebra of 2 x 2 matrices ... This doubles the trace, so define a new trace on the algebra of 2 x 2 matrices which is half the usual one. Now keep doing this, doubling the dimension each time, using the above formula to define a map from the $2^n \times 2^n$ matrices into the $2^{n+1} \times 2^{n+1}$ matrices, and normalizing the trace on each of these matrix algebras so that all the maps are trace-preserving. Then take the *union* of all these algebras... and finally, with a little work, complete this and get a von Neumann algebra ... this von Neumann algebra is a factor. It's pretty obvious that the trace of a projection can be any fraction in the interval $[0,1]$ whose denominator is a power of two. But actually, *any* number from 0 to 1 is the trace of some projection in this algebra - so we've got ... a type II_1 factor. This isn't the only II_1 factor, but it's the only one that contains a sequence of finite-dimensional von Neumann algebras whose union is dense in the weak topology. A von Neumann algebra like that is called "hyperfinite", so this guy is called "the hyperfinite II_1 factor" ... the algebra of $2^n \times 2^n$ matrices is a Clifford algebra, so the hyperfinite II_1 factor is a kind of infinite-dimensional Clifford algebra.

But the Clifford algebra of $2^n \times 2^n$ matrices is ... another name for the algebra generated by creation and annihilation operators on the fermionic Fock space over C^{2n} ...

the hyperfinite II_1 factor is the smallest von Neumann algebra containing the creation and annihilation operators on a fermionic Fock space of countably infinite dimension.

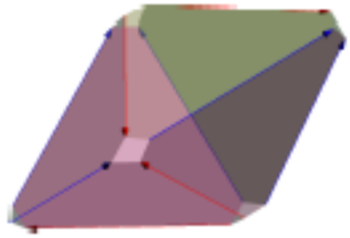
... the hyperfinite II_1 factor is the right algebra of observables for a free quantum field theory with only fermions. ...

The most mysterious factors are those of type III ... pretty much all the usual field theories on Minkowski spacetime have type III factors as their algebras of "local observables" - observables that can be measured in a bounded open set. ...".

Torsten Asselmeyer-Maluga and Jerzy Krol in arXiv 1001.0882 said:
"... non-standard smooth R4 's exist as a 4-dimensional smooth manifolds ...
a small exotic structures of the R4 is determined by the so-called Akbulut cork ...
and its embedding given by an attached Casson handle.
The boundary of the cork is a homology 3-sphere containing a 3-sphere S3
such that the codimension-1 foliations are determined by the foliations of S3 ...
we ... relate the exotic R4 to ...the hyperfinite factor III1 von Neumann algebra ...
we obtain a foliation of the horocycle flow ... which determines the factor II∞
...
we are looking for a classical algebraic structure which would give the ...
noncommutative algebra of observables as a result of quantization
...
The classical structure ... has the structure of a Poisson algebra ... idempotents were ...
constructed as closed curves in the leaf of the foliation of S3 ...
a quantization procedure of the ... Poisson algebra ... is the skein algebra ... directly
related to the factor III1 von Neumann algebra derived from the foliation of S3 ...
the skein algebra is ... the factor II1 algebra Morita equivalent to the factor II∞ which in
turn determines the factor III1 of the foliation ...
the ... main building blocks of ... 4-exotic smooth structures ... i.e., Casson handles,
determine the factor II1 algebras ...
a Casson handle is represented by a labeled finitely-branching tree Q ...
Every path in this tree represents one leave in the ...horocycle ... foliation of the S3 .
Two different paths in the tree represent two different leaves in the foliation.
Then we have to consider two paths in the tree Q ,
the reference path for the given leaf and a path for the another leaf of the foliation.
Thus, a pair of two paths corresponds to one element of the algebra ...
this algebra is given by ... Clifford algebra ... i.e. by the hyperfinite factor II1 algebra
...
We do not have explicit descriptions of a RAQFT ... algebraic relativistic QFT ... on an
exotic R4 or even classical field theory on it since we do not have an exotic metric nor
the global exotic smooth structures glued from local coordinate patches.
...
all we need is the knowledge about the existence of theories which have the quantum
algebra of observables spanned on the factor III1 and the classical algebra spanned on
a Poisson algebra
...
The 4-exotics approach is essentially 4-dimensional. The factor III1 von Neumann
algebra is unique. When one wants to vary different exotic R4 's in this approach, the
net of algebras suitably embedded into each other should be probably considered. ...".

Figure-eight 4_1 knot geometry
may give Exotic R4 an explicit RAQFT based on E8 Physics.

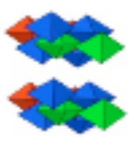
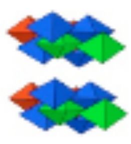
The figure-eight 4_1 knot has complement structure of Two Ideal Tetrahedra
(image from Martin Deraux ICERM Workshop Exotic Geometric Structures 2013")

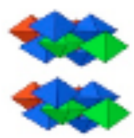


How do the figure-eight knot complement Tetrahedra fit in the 3 types of 3-dim space:
Euclidean - Spherical S3 - Hyperbolic H3
?

Euclidean 3-dim space is not tiled by Tetrahedra but Pairs of Tetrahedra can represent 8-dim half-spinors of the Real Clifford Algebra Cl(8)

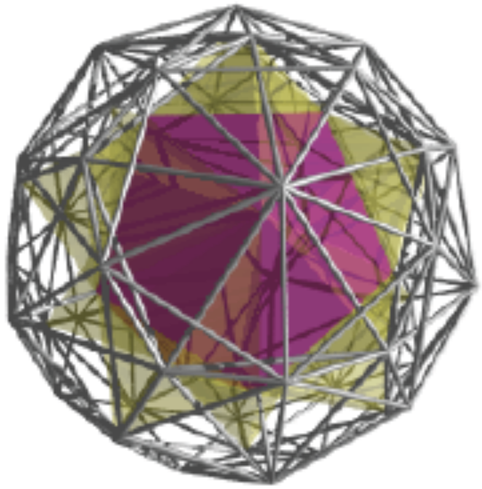
with pairs of Pairs  representing 16-dim full spinors of Cl(8).
By Clifford 8-Periodicity the tensor product of two Cl(8) full spinor pairs of Pairs

 x  form  +  = 128+128 =
= 256-dim full spinors of Cl(8)xCl(8) = Cl(16)

One set of 128-dim Cl(16) half-spinors  is the spinor/fermion part of the 248-dim Lie algebra E8 . The other part of E8 also comes from Cl(16) as the 120-dim Spin(16) bivector Lie Algebra of Cl(16) so that
248-dim E8 = 120-dim Spin(16) + 128-dim half-spinor of Spin(16) is contained in Cl(16).
The E8 structure in Cl(16) allows construction of a realistic Local E8 Physics Lagrangian.
By Clifford 8-Periodicity any Real Clifford Algebra no matter how large can be contained in a tensor product of copies of Cl(16).
Since each E8 Lagrangian is Local, it is necessary to combine Local Lagrangian Regions to form a Global Structure describing a Global E8 Algebraic Quantum Field Theory (AQFT) by completing the union of all tensor products of all the Cl(16), giving a generalized Hyperfinite II1 von Neumann factor Algebraic Quantum Field Theory that is the desired explicit RAQFT for Exotic R4.

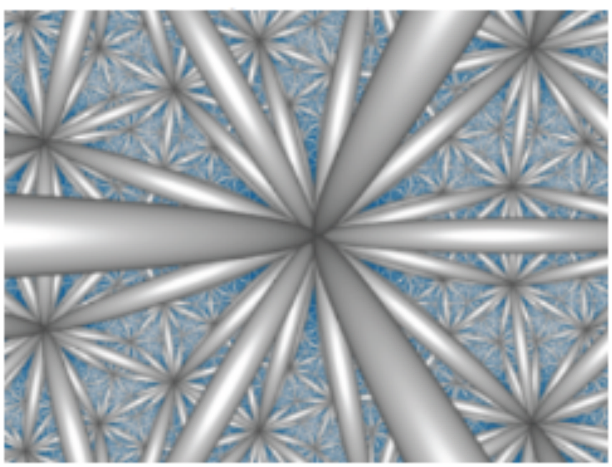
Although the process does not produce an exact Euclidean 3-dim Tiling by Tetrahedra it does produce the densest known packing of 3-dim Euclidean space by Tetrahedra as described in arXiv 1001.0586 by Chen, Engel, and Glotzer (from which the above tetrahedra pair system images were taken).

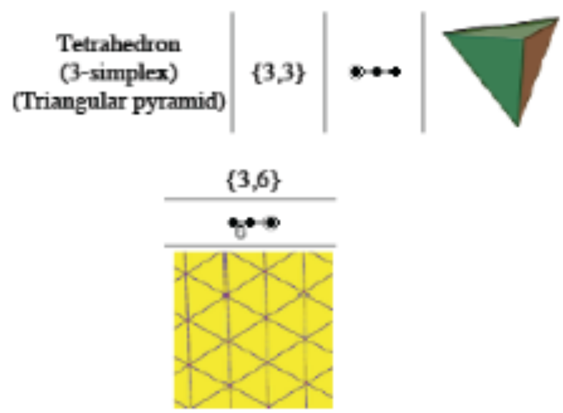
Spherical 3-dim space is tiled by 600 Tetrahedra as the 600-cell with 120 vertices.
(image from Wikipedia)



Two copies of the 600-cell give 120+120 = 240 vertices of the Root Vectors of E8
so the Spherical 3-dim S3 space tilling by figure-eight knot Complement Tetrahedra
represents the E8 Lie Algebra symmetry of E8 Physics.

Hyperbolic 3-dim space is tiled by Tetrahedra of the {3,3,6} Tetrahedral Honeycomb
(images from Wikipedia





Elisha Falbel said in J. Diff. Geom. 79 (2008) 69-110 :
"A spherical CR structure on the complement of the figure eight knot with discrete holonomy":
"... the fundamental group of ... the complement of the figure eight knot ... ha[s] a discrete representation in PSL(2,C) ...[and]... in PSL(2, Z[w]) where Z[w] is the ring of Eisenstein integers ...
the complement of the figure eight knot has a (branched) spherical CR structure with discrete holonomy such that the holonomy of the boundary torus is parabolic and faithful ... We also prove a rigidity theorem ... that it is the only one with faithful purely parabolic torus holonomy ... An interesting related feature of the representation is that its limit set is S3 ...".

John Milnor said in Bull. AMS 6(1982) 9-24 :
"... a hyperbolic manifold is a Clifford-Klein manifold with curvature equal to -1. ...
Consider the figure eight knot K ...
... The fundamental group II of the complement S3 - K is generated by two loops ...
II ...[is]... isomorphic... ...to a subgroup ... of PSL2Z[w] of index twelve ...
[where]... w = (-1 +/- sqrt(-3)) / 2 ...
the complement S3 - K is ... homeomorphic to the hyperbolic manifold H3 / II ...
Rigidity Theorem. If two hyperbolic manifolds of finite volume, with dimension ≥ 3 ,
have isomorphic fundamental groups, then they must ... be isometric to each other ...
It follows that geometric invariants such as volume, the lengths of closed geodesics,
and the eigenvalues of the Laplacian operator, are also topological invariants. ...".

John R. Parker said in his review of William M. Goldman's 1998 book "Complex Hyperbolic Geometry":
The unit ball in Cn has a natural metric of constant negative holomorphic sectional
curvature, called the Bergman metric. As such it forms a model for complex hyperbolic
n-space HnC analogous to the ball model of real hyperbolic space HnR
The main difference is that the real sectional curvature is no longer constant, but is
pinched between two negative numbers whose ratio is 4. ...
The geometry of HnC is not a completely straightforward generalisation of HnR
Aspects such as negative curvature
and the fact that maximal parabolic subgroups are nilpotent rather than Abelian
tend to make it hard to generalise real hyperbolic results to the complex case
However, the complex structure gives more tools for solving problems ...
It is not the case that results from HnR either generalise to HnC or else break down.
Using analogy as a guide, one can often formulate qualitatively similar results, but the
methods of proof a[re] usually rather different ...
Just as the internal geometry of real hyperbolic space
may be studied using conformal geometry on the boundary,
so the internal geometry of complex hyperbolic space
may be studied using CR-geometry on the Heisenberg group ...
The boundary of complex hyperbolic n-space is
the one point compactification of the (2n - 1)-dimensional Heisenberg group
in the same way that the boundary of real hyperbolic n-space is
the one point compactification of Euclidean (n - 1)-space ...".

Since "... the complement of the figure eight knot has a (branched) spherical CR structure with ... limit set ... S3 ..."
and S3 is the boundary of the spherical 4-ball B4 and the hyperbolic H4R
and real hyperbolic H4R corresponds to complex hyperbolic H4C
and complex hyperbolic H4C corresponds to the 7-dimensional Heisenberg group h3
with graded structure h3 = 3 + 1 + 3

Each 3 of the Heisenberg group h3 corresponds to the 3-sphere S3

S3 lives inside the 7-sphere S7 by the Quaternionic Hopf fibration S3 -> S7 -> S4

The 7-sphere S7 expands to the D4 Lie Algebra Spin(8)

The Spin(8) is the bivector Lie Algebra of the Real Clifford Algebra Cl(8)
Cl(8) has graded structure 1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1

The vectors, bivectors, and trivectors of Cl(8) are 8+28+56 = 92-dimensional
The 5-vectors, 6-vectors, and 7-vectors of Cl(8) are 56+28+8 = 92-dimensional

The Heisenberg group h92 has 7-graded structure 8 + 28 + 56 + 1 + 56 + 28 + 8
where 8+28+56 and 56+28+8 correspond to 3 and 3 of h3 = 3 + 1 + 3

The 92+1+92 + 63 = 248-dimensional semi-direct product h92 x SL(8,R)
is the Maximal Contraction of 248-dimensional E8

Therefore h92 x SL(8,R) represents the structure of the Exotic R4 RAFQT with creation and annihilation operators of the Heisenberg Group h92