First law of motion in periodic time

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Abstract

First law of motion operative in a time periodic universe with S^1 time is formulated. The inertial paths of the particles are defined as circles, with radius $R = T\,|\mathbf{v}|\,/(2\pi)$, where T is the time period of the universe, and $\mathbf{v}$ is the velocity of the particle. This law reduces to the Newton's first law of motion in the limit $T \to \infty$, when the radii $R \to \infty$, and so the circles open out and become indisthinguighable from the straight line trajectories of the Newtonian universe.

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Newton postulated a linear time running from past infinity to future infinity In absence of an external force, the particles traveled on straight lines with uniform velocities [1]. In the General Theory of Relativity (GTR), inertial motion in absence of external forces, was defined in terms of geodesics, on which the particle paths could be curved, and the particle velocities could vary [2]. Further there existed possibilities of compact topologies for space-time dimensions. In modern parlance, the Newtonian space time is flat with $R^3 \times R^1$ topology and posseses Galilean invariance, which is a special case of the speed of light $c \to \infty$. Segal [3], taking inspiration from the fact that Galilean group is a special case of Lorentz group pointed out a program for search for invariance groups with finite parameters.

This paper puts forth Newtonian universe as a special case of a universe with periodic time T, i.e., S^1 topology of time. The Newtonian universe emerges when $T \to \infty$. In [4], issue of eternal return is related to the S^1 topology of time. Therefore in a Newtonian universe with S^1 time, there occurs the issue of commensurate closure of all particle trajectories - both inertial and non-inertial. Trivially, non-inertial trajectories arise, when the particles are interacting, or there are forces acting on the particles.

We will represent the usual Newtonian universe by $N^{R^3 \times R^1}$ - where the superscript refers to topology of its space time. Accordingly, Newtonian universe with S^1 time will be represented by $N^{R^3 \times S^1}$. It is clear that in $N^{R^3 \times S^1}$, with time periodicity T, if all particles not under influence of any external force, are travelling with any speed $v \in [0, \infty]$ on a circle with radius R, given by -

$$R = \frac{vT}{2\pi} \tag{1}$$

then any initial configuration of such particles will reccur after time period T. We shall refer to this as *the first law of motion in periodic time.*

However, a circle in three dimensional space is specified as intersection of a sphere, with the equation -

$$(x-a)^2 + (y-b)^2 + (z-c)^2 = R^2 = \left(\frac{vT}{2\pi}\right)^2 \tag{2}$$

and a plane, with the equation -

$$n_x(x-a) + n_y(y-b) + n_z(z-c) = 0 \tag{3}$$

Here, (a, b, c) define the center of the sphere, and (n_x, n_y, n_z) define the orientation of the plane.

Now consider a particle located at coordinates (x, y, z) with velocity $\mathbf{v} = (v_x, v_y, v_z)$. What can one say about its past and future trajectory? In $N^{R^3 \times R^1}$, 6 parameters (3 of the coordinates, and 3 of velocity) suffice to determine both its past and future trajectory. However, in $N^{R^3 \times S^1}$, one would have to specify additional parameters determing the the circle on which the particle is travelling. The coordinates and velocity would only specify a set of spheres, on which past and future trajectory of the particle lies. The envelope of these spheres would be a torus with equal radii $R = vT/2\pi$ for both the S^1 factors. Thus, one would need an additional parameter for specifying the orientation of the plane, on which the velocity vector lies - and whose intersection with this torus gives the the circle(s) of particle's trajectory (as this intersection between torus and plane, would actually give 2 circles). Accordingly, one would need an additional choice parameter for choosing which one of these two circles - would the particle propagate along. These additional 2 parameters become redundant in the limit $T \to \infty, R \to \infty$ which leads to $N^{R^3 \times R^1}$.

It would be interesting to consider $N^{R^3 \times S^1}$, when particles are interacting, and hence experiencing forces. The particle paths now can be regarded as perturbations of circles. Constraint of recurrence requires that the trajectories of all particles should be having commensurate closure. Let γ be one such trajectory parameterized with s. Let $v(s)$ be the speed of the particle at any point s on γ. Commensurate closure of all particle paths now boils down to the condition -

$$\oint_\gamma \frac{ds}{v(s)} = T \tag{4}$$

Guillemin [5] has studied deformations of Lorentzian manifolds with $S^2 \times S^1$ topology of space time. It would be interesting to study issue of non-inertial motion (arising from particle interaction), as deformations of the $(2+1)$-dimensional $N^{R^2 \times S^1}$.

References

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