Linkage of new formalism for cosmological "constant" with the physics of wormholes

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Abstract. Wormhole physics as embodied by a modified Wheeler De Witt equation as given by Crowell inter connects with how to write a quintessence version of the 'cosmological constant'. The resulting value for the 'cosmological constant' is for wormholes for reasons stated in this document, using a modified form of the Weiner – Nordstrom metric, with no charge Q.

Key words: Gravitational waves (GW), modified Wheeler De Witt equation, Wormholes

1.Introduction

We examine what Crowell's worm hole version of the Wheeler De Witt equation portends to. The novel feature of his formalism entails both a damping term in the resulting Schrodinger equation, and a pseudo time component on the right hand side, equivalent to initial energy. The author in lieu of this highly complex evolution equation for a wave function uses a planar approximation to obtain k in exp(ikr), which is then used to obtain a quadratic equation for k, which has functional dependence upon a 'cosmological constant' term , and curvature, plus the initial energy used for the worm hole. The resulting k quadratic equation has a 2^{nd} term which is complex, with the result that the resulting allowed k equation has a real and complex part contribution. How this is analyzed plays a role in formulation of the cosmological 'constant'. The final piece is how the cosmological 'constant' can approach for worm holes, the temperature dependence derived by Park et. al. in 2002, in the case of worm holes. The temperature dependence probably does not apply to flat space, for reasons identified in the manuscript, which is in sync with current supernova data from the early universe supporting no variation in the cosmological 'constant'. The first part will be in looking at how the WdW equation initially arises, then how the WdW has modifications put in by Crowell, namely a damping term in the left hand side and a non zero pseudo energy term. The resulting expression for the k term in the presentation of 'momentum' will guide us into specific recommendations as to the vacuum energy and modified 'cosmological constant' term.

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2. Outlining the WdW equation by Crowell, which may permit formation of a wormhole. First by explaining its similarities and differences from the traditional WdW equation.

We start off with looking the WdW equation for which there is no energy term, i.e. then so called wave function of the universe being formed [1] due to there being no right hand sided energy, which is equivalent to saying the universe has no independent time component. From there the first order derivative with respect the wave function appears, plus energy which is equivalent to dispersive behaviour and also possibly, input energy into the wormholes. All of which will be discussed.

2.1 *Traditional wormhole physics, from first principles*

In doing this, start off with [1], consider how the initial wave function is formed via the use of a constant vacuum energy, as given by $\rho_{vacuum} = const.$ which means that the Friedman equations, transit to the form given by [1]

$$\begin{aligned}
&\frac{\dot{a}^2 + c^2}{a^2} = \frac{8\pi G}{3} \cdot \rho_{vacuum} \equiv \frac{\Lambda}{3} \\
&2\frac{\ddot{a}}{a} + \frac{\dot{a}^2 + c^2}{a^2} = 8\pi G \cdot \rho_{vacuum} \equiv \Lambda \\
&\Rightarrow \frac{\ddot{a}}{a} \equiv \frac{\Lambda}{3} \Rightarrow a_0 = c\sqrt{\frac{3}{\Lambda}} \\
&\Leftrightarrow a \equiv a_0 \cosh\left[\frac{ct}{a_0}\right]
\end{aligned} \tag{1}$$

The solution has a wave function which is infinitely large, in the infinite past, then goes to an infinite value in the far, far future, with a denied region for a value of $0 < a < a_0$, in which then one has to come up with quantum tunneling to get out of this problem. Reference [1] eventually argues to obtaining a Schrodinger equation without an energy term which is written up to look like, if $z = a/a_0$, and $\breve{a} = \left[\frac{9\pi c^5}{2\hbar G\Lambda}\right]$, with Λ a cosmological 'constant', and $Q^2(z) \equiv \breve{a}^2 \cdot (z^4 - z^2)$ so that then quantization of the momentum term, leading to [1]

$$\frac{d^2\Psi}{dz^2} + Q^2(z)\Psi = 0 \tag{2}$$

Eqn. (2) is solved via the WKB approximation, which if $w(z) = \frac{\breve{a}}{3} \cdot (z^2 - 1)^{3/2}$, and N_0 a normalization constant. Then[1]

$$\Psi(z) = \begin{cases} \frac{1}{2} N_0 \left|Q(z)\right|^{-1/2} \cdot \exp(\left|w(z)\right|); z < 1, \quad (a < a_0) \\ N_0 \left|Q(z)\right|^{-1/2} \cdot \cos(\left|w(z)\right| - \pi/4); z > 1, (a > a_0) \end{cases} \tag{3}$$

Where this gets nonsensical if a cosmological 'constant' is tiny is in transition probability from $a < a_0$ to $a > a_0$

$$T_{transition} \equiv 1 \Big/ \left[1 + \exp\left(-3\pi c^5 / \hbar G\Lambda\right)\right] \tag{4}$$

In the case of Eq.(3) usually the 'cosmological constant' is tiny, so Eq.(4) is nonsense. We use it for our wormhole problem in which it has a very different implication and is one of the formulas referred to. So next, let us refer to the Crowell treatment of wormholes which has very different wavefuction behavior than Eq.(3).

2.2. *Referral to the Crowell version of the WdW equation for worm holes, and with no charge Q in the Weiner-Nordstrom metric*

If there is, for a wormhole an emergent field evolving in time as $\Phi = \Phi_0 \cos[\omega \cdot t]$, with Φ_0 an initial configuration, and also with a line element given by $F(r) = 1 - \frac{2 \cdot \mathrm{M}}{r} + \frac{Q}{r^2} - \frac{\Lambda \cdot r^2}{3}$ with Q=0 leads to [2]

$$
\begin{aligned}
&-\left(\frac{\partial F}{\partial r}\right)^{-1} \cdot \left(\frac{\partial^2 \Psi}{\partial r^2}\right) + \frac{1}{2} \cdot \left(\left(\frac{\partial F}{\partial r}\right)^{-2} \left(\frac{\partial^2 F}{\partial r^2}\right) + \left(\frac{\partial F}{\partial r}\right)^{-1}\right) \cdot \frac{\partial \Psi}{\partial r} + r \cdot R^{(3)} \cdot \Psi \\
&= \left(\left(\frac{\partial F}{\partial r}\right) \cdot \Phi - \ddot{\Phi} \cdot r\right) \cdot \Psi
\end{aligned}
\tag{5}
$$

Then in general the evolution equation for the WdW with wormholes, as given by Crowell [2], assuming Q=0 is, if we set $\Psi \approx \Psi_0 \exp(i \cdot k \cdot r)$ and the following functional derivatives of F(r) to obtain the following quadratic equation

$$
\begin{aligned}
G_1(r) &= \frac{2 \cdot \mathrm{M}}{r^2} - \frac{2\Lambda \cdot r}{3} \\
G_2(r) &= -\frac{4 \cdot M}{r^3} - \frac{2\Lambda}{3}
\end{aligned}
\tag{6}
$$

$$
k^2 + \frac{i}{2} \cdot \left[\frac{G_2(r)}{G_1(r)} + \frac{1}{r}\right] \cdot k + G_1(r) \cdot \left[r \cdot R^{(3)} - \left(G_1(r) - \omega^2 \cdot r\right) \cdot \Phi\right] = 0 \tag{7}
$$

Pick

$$
G_3(r) = G_1(r) \cdot \left[r \cdot R^{(3)} - \left(G_1(r) - \omega^2 \cdot r\right) \cdot \Phi\right] \tag{8}
$$

Then, the quadratic equation for Eq.(7) yields having

$$
k = -\frac{i}{2} \cdot \left[\frac{G_2(r)}{G_1(r)} + \frac{1}{r}\right] \cdot \left(1 \pm i \cdot \sqrt{1 - \frac{16 G_3(r)}{\left[\frac{G_2(r)}{G_1(r)} + \frac{1}{r}\right]^2}}\right) \tag{9}
$$

Then the worm hole for the WdW has the following suggested construction, namely, if $\Psi \approx \Psi_0 \exp(i \cdot k \cdot r)$

$$
\Psi \approx \Psi_0 \exp\left(\frac{1}{2} \cdot \left[\frac{G_2(r)}{G_1(r)} + \frac{1}{r}\right] \cdot \left(1 \pm i \cdot \sqrt{1 - \frac{16 G_3(r)}{\left[\frac{G_2(r)}{G_1(r)} + \frac{1}{r}\right]^2}}\right) \cdot r\right) \tag{10}
$$

Depending upon the following, there are some restrictions put upon Λ, i.e. a direct query as to what happens when

$$1-\frac{16G_3(r)}{\left[\frac{G_2(r)}{G_1(r)}+\frac{1}{r}\right]^2}>0$$

or (11)

$$1-\frac{16G_3(r)}{\left[\frac{G_2(r)}{G_1(r)}+\frac{1}{r}\right]^2}<0$$

3. Outlining restrictions the WdW equation by Crowell, puts upon Λ due to Eq.(11)

We look at the two cases outlined below, one < inequality and another a > inequality

3.1 . *What if the first inequality in Eq. (11) holds ?*

We assume, due to elementary analysis that this always is the case ? I.e. breaking it down, it comes to looking at

$$16\cdot G_1(r)^2\cdot\left[r\cdot R^{(3)}-\left(G_1(r)-\omega^2\cdot r\right)\cdot\Phi\right]>\left[-\frac{6\cdot M}{r^3}\right] \tag{12}$$

M is the total mass of space-time in the vicinity of the worm hole. Then the k in Eq. (9) has a real and imaginary component. Consistent with the worm hole needing energy input, i.e. the Hamiltonian does not equal total energy.

3.2. *Second inequality in Eq. (11) is assumed not to hold. No restrictions on the Λ so far.*

We then will look at the sign of real **k** in terms of $\frac{G_2(r)}{G_1(r)}+\frac{1}{r}$.

4. Examining if $\frac{G_2(r)}{G_1(r)}+\frac{1}{r}$ is greater than or less than zero.

$$\frac{\frac{2\cdot M}{r^3}+\frac{\Lambda}{3}}{\frac{2\cdot M}{r^3}-\frac{2\Lambda}{3}}>\frac{1}{2} \tag{13}$$

Then, Λ is then tending to be very small. I.e. this favors the invariant cosmological constant hypothesis, and then would mean that the k would be picked so the real part of the wavefunction Ψ would be shrinking.

$$\frac{\frac{2\cdot M}{r^3}+\frac{\Lambda}{3}}{\frac{2\cdot M}{r^3}-\frac{2\Lambda}{3}}<\frac{1}{2} \tag{14}$$

Then, Λ is then tending to be much larger. I.e. this favors the temperature dependent cosmological constant hypothesis, and then would mean that the k would be picked so the real part of the wavefunction Ψ would be growing.

5. Placing in a model for quintessence, i.e. an update of the Park article.

What is suggested is that a temperature dependence is relevant for the ‘cosmological constant’ when in the vicinity of wormholes, i.e. why is this relevant ? To begin with on the basis of the large cosmological ‘constant’ value so assumed, there would be a large energy flux through a worm hole. In the derivation so done, the energy immediately relevant to a worm hole is by [2] , as by Crowell, given, or stated as:

$$E(effective-energy-wormhole)=\left[r\cdot R^{(3)}-\left(G_1(r)-\omega^2\cdot r\right)\cdot\Phi\right] \tag{15}$$

The full expression is, then

$$\begin{aligned} E(effective-energy-wormhole) &=\left[r\cdot R^{(3)}-\left(G_1(r)-\omega^2\cdot r\right)\cdot\Phi\right] \\ &= r\cdot R^{(3)}+\left[\left(\omega^2+2\cdot\Lambda\right)\cdot r-\frac{2M}{r^2}\right]\cdot\Phi_0\cdot\cos(\omega\cdot t) \end{aligned} \tag{16}$$

As the frequency could be ultra low for matter-energy traversing the worm hole , and curvature $R^{(3)}$ could go to near zero, if the radius r, initially could be very small, and M very large, it then comes down to a MINIMUM value of

$$\Lambda\geq\frac{M}{r^3} \tag{17}$$

In order to keep the wormhole effective energy greater than or equal to 0, then the cosmological 'constant', given small r and huge M would have to be large. So how as to ascertain how to obtain such a value to keep the bounds of Eq.(16) intact?

Ergo, use the following argument. Namely look at the vacuum energy density which is according to [3] equal to

$$\rho_{vacuum-energy}=\frac{\Lambda c^4}{8\pi G} \tag{18}$$

This is also to be compared to the ZPE energy, of say energy density per unit volume of radiation in [4] for frequencies in the interval $\left(\omega,\omega+d\omega\right)$ given by

$$\rho\left(\omega\right)d\omega\doteq\frac{\hbar\cdot\omega^3}{2\pi^2c^3}\cdot\coth\left[\frac{\hbar\omega}{2k_BT}\right]\cdot d\omega \tag{19}$$

Compare and equate Eq. (17) and Eq. (18) and then one has

$$\Lambda\doteq\frac{4\hbar\cdot G\cdot\omega^3}{\pi c^7}\cdot\coth\left[\frac{\hbar\omega}{2k_BT}\right] \tag{20}$$

If one makes the following identification, then one has , if

$$\hbar\omega\approx E_{Temperature}\doteq\frac{3}{2}k_BT \tag{21}$$

Then,

$$\Lambda\doteq\frac{36\cdot G\cdot\left(k_BT\right)^3}{8\cdot\pi c^7\hbar^3}\cdot\coth\left[\frac{3}{4}\right] \tag{22}$$

If so, comparing it to the Park value of [5]

$$\Lambda_{Park(2002)}\doteq c_1\cdot T^{\beta} \tag{23}$$

$$c_1\doteq\frac{36\cdot G\cdot\left(k_B\right)^3}{8\cdot\pi c^7\hbar^3},\ \beta=3 \tag{24}$$

The values so obtained are assuming a vacuum energy as equivalent to energy density of radiation, and in doing so, one has, in the case of wormholes, due to Eq.(17) a statement about temperature dependence, and Λ

6. Conclusions

We in making this inquiry have come up with one very specific result. I.e. for a worm hole, the Eq.(17) is a minimum condition for keeping a nonnegative energy for the wormhole, and this requires that there be a cosmological 'constant' significantly larger than the cosmological 'constant' parameter chosen in present day cosmology. Notice some numerology, the radius r and mass M, are for the wormhole considerably less than the two values for the radius of the universe, and the mass of the universe, namely given as scaling as mentioned by Valev [6], with his *H*, not a Hamiltonian, but instead the Hubble expansion rate *H*

$$r(radius - of - universe) \sim cH^{-1} \tag{25}$$

Also, the mass of the Universe, as given by Valev [6] is

$$M = (\text{Mass} - of - universe) \sim c^3 \cdot 2^{-1} \cdot \left(G \cdot H\right)^{-1} \tag{26}$$

These is here, two uppermost values of r and M in Eq. (25) and Eq.(26) many orders of magnitude larger than what is used in Eq. (17), with the following consequence. For large scale structures, Eq.(17) does not apply and there is a uniform low value 'cosmological constant'. For the scope of a worm hole, values ten to the minus 40 or so power smaller apply, for r and M, and so Eq.(17) is making the case for a temperature dependent , significantly larger than the present day value for the 'cosmological constant'. We hope that in the future, more specific bounds are created which allow for refinement of Eq.(17) plus an understanding of what magnitude of space time mass and radii are necessary for the breakdown of Eq.(17) so that we have a precise delineation of when Eq.(22) to Eq.(24) apply and do not apply in cosmological astrophysical analysis. This also may entail expanding upon issues brought up by Darabi and Jalalzadeh [7] as to relations to a holographic principle, which will be delved into in the follow up of this article . Note that we are assuming gravitational waves , as given by , Maggiore[8] that would be the frequency as brought up for space-time 'information' to be routed through this wormhole. If a large cosmological 'constant' due to Eq. (17) is employed, then the tunneling probability is nearly 'unity', or 100% , meaning that the transit of GW 'information' from the mouth to the exit of a wormhole would then be effectively unimpeded in Eq.(4). The follow up will also have background in entanglement and entropy also used, as well [9]

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