Limitations of Perturbative Renormalization and the Challenges of the Standard Model

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Abstract

In contrast with the paradigm of effective Quantum Field Theory (EFT), realistic Renormalization Group (RG) flows approaching fixed points are neither perturbative nor linear. We argue that overlooking these limitations is necessarily linked to many unsolved puzzles challenging the Standard Model of particle physics (SM). Here we show that the analysis of non-linear attributes of RG flows near the electroweak scale can recover the full mass and flavor structure of the SM. It is also shown that this analysis brings closure to the "naturalness" puzzle without impacting the cluster decomposition principle of EFT.

Key words: Renormalization Group, Standard Model, Effective Field Theory, Naturalness.

1. Introduction

In his 1979 seminal paper on "Phenomenological Lagrangians" [1], Steven Weinberg has formulated the fundamental principles that any sensible EFT must comply with in order to successfully explain the physics of the subatomic realm: Quantum Field Theory (QFT) has no content besides *unitarity*, *analyticity*, *cluster decomposition* and *symmetries*. This conjecture implies that, in order to compute the *S*-matrix for any field theory below some scale, one must use the most general effective Lagrangian consistent with these principles expressed in terms of the appropriate asymptotic states [2].

Closely related to Weinberg's conjecture are two key aspects of EFT that deal with the separation of heavy degrees of freedom from the light ones [3]. One is the *Decoupling Theorem* (Appelquist-Carrazone) stating that the effects of heavy particles go into local terms in a field theory, either renormalizable couplings or in non-renomalizable effective interactions suppressed by powers of the heavy scale. The other is *Wilson's Perturbative Renormalization Program* [4] who teaches how to separate the degrees of freedom above and below a given scale and then to integrate out all the high-energy effects and form a low-energy field theory with the remaining degrees of freedom below the separation scale.

The idea of scale separation in EFT is typically illustrated by considering the perturbative expansion of amplitudes in powers of momenta Q over a large scale Λ_{UV}, the latter setting the upper limit of validity for the EFT [2, 5]

$$M(Q/\mu, g_n, \Lambda_{UV}) = \sum_{\rho} (Q/\Lambda_{UV})^{\rho} f(Q/\mu, g_n) \tag{1}$$

Here, μ represents the RG scale, g_n are the low-energy couplings, the function f is of order unity $O(1)$ (expressing the "*naturalness*" of the theory) and the summation index ρ is bounded from below. The contribution of the large scale is naturally suppressed as $\Lambda_{UV} >> Q$.

In this work we re-examine Wilson's Renormalization ideas as traditionally viewed from the standpoint of EFT. The motivation stems from the fact that, although a fully consistent and well supported theoretical framework, the SM continues to be plagued by numerous conceptual challenges [6, 7]. Our basic premise is that realistic Renormalization Group (RG) flows approaching fixed points cannot be restricted to be either *perturbative* or *linear*. We argue herein that imposing these upfront restrictions is inevitably linked to the many challenges left unanswered within the SM. It is shown that the analysis of non-linear attributes of RG flows near the electroweak scale can recover the complete mass and flavor structure of the SM. It is also shown that this analysis brings closure to the "*naturalness*" puzzle without impacting the principle of scale separation of EFT.

The structure of the paper is as follows: Section two details the general construction and limitations of the RG program, with emphasis on the conclusion that non-renormalizable interactions vanish at the low energy scale. The idea of dimensional regularization and its implications on the emergence of fractal space-time in QFT form the topic of section three. A pointer to references that discuss the utility of fractal space-time in solving some of the main challenges confronting the SM is included in the last section.

2. Limitations of the RG program

As local QFT residing on Minkowski spacetime is expected to break down at very short distances due to (at the very least) quantum gravity effects, any physically sensible theory must include a high-energy cutoff (Λ_0). The *continuum limit* is defined by a cutoff approaching infinity ($\Lambda_0 \to \infty$). To simplify the presentation we follow [4] and consider a local scalar field theory in four dimensional spacetime where all field modes above some arbitrary momentum scale $\Lambda < \Lambda_0$ have been integrated out. The Lagrangian of such an effective theory assumes the form

$$L_\Lambda = \sum_n a_n(\Lambda) O_n(\varphi_\Lambda) \tag{2}$$

where $O_n(\Lambda)$ represent the set of local field operators, including their spacetime derivatives, and $a_n(\Lambda)$ the set of coupling parameters. If $O_n(\Lambda)$ have mass dimensions $4-d_n$, $a_n(\Lambda)$ carry mass dimensions d_n and one can cast all couplings in a dimensionless form as in

$$g_n(\Lambda) = a_n(\Lambda)\Lambda^{-d_n} \tag{3}$$

The behavior of local operators $O_n(\Lambda)$ depends on their mass dimensions: relevant operators correspond to $d_n > 0$, marginal operators to $d_n = 0$ and irrelevant operators to $d_n < 0$. All mass dimensions are assumed to be scale independent. Since Λ is arbitrary, we may fix the dimensionless couplings (3) at some reference scale chosen to lie in the deep ultraviolet region and yet far enough to the cutoff, say $\Lambda_{UV} < \Lambda_0$

$$\overline{g_n} = g_n(\Lambda_{UV}) \tag{4}$$

The flow of the coupling parameters with respect to a sliding RG scale $\mu < \Lambda_{UV}$ is then described by the system of partial differential equations

$$\mu \frac{\partial}{\partial \mu} g_n(\mu) = \beta_n(g_n; \mu/\Lambda_{UV}) \tag{5}$$

The above flow equations imply that the couplings measured at the sliding scale μ depend on the high-energy parameters $\overline{g_n}$ and on the ratio μ/Λ_{UV} as in

$$g_n(\mu) = g_n(\overline{g_n}; \mu/\Lambda_{UV}) \tag{6}$$

We assume below that there are N relevant and marginal operators with mass dimensions less than or equal to 4. The operators belonging to this set are denoted by the Roman indices $a,b,...$, whereas the irrelevant operators with dimension greater than 4 are indicated by Greek indices $\alpha,\beta....$ The Roman characters $m,n,r...$ describe the general set of operators and couplings.

It can be shown that in the regime of ***weakly coupled perturbation theory***, the RG flow (5) projects an arbitrary initial surface in the UV coupling space $\{\overline{g_n}\}$ to a N - dimensional surface of $\{g_n(\mu)\}$, a given point of which is uniquely specified by N low-energy parameters, up to corrections that decay as inverse powers of the ratio μ/Λ_{UV} [4]. The proof relies exclusively on a ***linear stability analysis*** of flow equations (5) and leads to the following relationships, valid for $\mu << \Lambda_{UV}$

$$\boxed{\delta g_\alpha(\mu) \sim G_{\alpha a} G_{ab}^{-1} \delta g_b(\mu) + O(\delta^* g_\alpha)} \tag{7}$$

where

$$\delta^* g_\alpha \sim (\frac{\mu}{\Lambda_{UV}})^{|d_\alpha|} \tag{8}$$

As mentioned above, α denotes the index of irrelevant couplings and operators present in the theory. Here, $\delta^* g_\alpha$ represents the set of first order variations in the irrelevant couplings

$$\delta^* g_\alpha(\mu) = \delta g_\alpha(\mu) - G_{\alpha a} G_{ab}^{-1} \delta g_b(\mu) \tag{9}$$

The matrix $G_{nm}(\mu)$ defines the variation of the low-energy parameters g_n under variations of the initial high-energy parameters $\overline{g_m}$ specified by (4), that is,

$$G_{nm}(\mu) = \frac{\partial g_n(\mu)}{\partial \overline{g_m}(\mu)} \tag{10}$$

The finite $N \times N$ sub-matrix G_{ab} contains rows and columns restricted to the marginal and relevant couplings. *Relation (7) states that the contribution of irrelevant couplings and operators at low energy (indexed by* α *) may be entirely absorbed in variations of the marginal and relevant couplings (indexed by b).*

Despite being rigorously derived, (7) is founded on a set of *simplifying assumptions* which disqualifies it from being a universal result. In particular,

1) The matrix G_{ab} is constrained to be *nonsingular*, which fails to be true for isolated sets of measure zero in coupling space [4].

2) The theory is considered *weakly coupled* to make the perturbation analysis applicable [4].

3) The linear stability of the flow equations is assumed to hold true in general. With reference to planar flows, this is a legitimate approximation only if the fixed points do not fall in the category of *borderline equilibria* (such as centers, degenerate nodes, stars or non-isolated attractors or repellers) [8]. Examples of such non-isolated fixed points are discussed in [9-12]

4) The flow equations are assumed to correspond to Markov processes, that is, they are *immune to memory effects* [13].

5) Bound states are excluded from this approach, as they require an entirely *non-perturbative treatment* [4].

It is somehow surprising that many QFT textbooks do not explicitly point out the limitations that these assumptions place on the validity of field theories in general. The widespread belief is that they do not appear to directly impact the cluster decomposition principle and all SM predictions up to the low-TeV scale probed by the LHC. However, in light of all unsettled questions confronting the SM, one cannot help but wonder if some important piece of the puzzle is not lost in overlooking these limitations. For example, over past decades the prevailing consequence of the concept of "*naturalness*" for model building has been the cancellation of quadratic divergences to the SM Higgs mass [14]. According to this paradigm, the SM itself is an unnatural theory, mandating new physics somewhere near the low-TeV scale. At the same time the LHC, flavor physics, electroweak precision results and evaluation of the electron dipole moment all point to the absence of any new phenomena in this range, which is however necessary to accommodate the observation of both neutrino oscillations and cold Dark Matter [14].

It seems that a paradigm shift is clearly needed to understand both the SM and the physics lying beyond it. Tackling this challenge from a novel perspective on the RG program forms the topic of the next two sections.

3. Continuum field theory as "effective" model of spacetime

A rather counterintuitive outcome of field theory is that the exact continuum limit of a local QFT formulated on flat spacetime has, strictly speaking, *no correlate to physical reality* [4]. The Minkowski metric of Special Relativity underlies the most basic aspect of QFT, namely the space-like commutativity of local observables, yet is considered only an "emergent" phenomenon and an approximate description of an underlying fundamental theory. But the basis for such a theory is currently far from being settled, despite claims to the contrary made by *asymptotically safe* and *UV complete models*.

It is instructive to recall that, in the context of perturbative RG, the idea of *continuous dimension* for a four-dimensional spacetime ($D = 4 - \varepsilon$, with $\varepsilon << 1$) was first introduced by Wilson and Fisher and initially used to compute physical quantities of interest as expansions in powers of the dimensional parameter ε. Later on, Veltman and ’t Hooft have shown how this idea can be incorporated in QFT and developed into a reliable renormalization technique. The connection between dimensional and cutoff regularizations is given by

$$\log \frac{\Lambda_{UV}^2}{\mu^2} = \frac{2}{\varepsilon} - \gamma_E + \log 4\pi + \frac{5}{6} \qquad (11)$$

We find it convenient to present (11) is a slightly different form, that is,

$$\varepsilon \sim \frac{1}{\log(\frac{\Lambda_{UV}^2}{\mu^2})} \qquad (12)$$

It is apparent from (12) that the four-dimensional space-time is recovered in either one of these limits:

a) $\Lambda_{UV} \to \infty$ and $0 < \mu << \Lambda_{UV}$,

b) $\Lambda_{UV} < \infty$ and $\mu \to 0$.

However, both limits are in conflict with our current understanding of the far UV and the far IR boundaries of field theory. Theory and experimental observations alike tell us that the notions of infinite *or* zero energy are, strictly speaking, meaningless. This is to say that either infinite energies (point-like objects) or zero energy (infinite distance scales) lead to divergences whose removal requires the machinery of the RG program. Indeed, there is always a finite cutoff at both ends of either energy or energy density scale (far UV = Planck scale, far IR = finite radius of the observable Universe or the non-vanishing energy density of the vacuum set by cosmological constant). It follows from these considerations that the limit $\varepsilon \to 0$ works as a highly accurate approximation and realistic models near or beyond the SM scale must account for space-time geometries having continuous dimensionality. Fractal space-time defined by the continuous dimension $D = 4 - \varepsilon$ asymptotically approaches ordinary space-time near or below the SM scale, that is, for $\mu \leq \mu_{SM}$.

4. Toward a resolution of the SM challenges

Refs. [15-18] describe how the concept of fractal space-time defined by $D = 4 - \varepsilon$ can be used to bring closure to some of the main challenges left open by the SM.

We end our paper with the key observation that, since the continuum field theory is only an "effective" space-time model, the effects induced by the dimensional parameter $\varepsilon = 4 - D$, with $\varepsilon << 1$, *are not perceivable* in the computation of scattering amplitudes (1) at the SM scale. With reference to (12), the condition $\varepsilon << 1$ is equivalent to setting $\mu = \mu_{SM} = O(Q) << \Lambda_{UV}$ and the contribution of ε becomes strongly suppressed by the power expansion (1). *As a result, the cluster decomposition principle of EFT remains insensitive to the emergence of fractal space-time near or above the SM scale (* $\mu \geq \mu_{SM}$ *).*

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