Standard Model Gauge Groups
Matter & the Dark Sector

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Abstract

In the paper [1] it was shown that extending Riemann geometry to include an asymmetric metric resulted in a calculation of 6d for these spaces. The spinors on these spaces are 4d Weyl-Majorana and are now considered to be spin 1/2. It is shown that there are 24 spin 1/2 matter states, chiral -1 SU(4) is emergent which via spontaneous symmetry breaking results in the chiral Electroweak or SU(3) gauge groups. The SU(3) right handed i.e. chiral +1 gauge group is also emergent. The chiral gauge groups SU(4) and SU(3) emerge after inflation ends with the emergence of (1,3) space. Dark matter and Dark energy are also emergent from the tensor products and tensor sums of the 6d vector spaces (3,3), (4,2), (1,5). The cosmological density ratios are Dark energy 0.6831, Dark Matter 0.2676 and Matter 0.0493 which are in agreement with the Planck Mission results [2]. Our Universe is just one of many outcomes of inflation and tensor products and tensor sums of these 3 6d vector spaces.

1 INTRODUCTION

In [1], the extension of Riemann geometry by including an asymmetric metric resulted in the determination of the number of dimensions to be 6. Let p,q be n-tuples, then the 6d vector spaces with dim p x dim q $\neq$ 0 and dim p + dim q = 6 are (p,q) $\in$ { (3,3), (4,2), (1,5) } with 6 4d irreducible Majorana-Weyl Spinors of spin s=1/2 and chirality (-1,1). These vector spaces will be referenced as the fundamental vector spaces.
It follows that there are 24 matter spin 1/2 states, in agreement with the Standard model.
It will be shown that the Lie gauge groups of the Standard Model, Dark matter and Dark energy are a consequence of tensor products of the 3 fundamental vector spaces and tensor sums of these product spaces.

2 STANDARD MODEL GAUGE GROUPS

The gauge spaces are of the form (1,3)×(p,p)
Form the following tensor product spaces of the fundamental vector spaces:

$$(3,3)\otimes(4,2) = \{ (3,2), (4,3), (7,0), (0,6) \}$$
$$(1,5)\otimes(4,2) = \{ (1,2), (4,5), (5,0), (0,10) \}$$
$$(3,3)\otimes(1,5) = \{ (1,3), (3,5), (0,8), (4,0) \}$$

2 space-like or time-like, p-tuple and r-tuple, $(p)\otimes(r)=(p,r)=(p+r)$-tuple. The first 2 product spaces of each set have chiral -1 spin 1 spinors and the remaining 2 have chiral +1 spin 1 spinors.

Tensor sums of the chiral -1 vector spaces are

$$(3,2)\oplus(1,2)\oplus(1,3) = (5,7)$$
$$(4,3)\oplus(4,5)\oplus(3,5) = (11,13)$$

Inflation of (1,3), results in the following gauge spaces

$$\begin{pmatrix} (5,7) \\ (11,13) \end{pmatrix} \rightarrow \begin{pmatrix} (1,3)\times(4,4) \\ (1,3)\times(10,10) \end{pmatrix}$$

Complexification of (4,4) forms a $(\mathbb{C}^4, \mathbb{C}^4)$, a pair of 4d complex spaces.
It follows that spin 1/2 chiral -1 spinors are gauge invariant under the Lie group SU(4), the gauge group acting on chiral -1 spinors will be denoted as $SU_L(4)$.

For chiral +1 spinors, the only gauge space is from the tensor sum $(4,0)\oplus(0,6) = (4,6)$
Inflation of (1,3) results in the following gauge space:

$$(4,6) \rightarrow (1,3) \times (3,3)$$

Complexification of (3,3) forms a $(\mathbb{C}^3, \mathbb{C}^3)$, a pair of 3d complex spaces.
It follows that spin 1/2 chiral +1 spinors are gauge invariant under the Lie group SU(3), this gauge group acting on chiral +1 spinors will be denoted as $SU_R(3)$

Spontaneous symmetry breaking of $SU_L(4)$

The Higgs et al mechanism [3] has the following relation for the number of Higgs scalar bosons H

$$H=2n-N+M$$

where 2n is the real dimension of a n-dimensional complex scalar, N is the dimension of the gauge group and M is the dimension of the sub-group.
Assume there is only 1 Higgs boson per complex scalar field, H=1 and dim $SU_L(4) = 15$
It follows that there are only 2 sub-groups as follows:
2n=8,12, complexification of (0,4) and (0,6) to form $\mathbb{C}^4$, $\mathbb{C}^6$ complex scalars results in

$$\begin{pmatrix} \mathbb{C}^4 \\ \mathbb{C}^6 \end{pmatrix} SU_L(4) \rightarrow \begin{pmatrix} SU_L(3) + 7\ massive\ bosons \\ SU_L(2) \times U_Y(1) + 11\ massive\ bosons \end{pmatrix}$$

Hence the Standard model gauge groups are

$$SU_L(3) \times SU_R(3) \times SU_L(2) \times U_Y(1)$$

where $SU_L(3) \times SU_R(3) = SU_c(3)$ is the QCD gauge group [4] and $SU_L(2) \times U_Y(1)$ is the chiral Electroweak gauge group [5]. Thus the gauge groups of the Standard Model emerge from symmetry breaking after inflation of (1,3).

Spontaneous symmetry breaking of $\boldsymbol{SU_L(2) \times U_Y(1)}$
Inflation of (3,5) results in the following gauge space:

$$(3, 5) \rightarrow (1, 3)\,(2, 2)$$

The complexification of (2,2) results in $(\mathbb{C}^2, \mathbb{C}^2)$, a pair of 2d complex scalar fields, the Higgs fields. The Higgs mechanism breaks the Electroweak gauge group [3]:

$$SU_L(2) \times U_Y(1) \rightarrow SU_{WL}(2) \times U_{em}(1)$$

The Electroweak interaction spontaneously breaks down to the Weak and Electromagnetic interactions.

3 DARK MATTER

The tensor sum of the chiral -1 vector spaces is (16,20) and the tensor sum of the chiral +1 vector spaces is (16,24). Dark matter is formed from the following set of vector spaces:

$$\{ (16,20), (16,24), (16,20)\otimes(16,24) \} = \{ (16,20), (16,24), (32,0), (16,24), (16,20), (0,44) \}$$

Complexification of these spaces results in 228 complex scalars, which constitutes dark matter.

4 DARK ENERGY

Dark energy is the asymmetric metric states after inflation. The fundamental vector spaces have total of 3 asymmetric metrics. The tensor sum of the vector spaces (5,0)⊕(0,8) = (5,8) and (7,0)⊕(0,10) = (7,10) form gauge spaces after inflation of the form (0,3)x(p,p) as follows

$$\begin{pmatrix} (5, 8) \\ (7, 10) \end{pmatrix} \rightarrow \begin{pmatrix} (0, 3) \times (5, 5) \\ (0, 3) \times (7, 7) \end{pmatrix}$$

Complexification of (5,5) and (7,7) results in the pair of complex spaces $(\mathbb{C}^5, \mathbb{C}^5)$ and $(\mathbb{C}^7, \mathbb{C}^7)$ with the special unitary groups SU(5) and SU(7). The number of massless spin 1 states is (24*2+48*2)*2=288. The tensor sum of (0,3) vector space with (3,0) which is a subspace of (5,5) and (7,7) forms a 6d space with an asymmetric metric.
It follows there are a total of 291 asymmetric metric states, and that Dark energy being a massless spin 2 field has a degeneracy of 582.

5 COSMOLOGICAL DENSITY RATIOS

When Matter, Dark matter and Dark energy are in thermal equilibrium, Fermi-Dirac and Bose-Einstein statistics is applicable. Matter has degeneracy 48, dark matter degeneracy 228 and dark energy degeneracy 582. Using Fermi-Dirac and Bose-Einstein statistics, the ratio of matter to dark matter is

$$\frac{\Omega_b}{\Omega_d} = \frac{7}{38}$$

and the ratio of dark energy to matter is

$$\frac{\Omega_\Lambda}{\Omega_b} = \frac{97}{7}$$

With the total energy density after inflation at critical density, the ratios satisfy the condition

$$\Omega_\Lambda + \Omega_d + \Omega_b = 1$$

Solving these 3 equations gives the cosmological density ratios

$\Omega_\Lambda = 0.6831$, $\Omega_d = 0.2676$, $\Omega_b = 0.0493$ which are in agreement with the Planck Mission results [2]

6 COMPLEXIFICATION

Complexification of the vector spaces is achieved automatically if the fundamental vector spaces are over the complex numbers instead of the real numbers. The (1,3) spaces become complex spaces, $\mathbb{C}^{1,3}$ is the tensor sum of (1,3)⊕(1,3) over the real numbers. It follows that there are 2 Lorentz groups SO(1,3). The significance of these 2 Lorentz groups requires further investigation.

7 CONCLUSION

Extending Riemann geometry by including an asymmetric metric results in a determination of 6d for these spaces [1]. The Standard model gauge groups, dark matter and dark energy are emergent from the fundamental vector spaces { (3,3), (4,2), (1,5) }. Only chiral -1 Electroweak gauge group emerges while the QCD gauge group is achiral. Complexification is not required if the fundamental vector spaces are complex vector spaces.

References

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