Two conjectures that relates any Poulet number by a type of triplets respectively of duplets of primes

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Abstract. In one of my previous papers I conjectured that there exist an infinity of Poulet numbers which can be written as the sum of three primes of the same form from the following eight ones: 30k+1, 30k+7, 30k+11, 30k+13, 30k+17, 30k+19, 30k+23, 30k+29. In this paper I conjecture that any Poulet number not divisible by 5 can be written as a sum of three primes of the same form from the following four ones: 30k+1, 30k+3, 30k+7 or 30k+9 respectively as a sum from a prime and the double of the another one, both primes having the same form from the four ones mentioned above. Finally, I yet made any other two related conjectures about two types of squares of primes.

Conjecture 1:

Any Poulet number P not divisible by 5 can be written at least in one way as P = m + n + q, where m, n, q are primes, not necessarily all three distinct, of the same form from the following four ones: 30k+1, 30k+3, 30k+7 or 30k+9.

Note that the primes m, n, q can't be all three equal, because there is no a Poulet number divisible by 3 with only two prime factors.

Verifying the conjecture:
(for the first Poulet number from each of the forms 10*k + 1, 10*k + 3, 10*k + 7 respectively 10*k + 9)

For P = 341, we have:
: 341 = 7 + 17 + 307 = 7 + 107 + 227 = 7 + 137 + 197 = 7 + 167 + 167 = 17 + 17 + 307 = 17 + 47 + 277 = 17 + 67 + 257 = 17 + 97 + 227 = 17 + 127 + 197 (...).

For P = 1387, we have:
: 19 + 79 + 1289 = 19 + 89 + 1279 = 19 + 109 + 1259 = 19 + 139 + 1229 = 19 + 229 + 1129(...).

For P = 1729, we have:
: 13 + 23 + 1693 = 13 + 53 + 1663 = 13 + 103 + 1613 = 13 + 163 + 1553 13 + 173 + 1543 (...).

For P = 4033, we have:
: 11 + 331 + 3691 = 11 + 661 + 3391 = 11 + 691 + 3331(...).

Conjecture 2:

Any Poulet number P not divisible by 5 can be written at least in one way as P = 2*m + n, where m and n are distinct primes of the same form from the following four ones: 30k+1, 30k+3, 30k+7 or 30k+9.

Verifying the conjecture:
(for the first eight Poulet numbers not divisible by 5)

For P = 341, we have:
: 341 = 2*167 + 7 = 2*17 + 307 = 2*107 + 127 (...).
For P = 561, we have:
: 561 = 2*7 + 547 = 2*37 + 487 = 2*47 + 467 (...).
For P = 1387, we have:
: 1387 = 2*79 + 1229 = 2*139 + 1109 (...).
For P = 1729, we have:
: 1729 = 2*73 + 1583 = 2*103 + 1523 (...).
For P = 2047, we have:
: 2047 = 2*79 + 1889 = 2*379 + 1289 (...).
For P = 2701, we have:
: 2701 = 2*79 + 1889 = 2*379 + 1289 (...).
For P = 2821, we have:
: 2821 = 2*79 + 1889 = 2*379 + 1289 (...).
For P = 3277, we have:
: 3277 = 2*79 + 1889 = 2*379 + 1289 (...).

Comment:

Because sometimes seems to me that Poulet numbers behaves like squares of primes (though the only squares of primes that are Poulet numbers are the two known Wieferich primes) I also make two conjectures about primes.

Conjecture 1:

Any square of a prime of the form p^2 = 10*k + 1 can be written as p^2 = m + n + q, where m, n, q are primes, not necessarily all three distinct, of the form 10*k + 7.

Examples:
: 11^2 = 121 = 37 + 37 + 47; 19^2 = 361 = 7 + 37 + 317.

Conjecture 2:

Any square of a prime of the form p^2 = 10*k + 9 can be written as p^2 = m + n + q, where m, n, q are primes, not necessarily all three distinct, of the form 10*k + 3.

Examples:
: 7^2 = 49 = 13 + 13 + 23; 13^2 = 169 = 13 + 43 + 113.