A very interesting formula of a subset of Poulet numbers involving consecutive powers of a power of two

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Abstract. I studied Poulet numbers for some time but I’m still amazed by the wealth of the patterns that this set of numbers offers; it’s like everything that the prime numbers, in their stubbornly to let themselves understood and disciplined, refuse us, these exceptions of the Fermat’s “little theorem” allow us. This paper states a conjecture about a new subset of Poulet numbers that I discovered by chance.

Conjecture:
There is an infinity of Poulet numbers $P$ of the form $P = ((2^n)^k)((2^n)^{k + 1} + 2^n + 1) + 1$, where $k$ is a non-null positive integer, for any $n$ non-null positive integer.

Examples:
For $n = 1$ the formula becomes $P = 2^k(2^{k + 1} + 3) + 1$ and we obtained:
- $2^{4*}(2^5 + 3) + 1 = 561 = 3*11*17$, a Poulet number;
- $2^{7*}(2^8 + 3) + 1 = 33153 = 3*43*257$, a Poulet number.

For $n = 2$ the formula becomes $P = 4^k(4^{k + 1} + 5) + 1$ and we obtained:
- $4^{2*}(4^3 + 5) + 1 = 1105 = 5*13*17$, a Poulet number;
- $4^{3*}(4^4 + 5) + 1 = 16705 = 5*13*257$, a Poulet number.

For $n = 4$ the formula becomes $P = 16^k(16^{k + 1} + 17) + 1$ and we obtained:
- $16^{1*}(16^2 + 17) + 1 = 4369 = 17*257$, a Poulet number;
- $16^{2*}(16^3 + 17) + 1 = 1052929 = 17*241*257$, a Poulet number;
- $16^{3*}(16^4 + 17) + 1 = 268505089 = 17*241*65537$, a Poulet number.
For $n = 6$ the formula becomes $P = 64^k(64^k + 1) + 65 + 1$ and we obtained:
\[64^1(64^2 + 65) + 1 = 266305 = 5\times 13\times 17\times 241\text{, a Poulet number.}\]

For $n = 7$ the formula becomes $P = 128^k(128^k + 1) + 129 + 1$ and we obtained:
\[128^1(128^2 + 129) + 1 = 2113665 = 3\times 5\times 29\times 43\times 113\text{, a Poulet number.}\]

For $n = 8$ the formula becomes $P = 256^k(256^k + 1) + 257 + 1$ and we obtained:
\[256^1(256^2 + 257) + 1 = 16843009 = 257\times 65537\text{, a Poulet number.}\]