Color-Flavor Locking, Constituent Quarks and Quantum Chromodynamics

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Abstract

The Color-Flavor locking phenomenon is predicted to occur at ultra-high densities in Quantum Chromodynamics. In this paper, we show that it does actually exist in a particular quark model scenario at low energies. This then leads to a proper understanding of how a constituent quark, in contrast to a current quark, arises. This shows that increasing flavour from $SU(2)_F$ to $SU(3)_F$ is a non-trivial physical extension with its unique physical implications. As to the $SU(3)_F$ symmetry breaking, it predicts that the symmetry when it first appears, is already intrinsically broken, providing us with masses $m_s \rangle \; (m_u = m_d)$, as the constituent quarks as input for the fundamental representation.

Keywords : Non-relativistic Quark Model, Constituent Quarks, Quantum Chromodynamics, Color-Flavor Locking

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Color-Flavor locking (CFL) is a phenomenon predicted to occur in Quantum Chromodynamics (QCD) at ultra-high densities in hadronic matter. Herein, colour properties are correlated with the flavour properties in a one-to-one correspondence between the three colours and the three flavours [1,2]. However, drastic changes, as a consequence of CFL, are predicted to occur; for example, the gluons become electrically charged and the quark charges are also shifted.

In this paper, we however show that, remarkably, there is a phenomenologically consistent color-flavor locking occurring at normal densities in a particular quark model itself. This, however does not require any drastic changes (like charged gluons, etc) in the structure of QCD. As we shall show below, interestingly, this color-flavor locking leads also to an explanation of what the constituent quarks (in contrast to the current quarks) are. It also predicts an intrinsically broken $SU(3)_F$ symmetry as the very starting symmetry in the quark model.

In the standard quark model

$SU(6)_{SF} \otimes SU(3)_C \supset SU(3)_F \otimes SU(2)_S \otimes SU(3)_C$, the nucleonic ground state is

$$|N_S\rangle = \frac{1}{\sqrt{2}} \left(\chi^\rho \phi^\rho + \chi^\lambda \phi^\lambda \right) \psi^0_{00S} \tag{1}$$

where

$$\phi^\lambda_p = -\frac{1}{\sqrt{6}} \left(udu + duu - 2uud \right) \tag{2}$$

$$\phi^\rho_p = \frac{1}{\sqrt{2}} \left(udu - duu \right) \tag{3}$$

and similarly for the corresonding spin wave functions too.

The colour antisymmetric part is

$$\frac{1}{\sqrt{6}} \left[\left(BG - GB \right) R + \left(GR - RG \right) B + \left(RB - BR \right) G \right] \tag{4}$$

The above displays the full three quark exchange symmetry. However, surprisingly, there exists a truncated and severely curtailed quark model wave function which gives good and consistent results for one body operators in the quark model [4,5,6]. We will call it the Truncated Quark Model (TQM). In this TQM, for example, for proton state is,

$$p_{\uparrow} = \frac{1}{\sqrt{2}}\,(uud)\,\chi_{\uparrow}^{\lambda}\psi^{0} = \left[\frac{1}{\sqrt{2}}(uu)_{0}^{1}d^{\uparrow} - \frac{\sqrt{2}}{\sqrt{3}}(uu)_{1}^{1}d^{\downarrow}\right]\psi^{0} \tag{5}$$

where ψ^0 is the L=0 orbital state and the identical uu quark-pair couples to isospin 1 and hence S=1. This wave function is symmetric only in the first two labels while the third one has no particular symmetry with respect to the other two labels. As mentioned by Close [4], "I cannot overemphasise the crucial, hidden role that colour plays here in getting the flavour-spin correlations right." However, clearly the fully colour anti-symmetric wave function above (eqn. (4)) cannot provide the colour anti-symmetry for the above wave function, which is symmetric only in the first two quark labels. In fact, for the same TQM, what Lichtenberg [6, p. 236] says about the colour, is quite different, "In writing these degrees of freedom we shall omit the color degrees of freedom .... ". Hence, it is not at all clear as to how colour wave function arises in this scenario and what kind of wave function does it actually provide. This has been one of the puzzling weaknesses of the TQM wave function. However, clearly we cannot neglect colour in the wave function and solving this puzzle is one of the purpose of this paper.

Let us now list the wave function of the spin half octet baryons in TQM. There are six pairs of identical quarks in $SU(3)_F$ model, which in this TQM model are given as follows [4,5,6]

$$P \sim (uu)^1 d; N \sim (dd)^1 u; \Sigma^+ \sim (uu)^1 s;$$

$$\Sigma^- \sim (dd)^1 s; \Xi^0 \sim (ss)^1 u; \Xi^- \sim (ss)^1 d \tag{6}$$

The remaining pair is Σ^0&Λ^0 both with uds quark flavour content. In Σ^0 (ud) has I=1 and hence S=1 and so it is $(ud)^1 s$. For Λ^0 (ud) has I=0 and hence S=0, and thus it is $(ud)^0 s$.

These wave functions do very well for one-body operators like electric charge, magnetic moment, etc. as discussed in literature [4,5,6]. However, due to its curtailed symmetry of just two states (which is in conflict with the full three particle symmetry in the successful $SU(6)_{SF}$ quark model) and due to the conflict with the totally antisymmetric $SU(3)_C$ state, this model is not discussed much in literature. However, as we have seen, that the TQM works pretty well and so the question, as to why is it so, remains unanswered. We provide here a physically and mathematically consistent answer which reveals a deeper hidden symmetry in quark model and which also tells us that the extension from SU(2) symmetry to higher symmetry group SU(3) is not as trivial as just adding one more state to the corresponding fundamental vector-state.

For mesons the colour goes as $3_C \otimes \bar{3}_C = 1 + 8$. The colour singlet state is

$$\frac{1}{\sqrt{3}}\left(\bar{R}R + \bar{G}G + \bar{B}B\right) \tag{7}$$

and the octet states are

$$\bar{G}R, \bar{B}R, \bar{R}G, \bar{B}G, \bar{R}B, \bar{G}B, \frac{1}{\sqrt{2}}\left(\bar{R}R - \bar{G}G\right), \frac{1}{\sqrt{6}}\left(\bar{R}R + \bar{G}G - 2\bar{B}B\right) \tag{8}$$

Here the singlet is associated with the mesons in $SU(3)_F$ and the octet of this colour representation is discarded as irrelevant. Thus the octet colour state arises as a completely spurious state.

Note that it is a property of SU(3) that

$$\bar{B} \sim RG - GR; \bar{G} \sim BR - RB; \bar{R} \sim GB - BG \tag{9}$$

That is the anticolour state is represented as antisymmetric product of the other two colours.

Now in a baryon first we have, $3_C \otimes 3_C = \bar{3} + 6$ and then the above anticolour trilet, being equivalent to the antisymmetric product of two colours as given in eqn. (9), is used to obtain, $3_C \otimes 3_C \otimes 3_C = 1 + 8 + 8 + 10$.

In the baryon case above, one again associates the colour antisymmetric singlet to go with the $SU(6)_{SF}$ symmetric wave functions. So for the baryons, are these nonsinglet states again discarded? No, the colour-octet above contibutes to the singlet state of the six quarks as the hidden colour components for B=2; and the colour-decuptet plays its part in baryon number B=3 and B=4 nuclei, to explian interesting physical aspects of these nuclei [7]. And thus these nonsinglet states of the baryons play significant roles in hadronic physics. So, the statement that the colour-octet in the colour-anticolour states is irrelevant to physics, should not make us too complacent. There must be a place where this colour-octet plays a physically significant role. Below, we show that indeed, it does connect one-to-one with the TQM wave functions of the octet.

Let us put the six members of the octet (eqn. (6)) along with six members of the octet colour states (eqn. (8)) as follows:

$$\begin{array}{rcl} P \sim (uu)^1 d & : & \bar{R}G \\ N \sim (dd)^1 u & : & \bar{G}R \\ \Sigma^+ \sim (uu)^1 s & : & \bar{R}B \\ \Xi^0 \sim (ss)^1 u & : & \bar{B}R \\ \Sigma^- \sim (dd)^1 s & : & \bar{G}B \\ \Xi^- \sim (ss)^1 d & : & \bar{B}G \end{array} \tag{10}$$

We are struck by a remarkable correlation between quark and identical-diquark flavours with colour and anti-colour for the six states as

$$\begin{array}{ccc} u \leftrightarrow R & ; & (uu) \leftrightarrow \bar{R} \\ d \leftrightarrow G & ; & (dd) \leftrightarrow \bar{G} \\ s \leftrightarrow B & ; & (ss) \leftrightarrow \bar{B} \end{array} \tag{11}$$

Note that in proton for example, (uu) state is symmetric in flavour-spin space and now $\bar{R} \sim GB - BG$ ¡(eqn. (9)) provides the corresponding antisymmetry in colour space to make this state totally antisymmetric in the exchange of the first two quarks, correctly fulfilling the Pauli Exclusion Principle requirement of antisymmetry on the exchange of identical fermions.

This color-flavor and anticolor - identifical diquark locking is remarkable. What is it trying to tell us?

Note that as one goes from $SU(2)_I$ with two flavours (u,d) to 3 flavours (u,d,s), one naively just uses the full group $SU(3)_F$. Note that in this process we are missing something crucial. We know that there are three SU(2) subgroups in SU(3). So as one goes from SU(2) to SU(3) the above TQM wave function suggests that the full group symmetry should be:

$$SU(2)_I \to (SU(2)_I \otimes U(1) + SU(2)_U \otimes U(1) + SU(2)_V \otimes U(1)) \otimes SU(3)_C \tag{12}$$

That is (u,d) $SU(2)_I$ is extended to $SU(2)_I \otimes U(1)_s$ by adding an s-quark and similarly for the extensions $(d, s) \to (d, s)\, u$ and $(u, s) \to (u, s)\, d$. Let us assume that all the three subgroups are equivalent to each other.

The above six states in the TQM wave function in eqn. (6) and (10) map this pattern of extension from SU(2) to three SU(2) x U(1) 's. Here, we suggest that the extension from SU(2) to SU(3) is not a one-step process but goes through this intermediate step, as indicated above. And amazingly, all this ia arising due to a complete color-flavor locking.

Now, we see the significance of why in TQM model, the states are eigenstates of one-body operators. Take the total magnetic moment operators as a sum of one-body operators acting on quarks in position 1,2,3 respectively for proton as

$$\mu_p = \mu_{u1} + \mu_{u2} + \mu_{d3} \tag{13}$$

We shall disuss its values below. Here, let us note that each quark acts independently of the others. Similar situation for the evaluation of electric charges etc. Clearly the mass of the corresponding quarks arise from the coresponding magnetic moments. That is masses do not arise as eigenstaes of a broken or unbroken Hamiltonian in this formalism. Note that for each of these baryons the colour part being normalized as $\langle \bar{R}G|\bar{R}G\rangle = 1$, it filters out.

Now we see why these six states are eigenstates of one-body operators. These correlated states in the TQM are made up of more fundamental current quarks which give out three effective constituent quarks (renormalised quarks are they are locked up to colour in the above states) as colour-independent entities. Right away we see that these three independent quarks may be identified with the three constituent quarks of $SU(3)_F$ quark model. So this intermediate state for the extension of SU(2) to SU(3), is what, through color-flavor locking, anticolor-identical diquark locking, produces states which are equivalent to three independent quarks. Upto this level of exact symmetry in the groups, the six states are identical with the masses $m_u = m_d = m_s$, that is that they generate identical constituent quark masses.

So remarkably we find that the purpose of the TQM is to provide three independent quark state wave functions, which act as inputs in the fundamental representations of the larger $SU(3)_F$ group. This is clearly providing a completely different picture of the constituent quarks from the presently held canonical view of it being made up of a sea of quark-antiquark and gluons.

So far we have considered six states of the TQM in the 1/2 octet model. What are Σ^0 and Λ^0 for this exact group as discussd after eqn. (6)? As each SU(2) group is equally basic in the above structure, these states would be

$$\Sigma^0 = (ud + du)\, s + (ds + sd)\, u + (su + us)\, d \tag{14}$$

and

$$\Xi^0 = (ud - du)\, s + (ds - sd)\, u + (su - us)\, d \tag{15}$$

Now note that for Σ^0 as the state $(ud)^1 s$ (is actually a broken symmetry structure as we discuss below) the corresponding colour state is $\frac{1}{\sqrt{2}}\left(\bar{R}R - \bar{G}G\right)$ and $\frac{1}{\sqrt{6}}\left(\bar{R}R + \bar{G}G - 2\bar{B}B\right)$ goes with (the symmetry broken) Ξ^0 as given by $(ud)^0 s$.

Hence the colour state, corresponding to the above unbroken states of Σ^0 and Ξ^0 as give above in eqns. (14) and (15) are respectively,

$$\frac{1}{\sqrt{2}}\left(\bar{R}R - \bar{G}G\right) + \frac{1}{\sqrt{2}}\left(\bar{G}G - \bar{B}B\right) + \frac{1}{\sqrt{2}}\left(\bar{B}B - \bar{R}R\right) = 0 \tag{16}$$

and

$$\frac{1}{\sqrt{6}}\left(\bar{R}R + \bar{G}G - 2\bar{B}B\right) + \frac{1}{\sqrt{6}}\left(\bar{R}R + \bar{B}B - 2\bar{G}G\right) + \frac{1}{\sqrt{6}}\left(\bar{G}G + \bar{B}B - 2\bar{R}R\right) = 0 \tag{17}$$

So the product of the spin flavour times colour factor for these states is zero. Hence, for the case of exact symmetry of the three SU(2)XU(1) groups as given in eqn.(12), these states do not exist. Only the six states with their color-flavor locking exist and these then provide the constituent quarks as inputs for the SU(3) flavour quarks with equal masses for the three flavours of quarks.

Next let the symmetry of the above group be broken. This is dictated by physical reasons wherein it is known that $SU(2)_I$ of (u,d) state is well-known to be a good symmetry but in SU(2)xU(1) the s-quark breaks the symmetry so that the corresponding masses as arsisng in the magnetic moment for the s-quark is different from that of the u- and the d-quarks, Let this broken group be represented (with an extra set of braces) as below:

$$SU(2)_I \rightarrow \left(\{SU(2)_I \otimes U(1)\} + SU(2)_U \otimes U(1) + SU(2)_V \otimes U(1)\right) \otimes SU(3)_C \tag{18}$$

Now note that for the symmetry broken Σ^0 as the state $(ud)^1 s$, the colour state is $\frac{1}{\sqrt{2}}\left(\bar{R}R - \bar{G}G\right)$ and $\frac{1}{\sqrt{6}}\left(\bar{R}R + \bar{G}G - 2\bar{B}B\right)$ goes with (the symmetry broken) Ξ^0 as given by $(ud)^0 s$.

The magnetic moments in standard notation are as follows:

$$\begin{array}{ccc} Baryons & TQM-broken & experiment \\ p & \frac{(4\mu_u-\mu_d)}{3}=2.793 & 2.793 \\ n & \frac{(4\mu_d-\mu_u)}{3}=-1.862 & -1.913 \\ \Lambda & \mu_s=-0.614 & -0.614 \\ \Sigma^+ & \frac{(4\mu_u-\mu_s)}{3}=2.687 & 2.46 \\ \Sigma^0 & \frac{(2\mu_u+2\mu_d-\mu_s)}{3}=0.825 & -- \\ \Sigma^- & \frac{(4\mu_d-\mu_s)}{3}=-1.042 & -1.16 \\ \Xi^0 & \frac{(4\mu_s-\mu_u)}{3}=-1.439 & -1.25 \\ \Xi^- & \frac{(4\mu_s-\mu_d)}{3}=-0.508 & -0.65 \end{array} \tag{19}$$

Note that for the TQM group there are no Σ^0 and Λ^0 states. These exist only in the broken TQM with isospin given the physically demanded priviledged status. From the table one sees that these magnetic moments of the baryon octet, as should be, are the same as obtained in the $SU(6)_{FS}$ model. We take proton, neutron and Λ magnetic moments as inputs to fit the other magnetic moments - and these work quite well. These three states are also used to obtain the u-, d- and s-quark masses For magnetic moments of these states, it is clear that $m_s\rangle\,(m_u = m_d)$. So the evaluation of all one-body operators with these wave functions shows that we have three independent quark states with constituent quark masses $m_s\rangle\,(m_u = m_d)$ as an input for the fundamental represenation for the quarks in $SU(3)_F$ group.

This is against the conventional picture where one demands that $SU(3)_F$ is a good exact starting symmetry which thereafter is broken through, some perturbation terms in the Hamiltonian introduced say, through the second diagonal generator Y. The new picture given above is quite distinct from this.

So the complete group structure of how an intrinsically broken $SU(3)_F$ arises with these constituent quarks is as follows:

$$SU(2)_I \to (SU(2)_I \otimes U(1)_S + SU(2)_U \otimes U(1) + SU(2)_V \otimes U(1)) \otimes SU(3)_C$$
$$\to ((SU(2)_I \otimes U(1)_S) + SU(2)_U \otimes U(1) + SU(2)_V \otimes U(1)) \otimes SU(3)_C$$
$$\to SU(3)_F \tag{20}$$

With $SU(6)_{SF} \otimes SU(3)_C \supset SU(3)_F \otimes SU(2) \otimes SU(3)_C$ to provide the correct symmetric spin-flavour states in the quark model and also to get the colour antisymmetric state.

Hence, here we have given a new interpretation of the TQM wave function as the cause of providing the structure which leads through the one-body operators to creation of three independent constituent quarks with $m_s\rangle\,(m_u = m_d)$ for the $SU(3)_F$ group, which then is used in $SU(6)_{SF} \otimes SU(3)_C$ quark model.

So, we learn that the TQM wave function is not that of $SU(3)_F$ but that of the broken group above. It is this that determines that $SU(2)_I$ is special with respect to the other two SU(2) groups. So this gives spin 1/2 octet states and then gives the correct constituent quarks to go as input in $SU(3)_F$. Earlier, one jumped right away in a single step from the group $SU(2_F)$ to the group $SU(3)_F$. As shown here, this is too abrupt a step, and that there is an intermediate stage which takes us gently from the smaller group to the larger group, and that it is essential to be taken into account.

One knows that $SU(3)_F$ symmetry is broken. The classic question has been as to how this breaking occurs. In the more familiar explanation, conventionally, at first the symmetry is actually known to be exact and thereafter broken through a perturbation as

$$H_{strong} = H_{SU(3)} + \epsilon H_{medium-strong} \tag{21}$$

This breaks the symmetry. However, as pointed out by Ryder [8, p 180], "According to yet another school of thought, SU(3) is intrinsically broken, that, there is not a symmetric state of affairs followed by a perturbation which breaks the symmetry, but rather the symmetry when it first appears is already broken." The present paper shows that this intrinsically broken SU(3) scenario is actually the reality!

Note that the masses which arise from this unique sub-group structure are $m_s\rangle$ $(m_u = m_d)$ This then acts as input to $SU(3)_F \otimes SU(3)_C$ group. Now as $3_C \otimes 3_C \otimes 3_C = 1 + 8 + 8 + 10$ the decuptet is a pure $SU(3)_F$ result with the above input of the constituent quarks. Indeed, it turns out that the decuptet is well known to be satisfied by the mass formula $M = M_0 + bY$ where Y is proportional to the second digonal generator of $SU(3)_F$. This is clearly equivalnt to the above masses for the s-quark and the (u,d)-quarks as input from the TQM model as shown here. Indeed, this should be taken as a smoking-gun evidence in support of the idea presented in this paper.

But why then does the octet member not show this mass structure ? It is well known that this does not give the correct masses for the octet. Now we notice that the reason has to do with the fact that, unlike the decuplet, which is a pure state of the $SU(3)_F$ state, arising from the new constituent quarks, the octet of the TQM consists of seven members which are symmetric in the first two quark indices and the eighth member which is antisymmetric in the same. But the octet of $SU(3)_F$ as being a part of $SU(6)_{SF}$ group is a mixture of $8_{MS} + 8_{MA}$, giving the wave function in eqn. (1) of the quark model. Thus there has to be a further symmetry breaking for the octet members. So specific $SU(3)_F$ symmetry breaking terms are required.

We look at some group theory to understand what we have achieved here. In SU(3) assuming symmetry breaking of the spin 1/2 baryon octet as given by Okubo [9]

$$M_{(8)} = m_0 Tr(\bar{B}B) + \frac{1}{2} m_1 (\{\bar{B}, B\}\lambda_8) - \frac{1}{2} m_2 Tr([\bar{B}, B]\lambda_8) \tag{22}$$

This gives the well known Gell-Mann-Okubo mass formula

$$m_8(I, Y) = m_0 + aY + b[I(I+1) - \frac{1}{4}Y^2] \tag{23}$$

which works very well for the spin 1/2 baryon octet in SU(3).

For the case of the 3/2 decuptet the above mass formula reduces to a much simpler form

$$m_{10}(I, Y) = m_0{}' + a' Y \tag{24}$$

And this works very well for the decuptet. Note that this right away justifies our above assertion that the symmtery-broken TQM provide the three quarks which go as the fundamental representation in the fully symmetric decuptet. The above SU(3) mass fomula works for the decuptet because it matches the unmixed decuptet, indicating its pureness as being made of three unbroken constituent quark entities.

In fact Okubo has shown [9] that the above complex equation (eqns. (22) and (23)) reduces to the simpler form above (eqn. (24)) for all Irreducible Representations given as triangular diagrams like the decuptet. These are 3, 6, 10, 15, 21 etc. which are totally symmetric states of 1-, 2- 3-, 4-, 5- etc. quarks respectively. So for the 5-quarks states, as being made up of the fundamental 3-quark representation, the state of dimension 21 is given by the Young diagram of the 5-quarks symmetric product representation; $\square\square\square\square\square$. Clearly all these triangular states retain the pure 3-quarks constituent entities as inputs as ummixed states (including the decuptet above). However, these constituent quarks as inputs are not enough for the octet in SU(3) as it is made through specific octet symmetry breaking unique to the group SU(3).

Thus these consideratins show that as one goes from the smaller SU(2) flavour group to the bigger SU(3) flavour group, it is not a trivial extension from 2 to 3. But that there is an hidden intermediate symmetry structure, discussed as TQM, which lets us go from 2 to 3 with an intrinsically broken $SU(3)_F$ symmetry with inputs of three constituent quarks. Mass considerattion from the baryon magnetic moments based on broken TQM model, which also works well for all the triangular represntations of SU(3) like the decuptet, support this new model. Difference in masses with respect to the octet masses like the Gell-Mann-Okubo mass formula for SU(3), are along the expected lines of the new broken TQM. Color-flavor locking is a unique property of the TQM which allows the new constituent quark structure to be build up.

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