

# Analyzing black hole super-radiance emission of particles/energy from a black hole as a thought experiment to get bounds on the mass of a graviton

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## Abstract

Although very unlikely to be observed, the phenomena of particle emission by super radiance of particles/energy by a black hole is examined as a thought experiment. In doing so, the idea is to come up with bounds to the mass of a graviton. Values for the following perturbations of space-time represented as metric  $g_{\mu\nu}$  being perturbed from flat space values by  $h_{00}$ ,  $h_{0i}$ , and  $h_{ij}$  make the case, due to the mass dependence of the black hole, that super radiance would almost certainly not be observable, but the considerations so evidenced in giving mass bounds to emitted particles via Padmanabhan's derivation of super-radiance allows massive gravity to be consistent with black hole physics and GR.

**Key words:** *massive gravity, Planck scale, Pilot theory embedding structure*

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## 1. Introduction: Massive gravity and how to get it commensurate with black hole physics?

In general relativity the metric  $g_{ab}(x, t)$  is a set of numbers associated with each point which gives the distance to neighboring points. I.e. general relativity is a classical theory. As is designated by GR traditionalists [1], the graviton is usually stated to be massless. With spin two and with two polarizations. Adding a mass to the graviton results in 5 polarizations plus other problems [2]. What this document will do will be to try to establish massive gravitons as super – radiant emission candidates from black holes [3] and in doing so provide another frame work for their analysis which would embed them in GR. In doing so, one should keep in mind that this is a thought experiment and that the author is fully aware of how hard it would be to perform experimental measurements. In coming up with criteria as to graviton mass, we are also, by extension considering the Myers-Perry higher dimensional model of black holes [4] and commenting upon its applications, some of which are in [5]. All of which start with the implications of  $dE/dt < 0$ , leading to 'leakage' from a black hole. i.e. energy of the black hole 'decreases' in time.

## 2. what is super-radiance in black hole physics ?

This paper examines Padmanabhan's derivation of super-radiance [3], stating its application to the graviton, with mass, and making then a referral to the likelihood of measurement which ties in with the metric  $g_{\mu\nu}$  being perturbed from flat space values by  $h_{00}$ ,  $h_{0i}$ , and  $h_{ij}$  [4], thereby making the case, due to the mass dependence of the black hole, that super-radiance would almost certainly not be observable but would firmly embed massive gravitons in GR in spite of the view point offered in [1]. To do so would mean that [3] has: if  $dE/dt < 0$  for super-radiance, i.e. escape of matter/energy from a BH, we examine, if  $c_1$  is a constant, and  $\omega$  a frequency, and  $m$  a mass, and  $\Omega_H$  angular velocity

$$\frac{dE}{dt} = c_1 \cdot \omega \cdot [\omega - m \cdot \Omega_H] \quad (1)$$

Here we have that the angular velocity is defined by, if  $M_{BH}$  is the black hole mass, then [3], page 371

$$\Omega_H = \frac{a}{2M_{BH} \cdot r_H} \quad (2)$$

$$r_H = M_{BH} - \sqrt{M_{BH}^2 - a^2} \quad (3)$$

$$a = \sqrt{x^2 + y^2} \quad (4)$$

Then,

$$0 < \frac{\omega}{m} < \Omega_H \quad (4)$$

Becomes

$$0 < \frac{\omega}{m} < [\Omega_H] \sim \frac{1}{a} + \varepsilon^+ \quad (5)$$

In the case of black rings, and other such exotica, in higher dimensions, [5],  $a \neq 0$ , and yet in the case of pure singularities, we have that, instead,  $a \xrightarrow{\text{classical-BH}} 0$ . Leading to our first result, that classical black hole physics, if the mass  $m$  is not zero, would not have a restriction on the mass, relative to the frequency of emitted material due to [3], but that the situation would change if  $a \neq 0$ , leading to our first theorem.

**Theorem 1.** If  $a \neq 0$ , then the bound on  $m$ , due to super-radiance from a black hole is

$$\text{Case 1: } m > \omega \cdot \left( \frac{1}{a} + \varepsilon^+ \right) \quad (6)$$

If  $a \xrightarrow{\text{classical-BH}} 0$ , and  $m \neq 0$ , and frequency of emission from the black hole is not zero then the bound on  $m$  effectively does not exist, ie.

$$\text{Case 2: } 0 < \frac{\omega}{m} < \infty \quad (7)$$

Now what if we set the mass in Case 1 and Case 2 as due to and being a massive graviton? Note that then Case 1 is then implying there is a tendency toward ultra low GW frequencies from emitted black holes? So then we go to our second theorem

**Theorem 2** In the case that the mass,  $m$  is of a massive graviton, of about  $10^{-62}$  grams, then the extension of Case 1 of Theorem 1 leads to

$$a \leq \frac{m_g}{\omega_{\text{graviton}}} \quad (8)$$

Since the frequency of a graviton is non zero, this would lead to, in black hole physics, a statement as to the interior structure of black holes which could be experimentally inferred. We next then reference Perturbation models of space time, i.e.

### 3. Discussion of Myers-Perry BH models in our thought experiment.

The subsequent values by  $h_{00}$ ,  $h_{0i}$ , and  $h_{ij}$  make the case, due to the mass dependence of the black holes in the Myers-Perry black holes, that although we can deduce the two theorems above, experimental verification will be a challenge. [4] has

$$\begin{aligned} h_{00} &\approx \frac{16\pi G}{(d-2) \cdot \Omega_{d-2}} \cdot \frac{M_{BH}}{r^{d-3}} \\ h_{ij} &\approx \frac{16\pi G}{(d-2) \cdot (d-3) \cdot \Omega_{d-2}} \cdot \frac{M_{BH}}{r^{d-3}} \cdot \delta_{ij} \\ h_{0i} &\approx -\frac{8\pi G}{\Omega_{d-2}} \cdot \frac{x^k}{r^{d-1}} \cdot J^{ki} \end{aligned} \quad (9)$$

The coefficient  $d$  is for dimensions, usually 4 or above, and in this situation, with angular momentum  $J^{ki}$

$$\Omega_{d-2} = 2\pi^{(d-1)/2} / \Gamma\left(\frac{d-1}{2}\right) \quad (10)$$

$$J^{ki} = 2 \cdot \int x^k \cdot T^{i0} \cdot d^{d-1}x \quad (11)$$

The  $T^{i0}$  above is a stress energy tensor as part of a  $d$  dimensional Einstein equation given in [3] as

$$R_{jl} - \frac{1}{2} \cdot g_{jl} \cdot R = 8\pi G T^j_l \quad (12)$$

Also, the mass of the black hole is, in this situation scaled as follows: if  $\mu$  is a re scaled mass term[3]

$$M_{BH} = \mu \cdot \Omega_{d-2} \cdot (d-2) / 16\pi G \quad (13)$$

More generally, the mass of the black hole is a by product of Eq.(12) and is written as

$$M_{BH} \equiv \int T_{00} d^{d-1}x \quad (14)$$

We will next go to the minimum size of a black hole which would survive as up to 13.6 billion years, and then say something about the relative magnitude of the magnitude of the terms in Eq.(9) and then their survival today, and what that portends as to the strength of signals which may be received. The variance of black hole masses, from super massive BHs to those smaller than  $10^{15}$  *grams* will be discussed, in the context of Eq.(9), and stress strength, with commentary as to what we referred to earlier, namely strain for detecting GW is given by  $h(t)$  given below, with  $D^{ij}$  as the detector tensor, i.e. a constant term, so that by [2], page 336, we write

$$h(t) = D^{ij} h_{ij} \quad (15)$$

This Eq.(15) means that the magnitude of strain,  $h$ , is effected by Eq. (13), Eq.(14) and its magnitude, seen next.

#### 4. Discussion of the magnitude of Eq.(9) and its links to Eq.(15) via scaling arguments.

As stated earlier, it , the magnitude of strain mentioned in Eq.(15), depends upon the allowed mass of a black hole. The arguments in this section proceed to give threshold values as to the strength of a signal, given by Eq. (15) above, and to talk about consequences for such magnitudes.

Starting this out requires that we estimate what the mass of a black hole has to be to last 13.6 billion years . To do this, note from Kolb and Turner [6], a critical value for a primordial black hole existing that long would be

$$M_{BH} \Big|_{\text{min-life.time}} \propto 10^{15} \text{ grams} \quad (16)$$

Ford [7] has a key mode frequency for evaporation needed for evaporation of a black hole during this process given as a very high value, as on page 297 of [7], so

$$\omega_{BH} \approx \exp(M_{BH}^2) / M_{BH} \quad (17)$$

For a stellar black hole, as given by Ford [7] this would be  $\omega \sim 10^{10^{75}}$  grams i.e. vastly larger than the mass of the universe, which is insane, so we note that black holes of the size of the sun, namely

$$M_{\odot} \sim 1.9891 \times 10^{33} \cdot M_{BH} \Big|_{\text{min-life.time}} \propto 1.9891 \times 10^{33} \text{ grams} \quad (18)$$

Are stellar sized BHs which last far longer than the lifetime of the universe so far, which leads to for a 100 times the mass of the Sun BH to have strain values , if  $D^{ij}$  is approximately unity, with

$$M_{BH} \Big|_{\text{min-life.time}} \propto 10^{15} \text{ grams} \Leftrightarrow h_{00} \ \& \ h_{ii} \propto 10^{-40} \text{ for BHs at Z(redshift)} \sim 10 \quad (19)$$

Whereas super massive black holes, of about 100 times solar mass at Z(redshift)~10

$$M_{BH} \Big|_{100\text{-solar-mass}} \Leftrightarrow h_{00} \ \& \ h_{ii} \propto 10^{-20} \quad (20)$$

It is easy from inspection to infer from this that most early formed black holes would not be accessible and that only the giant ones would do. Note however, that stochastic noise from the black holes would remove almost all chance of experimental confirmation of Theorem 1 and Theorem 2 above. I.e. information / energy which is lost from super radiance would allow us to understand and perhaps reconcile why the entropy of super massive black holes, could be larger than the usual calculations for entropy for the entire 4 dimensional universe, i.e. see Carroll [8], and we will then propose a solution based upon an extension of free energy arguments given in [9].

#### 5. Details / discussions as to how to use Theorem 1 & Theorem 2 to understand how entropy of a SMBH could be larger than the entropy of the Universe (4 Dimensional)

Carroll in [8] gave a cogent book on GR states that the entropy of the universe is of the order of magnitude (non dimensional units) for four dimensional space-time

$$S \sim 10^{88} \quad (21)$$

Typically, though, entropy of super massive black holes is calculated as leading to a many times larger value for entropy of the entire universe via [10], namely as given in that reference, and summed up to be a larger value, i.e. using holographic arguments [11] and the last page of [10]

$$S_{\max} \sim \text{SCEH}(t \rightarrow \infty) = 2.88 \pm 0.16 \times 10^{122} k \quad (22)$$

Given that there are at least one to ten million SMBHs, usually in galaxies, this would lead to by [10] at least for a super massive black hole in the center of a galaxy, Eq. (22) will lead to:

$$S_{\text{Max-BH}} \sim 10^{115} k \quad (23)$$

This value for a super massive black hole is likely for a higher dimensional black hole, i.e. equal to or possibly more than 5 dimensions. As given by Gregory [12] on page 33

$$S_{\text{BH}} \Big|_{\text{higher-dim}} = 4\pi M_{\text{BH}}^2 \cdot \sqrt{\frac{8L}{27\pi M_{\text{BH}}}} \quad (24)$$

When  $L$  is sufficiently large, then we have an explanation as to how Eq. (24) could correspond to Eq. (23). So from now on, we will attempt to understand the consequences of a very large  $L$  value. To do this, we shall look again at what Theorem 1 and what Theorem 2 is really saying. Namely.

**Theorem 3:** For sufficiently large  $L$  in Eq.(24), the coefficient  $a$  is non zero, in Theorem 2 leading to a nonzero mass of the graviton. i.e. **an abrupt shift away from the Schwartzshield solution to a black hole, as referenced in [10], ie the case where  $a$  is zero[13]**

We claim, that the embedding of black holes in five dimensional space time is a way to make a connection with a multiverse, as given in the following supposition [14]

### 6: Extending Penrose's suggestion of cyclic universes, black hole evaporation, and the embedding structure our universe is contained within, i.e. using the implications of Eq.(24) for a multi verse.

That there are no fewer than  $N$  universes undergoing Penrose 'infinite expansion' (Penrose, 2006) [15] contained in a mega universe structure. Furthermore, each of the  $N$  universes has black hole evaporation, with the Hawking radiation from decaying black holes. If each of the  $N$  universes is defined by a partition function, called  $\{\Xi_i\}_{i=1}^N$ , then there exist an information ensemble of mixed minimum information correlated as about

$10^7 - 10^8$  bits of information per partition function in the set  $\{\Xi_i\}_{i=1}^N \Big|_{\text{before}}$ , so minimum information is

conserved between a set of partition functions per universe

$$\{\Xi_i\}_{i=1}^N \Big|_{\text{before}} \equiv \{\Xi_i\}_{i=1}^N \Big|_{\text{after}} \quad (25)$$

However, there is non uniqueness of information put into each partition function  $\{\Xi_i\}_{i=1}^N$ . Furthermore

Hawking radiation from the black holes is collated via a strange attractor collection in the mega universe structure to form a new big bang for each of the  $N$  universes represented by  $\{\Xi_i\}_{i=1}^N$ . Verification of this mega

structure compression and expansion of information with a non uniqueness of information placed in each of the  $N$  universes favors Ergodic mixing treatments of initial values for each of  $N$  universes expanding from a

singularity beginning. The  $n_f$  value, will be using (Ng, 2008)  $S_{entropy} \sim n_f$ . [16] . How to tie in this energy expression, as in Eq. (24) will be to look at the formation of a non trivial gravitational measure as a new big bang for each of the N universes as by  $n(E_i)$ . the density of states at a given energy  $E_i$  for a partition function. (.Poplawski, 2011) [17]

$$\left\{ \Xi_i \right\}_{i=1}^i \approx N \propto \left\{ \int_0^\infty dE_i \cdot n(E_i) \cdot e^{-E_i} \right\}_{i=1}^i . \quad (26)$$

Each of  $E_i$  identified with Eq.(26) above, are with the iteration for N universes (Penrose, 2006)[15] Then the following holds

**Theorem 4;**

$$\frac{1}{N} \cdot \sum_{j=1}^N \Xi_j \Big|_{j\text{-before-nucleation-regime}} \xrightarrow{\text{vacuum-nucleation-transfer}} \Xi_i \Big|_{i\text{-fixed-after-nucleation-regime}} \quad (27)$$

For N number of universes, with each  $\Xi_j \Big|_{j\text{-before-nucleation-regime}}$  for j = 1 to N being the partition function of each universe just before the blend into the RHS of Eq. (25) above for our present universe. Also, each of the independent universes given by  $\Xi_j \Big|_{j\text{-before-nucleation-regime}}$  are constructed by the absorption of one to ten million black holes taking in energy. **I.e. (Penrose, 2006)** [15]. Furthermore, the main point is similar to what was done in [18] in terms of general Ergodic mixing

**Theorem 5.**

$$\Xi_j \Big|_{j\text{-before-nucleation-regime}} \approx \sum_{k=1}^{Max} \tilde{\Xi}_k \Big|_{\text{black-holes-jth-universe}} \quad (28)$$

## 7. Using free energy to understand a phase transition to massive gravitons

What is done in **Theorems 3 to Theorem 5** is to come up with a protocol as to how a multi dimensional representation of black hole physics enables continual mixing of spacetime [18] largely as a way to avoid the Anthropic principle, as to a preferred set of initial conditions. We will then, largely based upon the [9] linkage of free energy , its derivatives, and entropy, attempt to understand from first principles as to why Eq.(23) has such an enormous entropy. We do this, assuming each asorbing black hole eventually will radiate particles and energies as given in [ 3 ] . In [9] there is a well developed protocol as to linking free energy, and entropy, (which is a way of replacing the [10] derivation of [19] as given in [10] using Bekenstein–Hawking horizon entropy equation) , for individual black holes, as to explaining how each individual black hole could have the enormous entropy as given by Eq.(23), and not by the assumption given in [19] with the assumption made of dividing Eq.(22) by 10 million as was done, earlier. In 1994, the Free energy and entropy of black holes [20] was explored by Hochberg with, if the function Z is a partition function, then

$$\beta = 1/\text{Temperature} \quad (29)$$

$$F = -\beta^{-1} \log(Z) \quad (30)$$

$$S = \beta E + \log(Z)$$

In [20] there is a consideration as to alleged transformation from a hot flat Euclidian space to a colder space,

$$\begin{aligned}\tilde{a} &\equiv \frac{\pi^2}{15 \cdot \hbar^3} \\ E_{HFS} &= \tilde{a} \cdot T_{temp}^4 \cdot V_{volume} \\ S_{HFS} &= \frac{4}{3} \cdot \tilde{a} \cdot T_{temp}^4 \cdot V_{volume} \equiv \frac{4}{3} \cdot E_{HFS} \\ F_{HFS} &= E_{HFS} - T_{temp} \cdot S_{HFS} \equiv -\frac{\tilde{a}}{3} \cdot T_{temp}^3 \cdot V_{volume} < 0\end{aligned}\tag{31}$$

In [10] there is discussion of a phase transition from a hot to cold flat space, i.e. the main point being that such a transition is a defacto phase transition. Of second order. In [10] the critical temperature is given as by its [1] Eq.(16) as:

$$T_{critical} = \frac{\Omega}{2\pi}\tag{32}$$

Up to a degree of proportionality, we assert that the numerator of Eq. (32) is within modulo relations the same as Eq.(2). And that this means that we can use Eq.(2) after using Eq.(32) to fix a value for the key parameter inputs into both Theorem 1 and Theorem 2, and in doing so begin to work with obtaining values for bounds to the mass of a graviton and its relationships to frequency of the graviton.

## 8. Using free energy to understand a phase transition to massive gravitons refined. i.e. the role of Appendix A in terms of partition functions. Future bridge to quantum mechanics?

Below in **Appendix A** is a way to include a reconciliation with Quantum mechanics, in nucleation of a massive graviton. This partition function , with additional work will be included in disterning the nature of the free energy , as either purely classical, or with quantum features. This will go a long way toward eventually reconciling if the graviton, as a massive particle, is in sync with both GR and quantum mechanics.

## 9. Conclusion. Is QM imbedded in a semi classical structure ? How about black hole physics, too ?

We argue that further refinements of **Theorem 1** and **Theorem 2** are a way to ascertain this question, i.e. and to also answer if gravity is semi classical. If gravity is indeed semi classical, then the entropy as given by Eq. (24) for higher dimensional black holes is likely due to a superstructure which will embed quantum mechanics within a deterministic structure. Furthermore, it will also tie into the question of a single universe repeating its self versus a multiverse, as was gone over in this paper, and also in [14] . And all this will require for implementation is making use of the following: The particle per phase state count is, (Maggiorie, 2000) [22]

$$n_f \sim h_0^2 \Omega_{GW} \cdot \frac{10^{37}}{3.6} \cdot \left[ \frac{1000Hz}{f} \right]^4\tag{33}$$

Secondly detector strain for device physics is given by ( Maggiorie, 2000) [ 22 ]

$$h_c \leq (2.82 \times 10^{-21}) \cdot \left( \frac{1Hz}{f} \right)\tag{34}$$

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### Appendix A: Highlights of J.-W. Lee's paper

The following formulation is to highlight how entropy generation blends in with quantum mechanics, and how the break down of some of the assumptions used in Lee's paper coincide with the growth of degrees of freedom. What is crucial to Lee's formulation, is Rindler geometry, not the curved space formulation of initial universe conditions.. First of all. (Lee, 2010)[ 23 ],

*“Considering all these recent developments, it is plausible that quantum mechanics and gravity has information as a common ingredient, and information is the key to explain the strange connection between two. If gravity and Newton mechanics can be derived by considering information at Rindler horizons, it is natural to think quantum mechanics might have a similar origin. In this paper, along this line, it is suggested that quantum field theory (QFT) and quantum mechanics can be obtained from information theory applied to causal (Rindler) horizons, and that quantum randomness arises from information blocking by the horizons*

To start this we look at the Rindler partition function, as by (Lee, 2010)[ 23 ]

$$Z_R = \sum_{i=1}^n \exp[-\beta H(x_i)] = \text{Trace} \cdot (\exp[-\beta H]) \quad (\text{A.1})$$

As stated by Lee [48] , ..we expect  $Z_R$  to be equal to the quantum mechanical partition function of a particle with mass  $m$  in Minkowski space time. Furthermore, there exists the datum that: Lee made an equivalence between Eq. (A1) and ( Lee, 2010)[ 23 ]

$$Z_Q = N_1 \int \phi x \cdot \exp\left[\frac{-i}{\hbar} \cdot I(x_i)\right] \quad (\text{A2})$$

Where  $I(x_i)$  is the action ‘integral’ for each path  $x_i$ , leading to a wave function for each path  $x_i$ ,

$$\psi \sim \exp\left[\frac{-i}{\hbar} \cdot I(x_i)\right] \quad (\text{A3})$$

If we do a rescale  $\hbar = 1$ , then the above wave equation can lead to a Schrodinger equation,

**The example given by (Lee , 2010) [23]**is that there is a Hamiltonian for which

$$H(\phi) = \int d^3x \cdot \left\{ \frac{1}{2} \cdot \left( \frac{\partial \phi}{\partial t} \right)^2 + \frac{1}{2} \cdot (\nabla \phi)^2 + V(\phi) \right\} \quad (\text{A4})$$

Here,  $V$  is a potential, and  $\phi$  can have arbitrary values before measurement, and to a degree,  $Z$  represent uncertainty in measurement. In Rindler co-ordinates,  $H \rightarrow H_R$  , in co-ordinates  $(\eta, r, x_2, x_3)$  with proper time variance  $ard\eta$  then

$$H_R(\phi) = \int dr dx_{\perp} ar \cdot \left\{ \frac{1}{2} \cdot \left( \frac{\partial \phi}{\partial r} \right)^2 + \frac{1}{2} \cdot \left( \frac{\partial \phi}{ar \partial \eta} \right)^2 + \frac{1}{2} \cdot (\nabla_{\perp} \phi)^2 + V(\phi) \right\} \quad (\text{A5})$$

Here, the  $\perp$  is a plane orthogonal to the  $(\eta, r)$  plane. If so then

$$Z = \text{tr} \exp[-\beta H] \mapsto Z_R = \text{tr} \exp[-\beta H_R] \quad (\text{A6})$$

**Now, for the above situation, the following are equivalent**

1.  $Z_R$  thermal partition function is from information loss about field beyond the Rindler Horizon
2. QFT formation is equivalent to purely information based statistical treatment suggested in this paper
3. QM emerges from information theory emerging from Rindler co-ordinate



Lee also forms a Euclidian version for the following partition function, if  $I_E(x_i)$  is the Euclidian action for the scalar field in the initial frame. I.e.

$$Z_Q^E = N_1 \int \phi x \cdot \exp \left[ \frac{-i}{\hbar} \cdot I_E(x_i) \right] \quad (A7)$$

There exist analytic continuation of  $\tilde{t} \mapsto it$  leading to  $Z_Q^E \mapsto Z_Q =$  Usual zero temperature QM partition function of  $Z_Q$  for  $\phi$  fields.

**Important Claim:** The following are equivalent

1.  $Z_R$  and  $Z_Q$  are obtained by analytic continuation from  $Z_Q^E$
2.  $Z_R$  and  $Z_Q$  are equivalent .

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