THE CONVENIENT NUMERICAL BASE FOR LENGTH AND TIME

BRIEF INTRODUCTION
If A and B are two points in the space, the following two statements
i) the points A, B are distinct in the space
ii) the points A, B are coincident in the space
can not be simultaneously true.
a) Let’s assume to istante to, the points A and B distinct in the space. Then, the time necessary to make A and B overlap must be $\Delta t \neq 0$. In fact if the time necessary to make A and B overlap was $\Delta t = 0$, we can also assert that at time $t_0$, the points A and B are coincident, since it is not necessary time to make A and B overlap and i), ii) are simultaneously true in spite of the initial assumption a).

BEGINNING
Let’s the points O, O', M coincident with the origin $x_0 = 0$ of IRS(O xyt) at time $t_0 = 0$ (according to the clock of O and O') and let O', M, in motion in the IRS(O xyt) along the axis $x > 0$ at constant speed, such that $|v_{O'}| < |v_M|$. According to the clock of O, we assume that the coordinate $t_x$ in the t-axis, is the time needed for M to cover any distance $x-x_0$ along the x-axis; let’s call it, time interval $t_x$.
1) Let $x_{O'}$ be the coordinate of M to time $t_{O'}$, according to the clock of O;
2) Let $x_M$ be the coordinate of point M to time $t_M$, according to the clock of O, when the coordinate of O' is $x_{O'}$.

Whether, to a generic point $x > 0$ of the x-axis, we assign from the origin O a spatial distance of 10m, If I communicate by phone that distance to my friend, for my friend 10m may represent any distance, because it is not defined the numerical base; for example: $(10)123$ m = 123m, $(10)12$ m = 12m. This distance becomes quantifiable if my friend know its numerical base b ($b > 1$ and integer).
Then, if at time $t_M$ according to the clock of O, we assign to the point M a spatial distance of 10m from the origin O, we define $X_M = (10)t_{M}$ and let’s assume the time interval $t_M$, as the numerical base of the spatial distance from the origin O, of any point of x-axis. Likewise, in the t-axis we define $t_M = (10)_{X_M}$ m. See picture 1.

*We choose the unit of measurement in such a way that $t_M$ and $X_M$ are integer and positive numbers: $t_M > 1$, $X_M > 1$. 
If $x_M$ is the coordinate of $M$ at time $t_M$ according to the clock of $O$, let $M(On)$ be the value of the time interval $\Delta t_n = t_M - t_n$, from the point $M$ to point $O_n$, then we have the two equalities:

$m$) $M(O) = (10)_{x_M}ds = (t_M)_{asse t}$

$n$) $M(O') = (10 - t_{o'})_{x_M}ds = (t'_M)_{asse t}$

Conversely, if let’s assume the time interval $t_m - t_n$ as the base of the spatial distance of the point $M$ from the origin $O$, we have:

$m')$ $M(O) = (10)t_m m = (x_M)_{asse t}$

$n')$ $M(O') = (10 + x_{o'})t_m - t_{o'} m = (x'_M)_{asse t}$ see point 1

with $(x_m)_{asse t} = (x'_M)_{asse t}$.

As a variation of the distance $x_M$ of point $M$ corresponds to a variation of time interval $t_M$, then for the representation $(10)t_m m$ and $(10)_{x_M}ds$ of the points, the following proportion must be apply:
Let's find the time of O' with respect to time of O.

If we replace the \((x'\text{asse } x, (x'\text{asse } t)\) coordinates, with the \((x'\text{asse } x = (10 - x_o')\text{asse } t, (x'\text{asse } t = (10 + x_o')\text{asse } t)\) coordinates, we write the formula 1.3) as

\[
\frac{(t'\text{asse } x'}{\sqrt{(t'\text{asse } t)} = \frac{(t\text{asse } x}{\sqrt{(t\text{asse } t)}} \frac{(X'\text{asse } x}{(X\text{asse } x)} \frac{(X'\text{asse } t}{(X\text{asse } t)}
\]

\[
t' = t \sqrt{\frac{(X'\text{asse } x}{(X\text{asse } x)} \frac{(X'\text{asse } t}{(X\text{asse } t)}
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To perform mathematical operations in 1.4), we must assign for each number, the same numerical base, i.e. we must express \((x'\text{asse } t)\) in the base \(t\).

When we talked about spatial distance, we realized that a spatial distance of 10m becomes quantifiable when we know its numerical base; if we argue that the numerical base of space, is the time it takes the point \(M\) to move from one point to another, as we define \(to' = (10)x_o'\) ds (see point 1), then a variation of time implies a variation of the spatial distance.

Then, if let's \(X_M\) in the base \(t_M - t_o'\), to obtain \(X_M\) in the base \(t_M\), we must add up to \(t_M - t_o'\) the time interval \(t_o'\) and therefore, as \(t_o' = (10)x_o'\) ds, we must add to the spatial distance \(x_M\) the spatial distance \(x_o'\).
So \((x'_M)_{t_{M}} = (x_M)_{t_{M}}\) becomes \(x_M + x_0'\) in the base \(t_M\), i.e. \((10 + x_0')_{t_{M}}\). Then the formula 1.4) becomes

\[
t'_M = t_M \sqrt{\frac{(X_M - X_0')(X_M + X_0')}{X_M^3}} = t_M \sqrt{\frac{X_M^2 - X_0'^2}{X_M^3}}
\]

\[
t'_M = t_M \sqrt{1 - \frac{X_0'^2}{X_M^2}} \quad 1.5)
\]

\(t'_M\) is the time marked by the clock of \(O'\), when the clock of \(O\) marks \(t_M\). Likewise from 1.1) we define:

\[
x'_M = X_M \sqrt{1 - \frac{t_0'^2}{t_M^2}} \quad 1.6)
\]

i.e. in the reference frame of \(O'\), \(x'_M\) is the distance covered by \(M\), from the origin \(O\) to point \(x_M\).

Therefore for the point \(O'\), the distance covered in 2) don't is \(x_0'\), but

\[
x'_O = X_0 \sqrt{1 - \frac{t_0'^2}{t_M^2}}
\]

**CONCLUSION**

1) The formula 1.1), which can also be written as \(v_M : 1 / v_M = v'_M : 1 / v'_M\), imply \(v_M = v'_M\). Therefore, let's given a point \(M\) in motion along the x-axis in the IRS(O xyt), that cover a distance \(x_M\) in time \(t_M\) according to the clock of \(O\), if \(t_M\) is the numerical base of space (in this case of points in x-axis), i.e. \(x_M = (10)t_{M}m\) or conversely \(t_M = (10)x_Mds\), the speed of \(M\) is constant in all IRS.

The same example should also apply to the y-axes.

The speed of \(M\) is constant because the point \(M\), with respect to other points in motion, possesses the following property: the time \(t_M\) necessary to \(M\) for cover any distance \(x_M\) is the numerical base of \(x_M\), such that \(x_M = (10)t_Mm\).
2) According to the clock of O, at time $t_M$, the spatial distance covered by points O' and M in the IRS(O xyt), is respectively $x_o'$ and $x_M$. Therefore, with $x_M/t_M = v_M = c$, the 1.5) represents Einstein’s time dilation formula and we can assert that the speed of point M is the speed of light. Likewise, substituting ds with m inside the square root, the 1.6) is Einstein’s length contraction formula.

3) As $b > 1$, in agreement with the theory of relativity, the assumption $|v_o'| \geq |v_M|$ is an absurd case. In fact, for $n'$, $|v_o'| \geq |v_M| \Rightarrow b \leq 0$.

So, take the time as a numerical base of space, the speed of light is an insuperable limit.

* in the case of negative coordinates we write $(-1)x_M = (10)t_M (-1)m$ instead of $x_M = (10)t_m m$.

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