Some mathematics inspired by 137.036

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The experimental value of the fine structure constant inverse from physics (approximately 137.036) is shown to also have an interesting role in pure mathematics. Specifically, 137.036 is shown to occur in the minimal solution to one of several slightly asymmetric equations (that is, equations whose left- and right-hand sides are very similar).

I. INTRODUCTION

The experimental value of the fine structure constant inverse from physics (approximately 137.036) [1, 2] will be shown to also have an interesting role in pure mathematics. Specifically, 137.036 will be shown to occur in the minimal solution to one of several slightly asymmetric equations.

An equation is slightly asymmetric if its left- and right-hand sides are very similar. Such equations may be produced by breaking the symmetry of a simple algebraic identity by applying to it a *substitution map* or *rewriting system* [3]. Hence, the equations that follow are less arbitrary than they might at first seem, as their form derives from two slightly asymmetric equations already analyzed, which were produced in the above manner [4].

II. A PAIR OF SLIGHTLY ASYMMETRIC EQUATIONS

We begin by introducing a pair of slightly asymmetric equations and inspecting their solutions. Let

$$\frac{(M-\epsilon_0)^3}{N^3} + (M-\epsilon_0)^2 = \frac{M^3 - M^0}{N^3} + M^2 \tag{1a}$$

and

$$\frac{(M-\epsilon_1)^3}{N^3} + (M-\epsilon_1)^2 = \frac{M^3 - M^{-3}}{N^3} + M^2 - M^{-3} \quad , \tag{1b}$$

where ϵ_0 and ϵ_1 are variables such that

$$0 < \epsilon_0 < 0.1 \tag{1c}$$

$$0 < \epsilon_1 < 0.1 \tag{1d}$$

and M and N are positive integer constants where

$$M = \frac{N^3}{3} + 1 \quad , \tag{1e}$$

so that necessarily

$$M \geq 10 \qquad N \geq 3 \quad . \tag{1f}$$

To understand these equations it is useful to identify values for ϵ_0 and ϵ_1 that fulfill Eqs. (1a) and (1b) for $M = 10$ and $N = 3$, which are the smallest integers allowed by Eq. (1e), "the minimal solution" referred to at the outset. Applying the above assignments to Eq. (1a) gives

$$\begin{aligned} \frac{(10-\epsilon_0)^3}{3^3} + (10-\epsilon_0)^2 &= \frac{10^3 - 10^0}{3^3} + 10^2 \\ &= 137 \quad , \end{aligned} \tag{2a}$$

and to Eq. (1b) gives

$$\begin{aligned} \frac{(10-\epsilon_1)^3}{3^3} + (10-\epsilon_1)^2 &= \frac{10^3 - 10^{-3}}{3^3} + 10^2 - 10^{-3} \\ &= \frac{999.999}{3^3} + 99.999 \\ &= 137.036 \quad . \end{aligned} \tag{2b}$$

These, in turn, give

$$\epsilon_0 = \frac{1}{839.932\,138\,792\,177\,197\ldots} \tag{2c}$$

$$\epsilon_1 = \frac{1}{29\,999.932\,142\,743\,338\,577\ldots} \quad . \tag{2d}$$

Observe that the above denominators are both somewhat close to integers, with decimal portions beginning with

$$0.9321 \quad .$$

Might this just be a coincidence? The next smallest positive integers fulfilling Eq. (1e), namely, $M = 73$ and $N = 6$, will help resolve this issue. Repeating the earlier substitutions now gives

$$\begin{aligned} \frac{(73-\epsilon_0)^3}{6^3} + (73-\epsilon_0)^2 &= \frac{73^3 - 73^0}{6^3} + 73^2 \\ &= 7130 \end{aligned} \tag{3a}$$

and

$$\begin{aligned} \frac{(73-\epsilon_1)^3}{6^3} + (73-\epsilon_1)^2 &= \frac{73^3 - 73^{-3}}{6^3} + 73^2 - 73^{-3} \\ &= 7130.004\,627\,047\,147\ldots \quad , \end{aligned} \tag{3b}$$

which in turn give

$$\epsilon_0 = \frac{1}{47\,522.990\,846\,536\,145\,793\ldots} \tag{3c}$$

$$\epsilon_1 = \frac{1}{85\,194\,722.990\,846\,537\,465\,332\ldots} \quad . \tag{3d}$$

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Again the denominators are close to integers (indeed, even closer), where their decimal portions now each begin with

$$0.990\,846\,53 \quad .$$

This suggests that coincidence is unlikely.

III. ANALYSIS OF DENOMINATORS

It is not hard to find a pattern in the denominators of Eqs. (2c), (2d), (3c), and (3d) *to the left* of their decimal points. For the denominators of ϵ_0 in Eqs. (2c) and (3c) we find that, respectively,

$$\begin{aligned}(N^3+2)^2-1 &= (3^3+2)^2-1 \\ &= 840 \\ &\approx 839.932\,138\,792\,177\,197\ldots \quad ,\end{aligned}$$

and

$$\begin{aligned}(N^3+2)^2-1 &= (6^3+2)^2-1 \\ &= 47523 \\ &\approx 47\,522.990\,846\,536\,145\,793\ldots \quad ;\end{aligned}$$

whereas for the denominators of ϵ_1 in Eqs. (2d) and (3d) we find that, respectively,

$$\begin{aligned}3M^4 &= 3\times 10^4 \\ &= 30\,000 \\ &\approx 29\,999.932\,142\,743\,338\,577\ldots \quad ,\end{aligned}$$

and

$$\begin{aligned}3M^4 &= 3\times 73^4 \\ &= 85\,194\,723 \\ &\approx 85\,194\,722.990\,846\,537\,465\,332\ldots \quad .\end{aligned}$$

Hence, one would tend to expect that the approximations

$$\begin{aligned}&\frac{\left(M-\dfrac{1}{(N^3+2)^2-1}\right)^3}{N^3}+\left(M-\frac{1}{(N^3+2)^2-1}\right)^2 \\ &\quad\approx \frac{M^3-M^0}{N^3}+M^2 \end{aligned} \tag{5a}$$

and

$$\begin{aligned}&\frac{\left(M-\dfrac{1}{3M^4}\right)^3}{N^3}+\left(M-\frac{1}{3M^4}\right)^2 \\ &\quad\approx \frac{M^3-M^{-3}}{N^3}+M^2-M^{-3} \end{aligned} \tag{5b}$$

should prove especially accurate. Theorems 1 and 2, which follow, will show that these approximations are, in fact, quite accurate.

Theorem 1. *Let*

$$\begin{aligned}\epsilon_A \;=\; &\left(\frac{(M-\epsilon_0)^3}{N^3}+(M-\epsilon_0)^2\right) \\ &-\left(\frac{M^3-M^0}{N^3}+M^2\right) \quad ,\end{aligned} \tag{6a}$$

where

$$\epsilon_0=\frac{1}{(N^3+2)^2-1} \quad , \tag{6b}$$

and where M and N are positive integer constants such that

$$M=\frac{N^3}{3}+1 \quad , \tag{6c}$$

so that necessarily

$$M\ge 10 \qquad N\ge 3 \quad . \tag{6d}$$

Then

$$\epsilon_A=\frac{2K^3+11K^2+18K+8}{K(K^2+4K+3)^3} \quad , \tag{6e}$$

where

$$K=N^3 \quad . \tag{6f}$$

Proof. Equations (6c) and (6f) allow substitution for M and N^3 in Eq. (6a) to get

$$\epsilon_A=\frac{L+H}{K(K^2+4K+3)^3} \quad ,$$

where L and H are composed of these *lower*- and *higher*-degree terms:

$$\begin{aligned}L \;=\; &26+137K^3+175K^2+111K \\ &+9\left(\frac{K}{3}+1\right) \quad -\;27\left(\frac{K}{3}+1\right)^2 \\ &-6\left(\frac{K}{3}+1\right)K \;-\;45\left(\frac{K}{3}+1\right)K^2 \\ &-72\left(\frac{K}{3}+1\right)^2K \\ H \;=\; &57K^4+12K^5+K^6 \\ &-44\left(\frac{K}{3}+1\right)K^3-\;3\left(\frac{K}{3}+1\right)^2K^4 \\ &-2\left(\frac{K}{3}+1\right)K^5\;-\;24\left(\frac{K}{3}+1\right)^2K^3 \\ &-16\left(\frac{K}{3}+1\right)K^4-\;66\left(\frac{K}{3}+1\right)^2K^2 \quad .\end{aligned}$$

Conveniently, all of the fourth, fifth, and sixth degree terms of H collectively cancel, so that

$$H=-112K^3-66K^2 \quad ,$$

whereas L simplifies to

$$L = 114K^3 + 77K^2 + 18K + 8 \quad .$$

This leads to further large cancellations, namely

$$\begin{aligned}\epsilon_A &= \frac{L+H}{K(K^2+4K+3)^3} \\ &= \frac{(114K^3+77K^2+18K+8)+(-112K^3-66K^2)}{K(K^2+4K+3)^3} \\ &= \frac{(114K^3-112K^3)+(77K^2-66K^2)+18K+8}{K(K^2+4K+3)^3} \\ &= \frac{2K^3+11K^2+18K+8}{K(K^2+4K+3)^3} \quad . \qquad \square\end{aligned}$$

Remark 1. It follows from Eqs. (6d) and (6f) that

$$\epsilon_A \leq \frac{47\,879}{16\,003\,008\,000}.$$

Theorem 2. *Let*

$$\begin{aligned}\epsilon_B \;=\; &\left(\frac{(M-\epsilon_1)^3}{N^3} + (M-\epsilon_1)^2\right) \\ &- \left(\frac{M^3 - M^{-3}}{N^3} + M^2 - M^{-3}\right) \quad , \end{aligned} \tag{8a}$$

where

$$\epsilon_1 = \frac{1}{3M^4} \quad , \tag{8b}$$

and where M and N are positive integer constants such that

$$M = \frac{N^3}{3} + 1 \quad , \tag{8c}$$

so that necessarily

$$M \geq 10 \qquad N \geq 3 \quad . \tag{8d}$$

Then

$$\epsilon_B = \frac{1}{3M^7N^3} + \frac{1}{9M^8} - \frac{1}{27M^{12}N^3} \quad . \tag{8e}$$

Proof. Equation (8a) expands and simplifies to

$$\begin{aligned}\epsilon_B = &\frac{-27M^{10}+9M^5-1}{27M^{12}N^3} + \frac{3M^4N^3-18M^9N^3}{27M^{12}N^3} \\ &- \left(-\frac{1}{M^3N^3} - \frac{1}{M^3}\right) \\ = &\frac{-27M^{10}+9M^5-1+3M^4N^3-18M^9N^3}{27M^{12}N^3} \\ &+ \frac{27M^9+27M^9N^3}{27M^{12}N^3} \quad . \end{aligned} \tag{9a}$$

Combining terms gives

$$\epsilon_B = \frac{9M^9N^3+27M^9-27M^{10}+9M^5+3M^4N^3-1}{27M^{12}N^3} \quad .$$

But, Eq. (8c) determines that the three largest terms of the above numerator sum to 0, which is to say that

$$\begin{aligned}&9M^9N^3 + 27M^9 - 27M^{10} \\ &\quad = 9M^9N^3 + 27M^9(1-M) \\ &\quad = 9\left(\frac{N^3}{3}+1\right)^9 N^3 + 27\left(\frac{N^3}{3}+1\right)^9\left(1 - \frac{N^3}{3} - 1\right) \\ &\quad = 9\left(\frac{N^3}{3}+1\right)^9 N^3 - \frac{27}{3}\left(\frac{N^3}{3}+1\right)^9 N^3 \\ &\quad = 0 \quad . \end{aligned} \tag{9b}$$

Hence,

$$\begin{aligned}\epsilon_B &= \frac{9M^5+3M^4N^3-1}{27M^{12}N^3} \\ &= \frac{1}{3M^7N^3} + \frac{1}{9M^8} - \frac{1}{27M^{12}N^3} \quad . \qquad \square\end{aligned} \tag{9c}$$

Remark 2. It follows from Eq. (8d) that

$$\epsilon_B \leq \frac{1\,709\,999}{729\,000\,000\,000\,000} \quad .$$

Remark 3. The cancellations that take place above derive from the first expression on the right-hand side of Eq. (8a) expanding into $4+3$ terms, while its second expression is comprised of $2+2$ terms. The four highest powers of M cancel irrespective of how M is defined, leaving the $3+2$ terms and $1+1$ terms at the top of Eq. (9a). Combining terms reduces the four largest terms to three, where, by the substitutions of Eq. (8c), these three cancel, leaving just the $2+1$ and $0+0$ terms of Eq. (9c). Hence, all $2+1=3$ terms of Eq. (9c) derive from uncanceled portions of the first expression on the right-hand side of Eq. (8a).

IV. ANALYSIS OF THEOREMS

Theorems 1 and 2 prove that Eqs. (5a) and (5b) are excellent approximations given the following definitions

$$\begin{aligned}\epsilon_0 &= \frac{1}{(N^3+2)^2-1} \\ \epsilon_1 &= \frac{1}{3M^4} \quad . \end{aligned}$$

Their effectiveness appears to stem from the cancellation of many higher degree terms, which, in turn, is a consequence of the restriction that $M = N^3/3 + 1$. So far, so good. But the denominators above are integers, and hence do nothing to fit the decimal portions of the denominators of Eqs. (2c), (2d), (3c), and (3d) (that is, the digits *to the right* of their decimal points).

TABLE I: This table has six rows, each row having four lines. In each row the value $\kappa(N)$ (first line) grows ever closer to the decimal portions of $1/\epsilon_0$ and $1/\epsilon_1$ (second and third lines), where ϵ_0 and ϵ_1 derive from Eqs. (1a) and (1b), respectively. The decimal portion of $1/\epsilon_2$ (fourth line) derives from Eq. (14a). Digits that all agree are in boldface.

$N=3$
$0.\mathbf{932}\,142\,857\,142\,857\ldots$
$0.\mathbf{932}\,138\,792\,177\,197\ldots$
$0.\mathbf{932}\,142\,743\,338\,577\ldots$
$0.\mathbf{932}\,028\,626\,207\,426\ldots$
$N=6$
$0.\mathbf{990\,846}\,537\,466\,069\ldots$
$0.\mathbf{990\,846}\,536\,145\,793\ldots$
$0.\mathbf{990\,846}\,537\,465\,332\ldots$
$0.\mathbf{990\,846}\,250\,946\,436\ldots$
$N=9$
$0.\mathbf{997\,265\,8}88\,165\,281\,832\,472\ldots$
$0.\mathbf{997\,265\,8}88\,154\,794\,576\,665\ldots$
$0.\mathbf{997\,265\,8}88\,165\,281\,305\,466\ldots$
$0.\mathbf{997\,265\,8}80\,509\,537\,517\,610\ldots$
$N=12$
$0.\mathbf{998\,844\,26}4\,373\,772\,720\,028\ldots$
$0.\mathbf{998\,844\,26}4\,373\,438\,806\,101\ldots$
$0.\mathbf{998\,844\,26}4\,373\,772\,717\,016\ldots$
$0.\mathbf{998\,844\,26}3\,795\,146\,638\,689\ldots$
$N=15$
$0.\mathbf{999\,407\,84}6\,001\,026\,996\,540\ldots$
$0.\mathbf{999\,407\,84}6\,001\,003\,938\,327\ldots$
$0.\mathbf{999\,407\,84}6\,001\,026\,996\,485\ldots$
$0.\mathbf{999\,407\,84}5\,923\,182\,446\,748\ldots$
$N=18$
$0.\mathbf{999\,657\,211}\,407\,306\,271\,338\ldots$
$0.\mathbf{999\,657\,211}\,407\,303\,682\,194\ldots$
$0.\mathbf{999\,657\,211}\,407\,306\,271\,336\ldots$
$0.\mathbf{999\,657\,211}\,392\,203\,788\,392\ldots$

V. IMPROVED DEFINITIONS

As it turns out, recasting Eqs. (6b) and (8b) as

$$\epsilon_0 = \frac{1}{[(N^3+2)^2-1]-1+\kappa(N)} \tag{10a}$$

$$\epsilon_1 = \frac{1}{3M^4-1+\kappa(N)} \tag{10b}$$

and letting

$$\kappa(N) = \frac{(N^3+0)(N^3+2)}{(N^3+1)(N^3+3)} \tag{11}$$

produces ϵ_0 and ϵ_1 that better approximate ϵ_0 and ϵ_1 as they appear in Eqs. (2c), (2d), (3c), and (3d).

So, if $M=10$ and $N=3$, then Eqs. (10a) and (10b) give

$$\begin{aligned}\epsilon_0 &= \frac{1}{[(3^3+2)^2-1]-1+\kappa(3)} \\ &= \frac{1}{839.\mathbf{932}\,142\,857\,142\,857\ldots} \\ \epsilon_1 &= \frac{1}{3\times 10^4-1+\kappa(3)} \\ &= \frac{1}{29999.\mathbf{932}\,142\,857\,142\,857\ldots} \quad .\end{aligned}$$

The decimal portions of the above denominators both equal $\kappa(3)$ and appear in line one under $N=3$ in Table I, whereas lines two and three hold the decimal portions of the denominators of Eqs. (2c) and (2d). The digits that all agree are in boldface.

In the same way, if $N=6$ and $M=73$, then Eqs. (10a) and (10b) give

$$\begin{aligned}\epsilon_0 &= \frac{1}{[(6^3+2)^2-1]-1+\kappa(6)} \\ &= \frac{1}{47\,522.\mathbf{990\,846}\,537\,466\,069\ldots} \\ \epsilon_1 &= \frac{1}{3\times 73^4-1+\kappa(6)} \\ &= \frac{1}{85\,194\,722.\mathbf{990\,846}\,537\,466\,069\ldots} \quad .\end{aligned}$$

The decimal portions of the above denominators both equal $\kappa(6)$ and appear in line one under $N=6$ in Table I, whereas lines two and three hold the decimal portions of the denominators of Eqs. (3c) and (3d). The digits that all agree are again in boldface. Equivalent calculations for $N=9$ through $N=18$ appear in lines one through three in Table I, where $\kappa(N)$ and the decimal portions of $1/\epsilon_0$ and $1/\epsilon_1$ from Eqs. (1a) and (1b), respectively, are seen to grow ever closer. In this way, $\kappa(N)$ helps approximate *both* ϵ_0 and ϵ_1: a curious result.

VI. ANOTHER SLIGHTLY ASYMMETRIC EQUATION

One immediately suspects that there may be other slightly asymmetric equations linked to $\kappa(N)$, as indeed there are. Let

$$\frac{(M-\epsilon_2)^3}{N^3} + (M-\epsilon_2)^2 = \frac{M^3-M^0}{N^3} + M^2 - M^0 \quad , \tag{14a}$$

so that for $M=10$ and $N=3$

$$\begin{aligned}\frac{(10-\epsilon_2)^3}{3^3} + (10-\epsilon_2)^2 &= \frac{10^3-10^0}{3^3} + 10^2 - 10^0 \\ &= 136 \quad ,\end{aligned} \tag{14b}$$

where Eq. (14a) differs from Eq. (1a) only in having M^0 twice on its right-hand side. The above equation gives

$$\epsilon_2 = \frac{1}{29.\mathbf{932}\,028\,626\,207\,426\ldots} \quad . \tag{14c}$$

The decimal portion of the above denominator appears in line four under $N = 3$ in Table I, with digits matching $\kappa(3)$ in boldface. Now consider that Eq. (2d) gave

$$\epsilon_1 = \frac{1}{29\,999.\mathbf{932}\,142\,743\,338\,577\ldots} \quad ,$$

almost exactly a thousandfold difference in denominators. Ideally, a single formula should cover both cases.

VII. EQUATIONS WITH DIFFERENT EXPONENTS

Given that Eq. (14a) uses M^0 on its right-hand side to produce $\sim 1/29.932\,028\,626\,207\,426$, and that Eq. (1b) uses M^{-3} to produce $\sim 1/29\,999.932\,142\,743\,338\,577$, it seems likely that a general formula using M^{-p} is possible.

To illustrate, let p be an integer such that

$$p \geq -1 \quad ,$$

where

$$\frac{(10-\epsilon)^3}{3^3} + (10-\epsilon)^2 = \frac{10^3 - 10^{-p}}{3^3} + 10^2 - 10^{-p} \quad , \tag{15a}$$

so that for $p = 0$ the above equation recovers Eq. (14b), which produces 136, whereas for $p = 3$ it recovers Eq. (2b), which produces 137.036.

Now, for $p = -1, 0, 1$, etc., Eq. (15a) gives $1/\epsilon$ as:

$$\begin{array}{r}
2.930\,960198\,505082\,351\ldots \\
29.932\,028626\,207426\,996\ldots \\
299.932\,131472\,504679\,103\ldots \\
2999.932\,141719\,061799\,505\ldots \\
29999.932\,142743\,338577\,196\ldots \\
299999.932\,142845\,762467\,404\ldots \\
\\
2999999.932\,142856\,004818\,551\ldots \\
29999999.932\,142857\,029053\,287\ldots \\
299999999.932\,142857\,131476\,757\ldots \\
2999999999.932\,142857\,141719\,104\ldots \\
29999999999.932\,142857\,142743\,339\ldots \\
299999999999.932\,142857\,142845\,76247\ldots \\
\vdots \\
29999999999999999.932\,142857\,142857\,142743\ldots
\end{array}$$

Note that, interestingly, the sequences in red from rows 1–6 repeat in rows 7–12. More importantly, the above values' decimal portions appear to approach

$$0.932\overline{142857} = 0.932 + \frac{1}{7000}$$

for ever larger p. Hence, one would tend to expect that each row, above, differs from

$$\underbrace{299\ldots99}_{p+1 \text{ nines}}.932 + \frac{1}{7000}$$

by an ever smaller amount as $p \to \infty$. But the decimal portion of the above value also can be stated compactly as

$$\begin{aligned}
\kappa(3) &= \frac{(3^3+0)(3^3+2)}{(3^3+1)(3^3+3)} = \frac{261}{280} \\
&= 0.932 + \frac{1}{7000} \quad ,
\end{aligned}$$

so that the above expression can be rewritten compactly as

$$\underbrace{299\ldots99}_{p+1 \text{ nines}} + \kappa(3) \quad .$$

As another illustration, let p be an integer such that

$$p \geq -1 \quad ,$$

where

$$\frac{(10-\epsilon)^3}{3^3} + (10-\epsilon)^2 = \frac{10^3 - 10^{-p}}{3^3} + 10^2 \quad , \tag{15b}$$

so that for $p = 0$ the above equation recovers Eq. (2a), which produces 137.

Now, for $p = -1, 0, 1$, etc., Eq. (15b) gives $1/\epsilon$ as:

$$\begin{array}{r}
83.932\,102158\,599679\,621\ldots \\
839.932\,138792\,177197\,926\ldots \\
8399.932\,142450\,695096\,415\ldots \\
83999.932\,142816\,498569\,040\ldots \\
839999.932\,142853\,078433\,212\ldots \\
8399999.932\,142856\,736414\,798\ldots \\
\\
83999999.932\,142857\,102212\,908\ldots \\
839999999.932\,142857\,138792\,719\ldots \\
8399999999.932\,142857\,142450\,700\ldots \\
83999999999.932\,142857\,142816\,4986\ldots \\
839999999999.932\,142857\,142853\,078433\,7\ldots \\
8399999999999.932\,142857\,142856\,736414\,8\ldots \\
\vdots \\
83999999999999999.932\,142857\,142857\,142853\ldots \\
\vdots \\
\underbrace{8399\ldots99}_{p+1 \text{ nines}} + \kappa(3)
\end{array}$$

Again, the sequences in red from rows 1–6 repeat in rows 7–12. More importantly, the above values' decimal portions likewise appear to approach $\kappa(3)$ for ever larger p.

VIII. A GENERAL FORMULA

Of course, Eq. (15a) can be expressed more generally in terms of M and N. Let

$$\frac{(M-\epsilon)^3}{N^3} + (M-\epsilon)^2 = \frac{M^3 - M^{-p}}{N^3} + M^2 - M^{-p} \quad , \tag{16a}$$

where M and N are positive integer constants such that

$$M = \frac{N^3}{3} + 1 \quad , \tag{16b}$$

and p is an integer where

$$p \geq -1 \quad .$$

Then, as $p \to \infty$, the decimal portion of $1/\epsilon$ appears to approach $\kappa(N)$ as a limit.

So, if $N = 6$, then Eq. (16b) gives $M = 73$, so that with $p = 15$, Eq. (16a) gives

$$\frac{(73-\epsilon)^3}{6^3} + (73-\epsilon)^2 = \frac{73^3 - 73^{-15}}{6^3} + 73^2 - 73^{-15} \quad .$$

This, in turn, gives

$$\begin{aligned} \frac{1}{\epsilon} = 1\,95113\ 36394\ 53428\ 71312\ 63685 \\ 03682.99084\ 65374\ 66069\ 06129 \\ 66353\ 13427\ 1825\ldots \quad , \end{aligned}$$

whose decimal portion is reproduced to 33 digits by

$$\begin{aligned} \kappa(N) &= \kappa(6) \\ &= \frac{(6^3+0)(6^3+2)}{(6^3+1)(6^3+3)} \\ &= \frac{15696}{15841} \\ &= 0.99084\,65374\,66069\,06129\,66353\,13427\,1826\ldots \quad . \end{aligned}$$

As the integer portion of $1/\epsilon$ equals $3 \times 73^{16} - 1$, or $3M^{p+1} - 1$, one can guess that for Eq. (16a) the compact equation

$$\epsilon = \frac{1}{3M^{p+1} - 1 + \kappa(N)} \tag{16c}$$

gives a very accurate approximation of ϵ when p is large.

One could go on multiplying examples of the above type indefinitely, but the general goal of demonstrating that 137.036 resides at a minimum associated with some interesting mathematics has, the author hopes, already been achieved. And although some key mathematical tasks have been left undone—e.g., proving that $\kappa(N)$, as used above, actually represents a limit—this is partly because the purpose here has been to raise more intriguing questions than are resolved.

IX. SUMMARY AND CONCLUSION

At the outset it was claimed that the experimental value of the fine structure constant inverse from physics (approximately 137.036) would be shown to have an interesting role in pure mathematics: specifically, that it would occur in the minimal solution to one of several slightly asymmetric equations. Equation (1b) is this "slightly asymmetric equation," where, as shown by Eq. (2b), the value 137.036 *does* occur when its positive integers M and N are at a minimum.

But, has 137.036 *also been shown to have an interesting role in pure mathematics?*

Admittedly, 137.036 does not appear to be nearly as important to mathematics as it is to physics, where it is a coupling constant that has fascinated a succession of illustrious physicists from Einstein onward [1]. Nevertheless, the various equations associated with 137.036 do provoke a measure of interest of their own, in particular the way that Eq. (2b) produces

$$137.036 = \frac{999.999}{3^3} + 99.999 \quad ,$$

while simultaneously producing

$$\begin{aligned} 137.036 = {} & \left(\frac{10}{3} - \frac{1}{3 \times 29\,999.932\ldots}\right)^3 \\ & + \left(10 - \frac{1}{29\,999.932\ldots}\right)^2 \end{aligned} \tag{17}$$

with the assistance of an unexpectedly round number: $\sim$29 999.932. Also suggestive is how the higher degree terms in Theorem 2 neatly cancel, leaving only the much smaller terms responsible for the above nearly round result; and the way that the function $\kappa(N)$ approximates the decimal portions of the denominators of so many variants of ϵ, as well as the decimal portion of the above denominator. And, finally, there is the (perhaps unexpected) economy of some of the equations, for instance, Eq. (16c).

Of course, this begs the broader question of whether $\sim$137.036 occurs concurrently in mathematics and physics as a matter of coincidence; but, as the experimental fine structure constant inverse measures 137.035 999 074 (44), which 137.036 fits within seven parts per billion [2], the degree of coincidence (if it is such) must be remarkable. Moreover, the four expressions on the right-hand side of Eq. (17)

$$\begin{array}{cc} \dfrac{10}{3} & \dfrac{1}{3 \times 29\,999.932\ldots} \\ 10 & \dfrac{1}{29\,999.932\ldots} \end{array}$$

can be reproduced from the sines squared of the quark and lepton angles, as shown in [5, 6]. Given the precision with which some of these six angles are known (e.g., the Cabibbo angle) the collective weight of evidence suggests that it may be *non*-coincidence that strains credulity the least.

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