Lorentz Violation and Modified Cosmology

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Abstract

We propose a modification of Einstein-Cartan gravity equations and study the related applications to cosmology, in an attempt to account for cosmological mass discrepancies without resorting to dark matter. The deviation from standard model of cosmology is noticeable when the Hubble parameter becomes comparable to or less than a characteristic scale.

Keywords. Lorentz violation, modified gravity, torsion, dark matter, characteristic Hubble scale.

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1 Introduction

A modification of Einstein-Cartan equations has recently been propounded[1]. The modified torsion explicitly breaks local Lorentz gauge symmetry[2], while preserving diffeomorphism invariance. In the weak field limit, galactic rotation curves are explained without invoking dark matter.

In this paper, we apply our modified gravity theory to cosmology. Friedmann equations are updated and their implications are discussed.

2 Gauge Theory of Gravity

In de Sitter gauge theory of gravity[3, 4], gravitational gauge field can be written as a Clifford-valued 1-form[5, 6, 7, 1]

$$A = \frac{1}{l}e + \omega, \tag{1}$$

$$e = e^a\gamma_a = e^a_\mu dx^\mu \gamma_a, \tag{2}$$

$$\omega = \frac{1}{4}\omega^{ab}\gamma_{ab} = \frac{1}{4}\omega^{ab}_\mu dx^\mu \gamma_{ab}, \tag{3}$$

where e is vierbein, ω is spin connection, $\mu, a, b = 0, 1, 2, 3$, $\omega^{ab}_\mu = -\omega^{ba}_\mu$, and $\gamma_{ab} \equiv \gamma_a\gamma_b$. Here we adopt the summation convention for repeated indices. Clifford algebra vectors γ_a observe anticommutation relations

$$\{\gamma_a, \gamma_b\} \equiv \frac{1}{2}(\gamma_a\gamma_b + \gamma_b\gamma_a) = \eta_{ab}, \tag{4}$$

where η_{ab} is of signature $(+, -, -, -)$.

The constant l is related to Minkowskian vacuum expectation value (VEV) of gravity gauge field

$$\bar{A} = \frac{1}{l}\bar{e} + \bar{\omega} = \frac{1}{l}\delta^a_\mu dx^\mu \gamma_a. \tag{5}$$

Gravity curvature 2-form is given by

$$F = dA + A^2 = R + \frac{1}{l}T + \frac{1}{l^2}e^2, \tag{6}$$

where spin connection curvature 2-form R and torsion 2-form T are defined by

$$R = d\omega + \omega^2 = \frac{1}{4}R^{ab}\gamma_{ab} = \frac{1}{4}(d\omega^{ab} + \eta_{cd}\omega^{ac}\omega^{db})\gamma_{ab}, \tag{7}$$

$$T = de + \omega e + e\omega = T^a\gamma_a = (de^a + \eta_{bc}\omega^{ab}e^c)\gamma_a. \tag{8}$$

$$\tag{9}$$

Here exterior $\wedge$ products between forms are implicitly assumed.

One can write down the action for general relativity as[5, 6, 1]

$$S_G = \frac{c^4}{8\pi G}\int \left\langle -ie^2 F\right\rangle \tag{10}$$

$$= \frac{c^4}{8\pi G}\int \left\langle -ie^2 (R + \frac{1}{l^2}e^2)\right\rangle \tag{11}$$

$$= \frac{c^4}{8\pi G}\int \left\langle -ie^2 (R + \frac{\Lambda}{24}e^2)\right\rangle \tag{12}$$

$$= \frac{c^4}{32\pi G}\int \epsilon_{abcd}e^a e^b (R^{cd} + \frac{\Lambda}{6}e^c e^d), \tag{13}$$

$$\tag{14}$$

where Λ is cosmological constant

$$\Lambda = \frac{24}{l^2}, \tag{15}$$

c is speed of light, G is Newton constant[1], i is Clifford unit pseudoscalar

$$i = \gamma_0\gamma_1\gamma_2\gamma_3, \tag{16}$$

and $\langle\cdots\rangle$ means Clifford scalar part of enclosed expression. The action of gravity is invariant under local Lorentz gauge transformations.

Field equations are derived by varying total action

$$S = S_G + S_M \tag{17}$$

with gauge fields e and ω independently, where S_M is matter part of the action. The resulted Einstein-Cartan equations read

$$\frac{c^4}{8\pi G}(Re + eR + \frac{\Lambda}{6}e^3) = \mathbb{T}i, \tag{18}$$

$$\frac{c^4}{8\pi G}(Te - eT) = \frac{1}{2}\mathbb{S}i, \tag{19}$$

where $\mathbb{T}$ is energy-momentum current 3-form, and $\mathbb{S}$ is spin current 3-form.

3 Lorentz Violation and Modified Gravity Equations

Local Lorentz gauge transformation is characterized by

$$\mathbb{R}_L(x) = e^{\frac{1}{2}\epsilon^{ab}(x)\gamma_{ab}}, \tag{20}$$

[1]See [5, 6] for how Newton constant G is related to l and VEV of gravity Higgs field.

where $\epsilon^{ab}(x) = -\epsilon^{ba}(x)$, and γ_{ab} are generators of Lorentz algebra. Gauge field 1-form $e(x)$, spin connection curvature 2-form $R(x)$, and torsion 2-form $T(x)$ transform as[2]

$$V(x) \quad \rightarrow \quad \mathbb{R}_L(x)V(x)\mathbb{R}_L(x)^{-1}. \tag{21}$$

The Einstein-Cartan equations are covariant under local Lorentz gauge transformation, thanks to above transformation property for $e(x)$, $R(x)$, and $T(x)$.

With the assumption of Lorentz symmetry violation, we study the remaining symmetry under local gauge transformation

$$\mathbb{R}_S(x) = e^{\frac{1}{2}\epsilon^{jk}(x)\gamma_{jk}}, \tag{22}$$

where $j, k = 1, 2, 3$. Gravity gauge fields

$$e_S = e^j\gamma_j = e^j_\mu dx^\mu \gamma_j, \tag{23}$$
$$e_T = e^0\gamma_0 = e^0_\mu dx^\mu \gamma_0, \tag{24}$$
$$\omega_T = \frac{1}{4}(\omega^{j0}\gamma_{j0} + \omega^{0j}\gamma_{0j}) = \frac{1}{2}\omega^{j0}_\mu dx^\mu \gamma_{j0}, \tag{25}$$

transform as

$$V(x) \quad \rightarrow \quad \mathbb{R}_S(x)V(x)\mathbb{R}_S(x)^{-1}, \tag{26}$$

while gauge field

$$\omega_S = \frac{1}{4}\omega^{jk}\gamma_{jk} \tag{27}$$

transforms differently as

$$\omega_S(x) \quad \rightarrow \quad \mathbb{R}_S(x)\omega_S(x)\mathbb{R}_S(x)^{-1} - d\mathbb{R}_S(x)\mathbb{R}_S(x)^{-1}. \tag{28}$$

With violation of Lorentz symmetry, we propose a change to Einstein-Cartan equation (19) in the form:

$$\frac{c^4}{8\pi G}(\tilde{T}e - e\tilde{T}) = \frac{1}{2}\mathbb{S}i. \tag{29}$$

Here modified torsion 2-form $\tilde{T}$ is defined by

$$\tilde{T} = T + \Delta T_S + \Delta T_T, \tag{30}$$

[2] See e.g. chapter IX.7 of [8] for discussions about local Lorentz gauge transformation in the context of differential forms.

where

$$\Delta T_S = (\alpha_S z)^{-\frac{1+\delta}{2}}(\omega_T e_S + e_S\omega_T), \tag{31}$$

$$\Delta T_T = (\alpha_T z)^{-\frac{1+\delta}{2}}(\omega_T e_T + e_T\omega_T), \tag{32}$$

$$z = |\frac{12(\frac{e}{l})^2(\omega_T\frac{e}{l} + \frac{e}{l}\omega_T)}{(\frac{e}{l})^4}| = |\frac{12le^2(\omega_T e + e\omega_T)}{e^4}|. \tag{33}$$

The modified torsion $\tilde{T}$ breaks local Lorentz gauge symmetry, while preserving diffeomorphism invariance. Because of the transformation property (26) for e_S, e_T, and ω_T, the modified Einstein-Cartan equations (18) and (29) are covariant under local gauge transformation (22)[3].

Three dimensionless parameters δ, α_S and α_T are to be determined by comparing predictions of our proposal with astronomical observations. If α_S and α_T are equal, the modification to torsion can be written as

$$\Delta T = \Delta T_S + \Delta T_T = (\alpha z)^{-\frac{1+\delta}{2}}(\omega_T e + e\omega_T), \tag{34}$$

where $\alpha = \alpha_S = \alpha_T$.

In our earlier paper[1], it has been shown that one can recover Modified Newtonian Dynamics[9, 10](MOND) in weak field limit by setting parameter δ to zero[4]. Galactic rotation curves are explained without invoking dark matter. The characteristic acceleration scale a_0 is given by

$$a_0 = \frac{c^2}{\alpha_S l}. \tag{35}$$

It is intrinsically linked to cosmological constant (15) as

$$a_0 = \frac{c^2}{\alpha_S}\left(\frac{\Lambda}{24}\right)^{\frac{1}{2}}. \tag{36}$$

4 Cosmology and Modified Friedmann Equations

In this section, we apply our modification to cosmology[5]. The spatially homogeneous and isotropic universe is described by Robertson-Walker (RW) metric

$$ds^2 = c^2dt^2 - a(t)^2\left(\frac{dr^2}{1-\kappa r^2/R_0^2} + r^2 d\Omega^2\right), \tag{37}$$

[3]Since ω_S transforms differently as (28), the modified torsion can not be dependent on ω_S individually.

[4]See [11, 12] for two examples of different relativistic MONDian theories. See [10] for a comprehensive list.

[5]See [13] for a review of other modified gravity theories and their applications in cosmology.

where $\Omega^2 = d\theta^2 + sin^2\theta d\phi^2$. With the above metric, (18) and (29) are reduced to modified Friedmann equations as

$$\tilde{H}^2 = \frac{8\pi G}{3}\rho + \frac{c^2}{3}\Lambda - \frac{\kappa c^2}{R_0^2 a^2}, \tag{38}$$

$$\frac{d(a\tilde{H})/dt}{a} = -\frac{4\pi G}{3}(\rho + \frac{3}{c^2}p) + \frac{c^2}{3}\Lambda, \tag{39}$$

where

$$\tilde{H}\left(1 + \left(\tilde{H}/h_0\right)^{-\frac{1+\delta}{2}}\right) = H, \tag{40}$$

$$H = \frac{\dot{a}}{a} = \frac{da/dt}{a}, \tag{41}$$

$$h_0 = \frac{c}{3\alpha_T l} = \frac{c}{3\alpha_T}\left(\frac{\Lambda}{24}\right)^{\frac{1}{2}}. \tag{42}$$

Here H is Hubble parameter, $\tilde{H}$ is modified Hubble parameter, h_0 is a characteristic Hubble scale, ρ is mass density, and p is pressure. Spin current $\mathbb{S}$ is assumed to be zero. It is noted that torsion modification ΔT_T is relevant for RW metric, while ΔT_S is relevant for Schwarzschild metric. Hence, h_0 and a_0 are dependent on α_T and α_S, respectively. The relation between the characteristic Hubble scale (42) and MOND acceleration scale (35) is

$$h_0 = \frac{1}{3c}\frac{\alpha_S}{\alpha_T}a_0. \tag{43}$$

Since the free parameter δ is estimated to be very close to zero[1], we will assume that $\delta = 0$ in the following analysis. The modified Hubble parameter $\tilde{H}$ is determined via equation (40) as

$$\tilde{H} = \mu(H/h_0)H, \tag{44}$$

with interpolation function[6]

$$\mu(x) \quad \rightarrow \quad 1 \qquad\qquad for \quad x \gg 1, \tag{45}$$

$$\mu(x) \quad \rightarrow \quad x \qquad\qquad for \quad x \ll 1. \tag{46}$$

In the limit of $H \gg h_0$, one has $\tilde{H} \simeq H$. Therefore, (38) and (39) are reduced to the usual Friedmann and acceleration equations[14, 15] as,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho + \frac{c^2}{3}\Lambda - \frac{\kappa c^2}{R_0^2 a^2}, \tag{47}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + \frac{3}{c^2}p) + \frac{c^2}{3}\Lambda, \tag{48}$$

[6]One can potentially regard (38) as a phenomenological model, with the modified Hubble parameter specified by the above interpolation function.

where $\ddot{a} = d^2a/dt^2$.

In the opposite limit of $H \ll h_0$, $\tilde{H}$ is given by

$$\tilde{H} \simeq \frac{H}{h_0} H. \tag{49}$$

The modified Friedmann equations then read

$$\frac{1}{h_0^2}\left(\frac{\dot{a}}{a}\right)^4 = \frac{8\pi G}{3}\rho + \frac{c^2}{3}\Lambda - \frac{\kappa c^2}{R_0^2 a^2}, \tag{50}$$

$$\frac{1}{h_0}\left(\frac{2\dot{a}\ddot{a}}{a^2} - \frac{\dot{a}^3}{a^3}\right) = -\frac{4\pi G}{3}(\rho + \frac{3}{c^2}p) + \frac{c^2}{3}\Lambda. \tag{51}$$

Let's study a simple case of one-component universe with $\kappa = 0$, $\Lambda = 0$, and matter density $\rho = \rho_0 a^{-3}$. We assume that it starts with $H \gg h_0$. From equation (47), $\dot{a}$ follows

$$\dot{a} \sim t^{-\frac{1}{3}}, \tag{52}$$

which is decelerating. Eventually, the decreasing Hubble parameter will enter the regime $H \ll h_0$. Therefore, according to (50), $\dot{a}$ should follow

$$\dot{a} \sim t^{\frac{1}{3}}, \tag{53}$$

which is accelerating. This scenario of late-time cosmic speed-up without cosmological constant is otherwise not possible in the standard model of cosmology[7].

For certain values of $\kappa > 0$ and $\Lambda > 0$, numerical simulations show that the universe can even experience two periods of decelerating and accelerating phases. The first cycle of deceleration and acceleration is dominated by matter and characterized by $H \gg h_0$ and $H \ll h_0$, respectively. The second cycle is driven by positive curvature and cosmological constant, respectively.

5 Modified Density Parameter

Dividing the new Friedmann equation (38) by $\tilde{H}^2$, one can get the density contributions for different components of the universe. The modified density parameter for baryonic matter is given by

$$\tilde{\Omega}_b = \frac{8\pi G}{3\tilde{H}^2}\rho_b = \frac{H^2}{\tilde{H}^2}\frac{8\pi G}{3H^2}\rho_b = \frac{H^2}{\tilde{H}^2}\Omega_b, \tag{54}$$

where $\Omega_b = \frac{8\pi G}{3H^2}\rho_b$ is the usual density parameter for baryonic matter.

[7]See [16] for a review of cosmological constant and its implications in cosmology. See e.g. [17, 18] for earlier theories of cosmic acceleration without cosmological constant.

In the standard model of cosmology, cold dark matter(CDM) is invoked as an additional source of matter, since Ω_b is lower than what is observed. Here we propose that there is neither galactic CDM (as shown in our earlier paper[1]) nor cosmological CDM. The modified density parameter $\tilde{\Omega}_b$ can be higher than the observed value Ω_b, thanks to the factor $H^2/\tilde{H}^2$. This may eliminate the need for CDM.

Now we try to determine the magnitude of $H^2/\tilde{H}^2$. With the estimated value of characteristic acceleration [10] in terms of Hubble constant H_0 (which is the present value of Hubble parameter $H_0 = H|_{t=t_0}$),

$$a_0 \simeq \frac{c}{6} H_0, \tag{55}$$

equation (43) gives

$$h_0 \simeq \frac{1}{18} \frac{\alpha_S}{\alpha_T} H_0. \tag{56}$$

With (56) and (40) (and $\delta = 0$), the present value of factor $(H^2/\tilde{H}^2)|_{t=t_0}$ can be calculated as 1.6 or 6.9 for $\alpha_S/\alpha_T = 1$ or 18 (i.e. $H_0/h_0 = 18$ or 1), respectively.

6 Conclusion

We propose a modification of Einstein-Cartan equations. Spin current is coupled to modified torsion, which breaks local Lorentz gauge symmetry and leaves diffeomorphism invariance intact.

We apply the new gravity theory to cosmology. The updated Friedmann equations are dependent on a modified Hubble parameter. The deviation from standard model of cosmology is noticeable when the Hubble parameter becomes comparable to or less than h_0. The characteristic Hubble scale h_0 is proportional to MOND acceleration scale a_0.

One of the implications is that there may be no need to invoke dark matter to account for cosmological mass discrepancies. Another interesting observation is that our model can accommodate late-time cosmic acceleration without cosmological constant.

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